Surveying I.

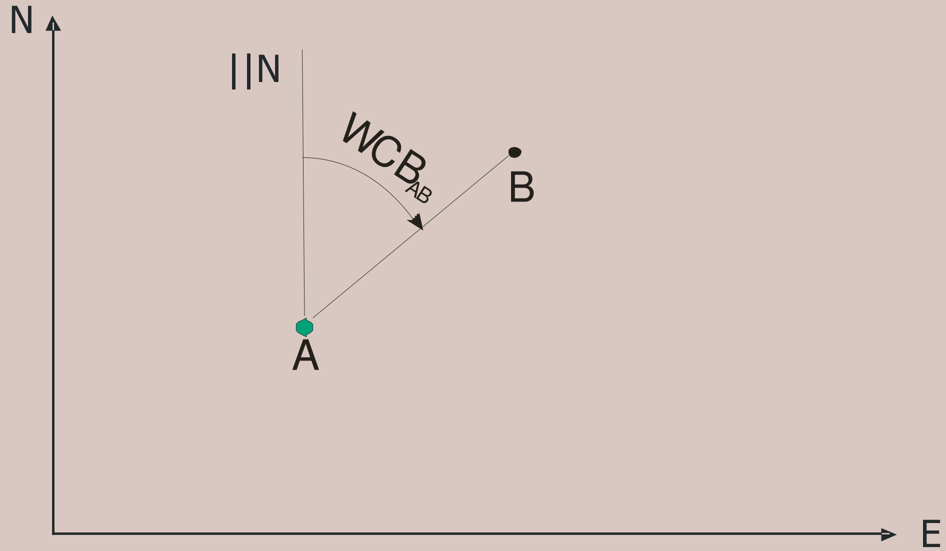
Plane surveying. Fundamental tasks of surveying. Intersections. Orientation.

The coordinate system

Northing axis is the projection of the starting meridian of the projection system, while the Easting axis is defined as the northing axis rotated by 90° clockwise.

The whole circle bearing

How could the direction of a target from the station be defined?



Whole circle bearing: the local north is rotated clockwise to the direction of the target. The angle which is swept is called the whole circle bearing.

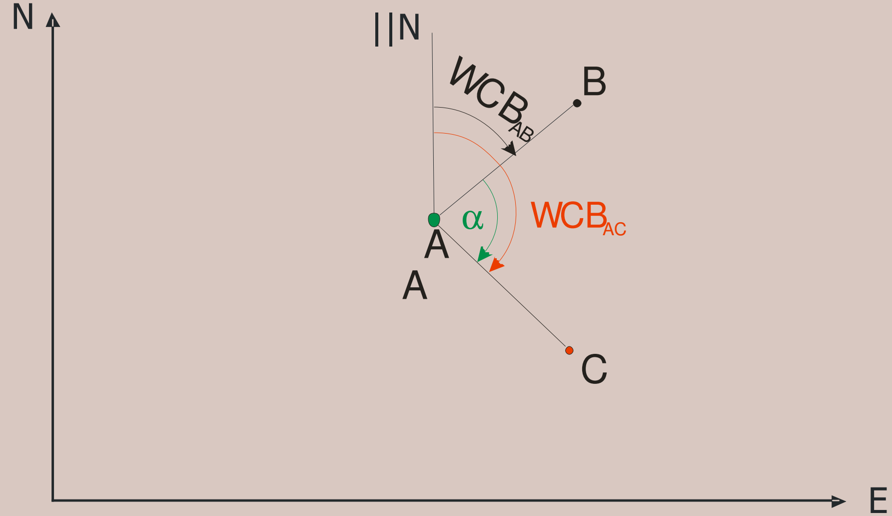
$$0° \leq WCB_{AB} < 360°$$

Transferring Whole Circle Bearings

WCB of reverse direction:

$$WCB_{BA} = WCB_{AB} \pm 180°$$

Transferring WCBs: WCB_{AB} is known, α is measured, how much is WCB_{AC}?

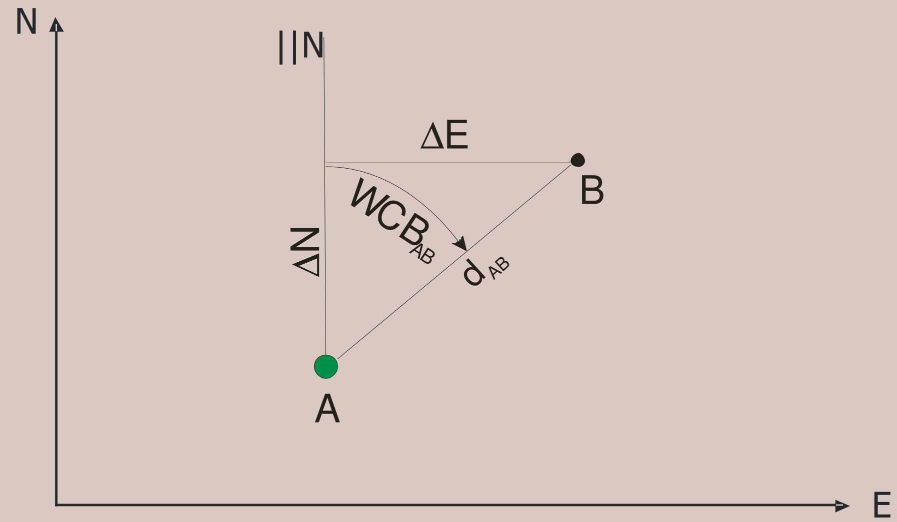


$$WCB_{AC} = WCB_{AB} + \alpha$$

$$or$$

$$WCB_{AB} = WCB_{AC} - \alpha$$

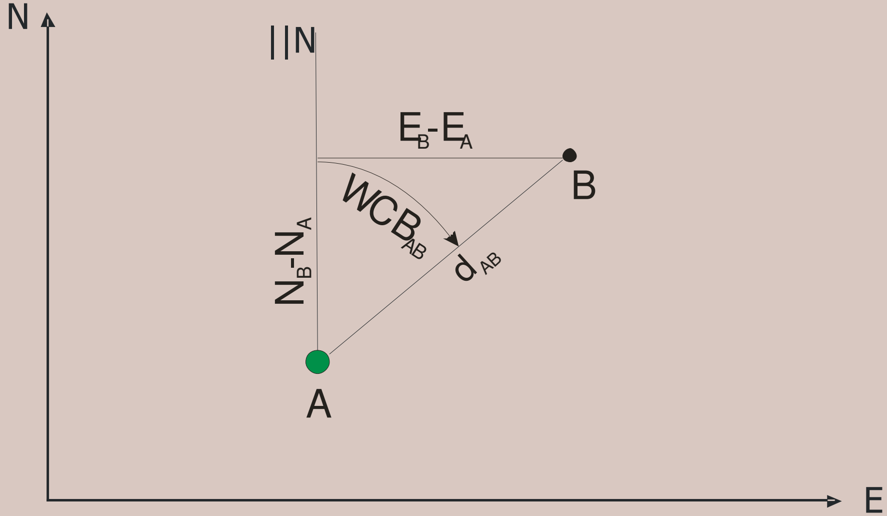
1st fundamental task of surveying



A(E_A, N_A), WCB_{AB} and d_{AB} is known,
B(E_B,N_B)=?

$$\Delta E_{AB} = E_B - E_A = d_{AB} \cdot \sin WCB_{AB}$$
$$\Delta N_{AB} = N_B - N_A = d_{AB} \cdot \cos WCB_{AB}$$
$$\Downarrow$$
$$E_B = E_A + d_{AB} \cdot \sin WCB_{AB},$$
$$N_B = N_A + d_{AB} \cdot \cos WCB_{AB}.$$

2nd fundamental task of surveying



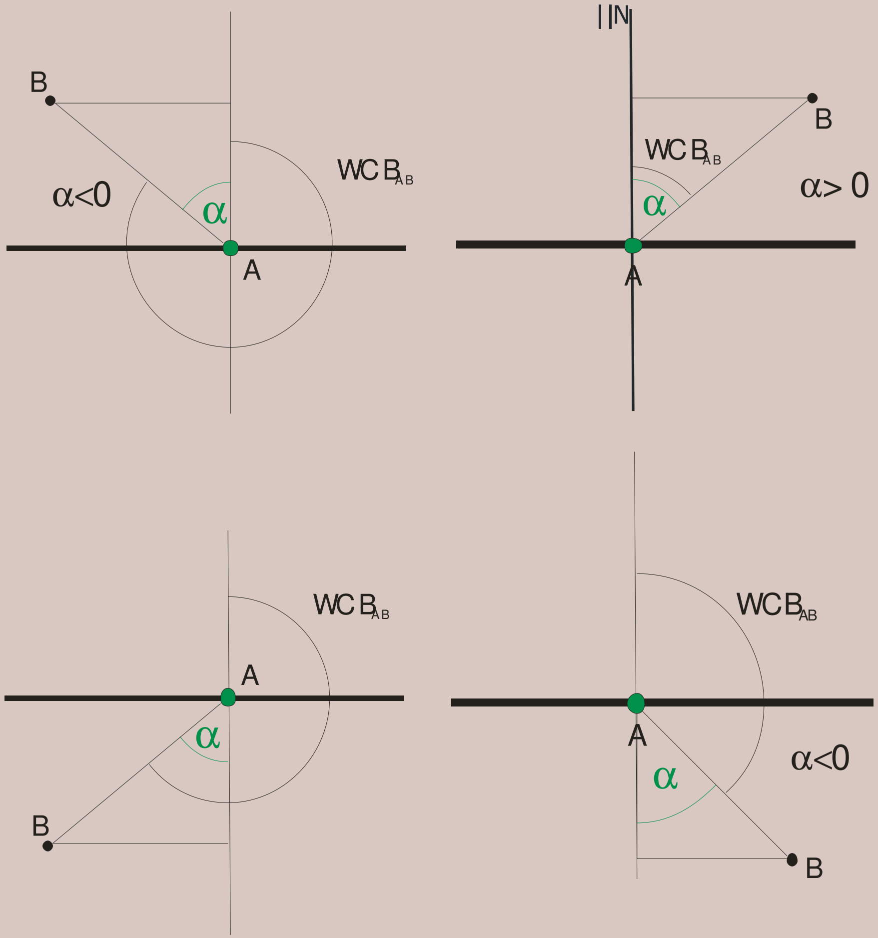
A(E_A, N_A), B(EB,NB) is known,
WCB_{AB}=? and d_{AB}=?

$$d_{AB} = \sqrt{(E_B - E_A)^2 + (N_B - N_A)^2}$$

$$\alpha = \arctan \frac{E_B - E_A}{N_B - N_A},$$

$$WCB_{AB} = \alpha + c$$

2nd fundamental task of surveying



$$d_{AB} = \sqrt{(E_B - E_A)^2 + (N_B - N_A)^2}$$

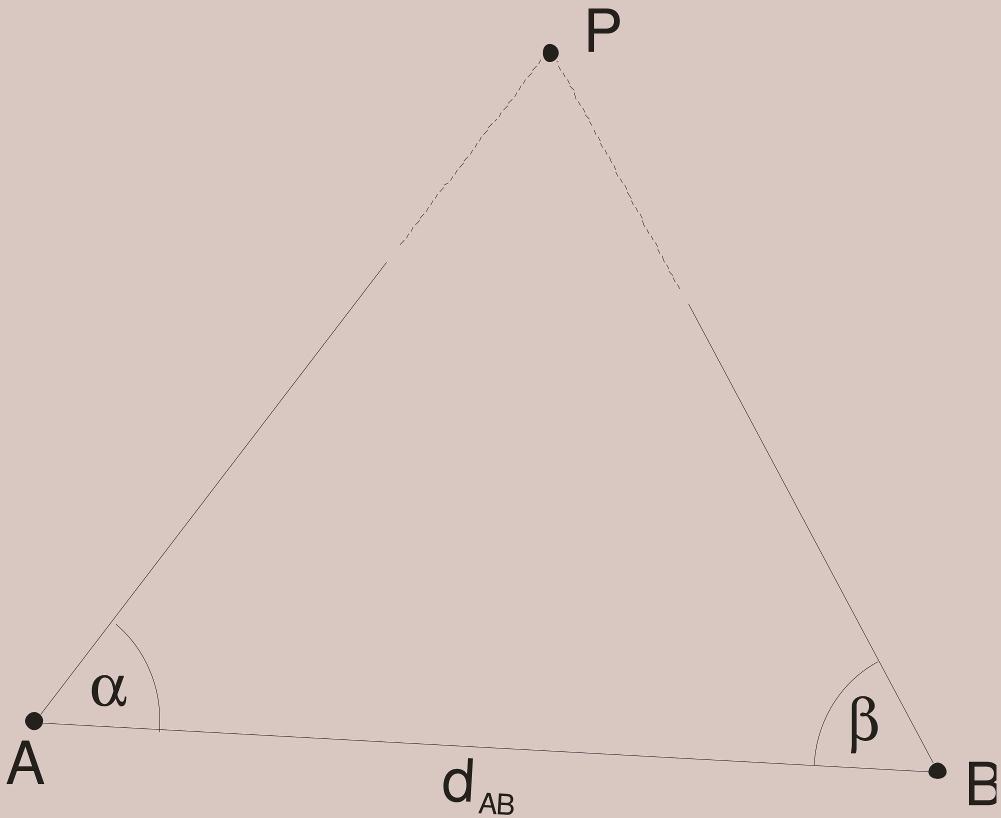
$$\alpha = \arctan \frac{E_B - E_A}{N_B - N_A},$$

$$WCB_{AB} = \alpha + c$$

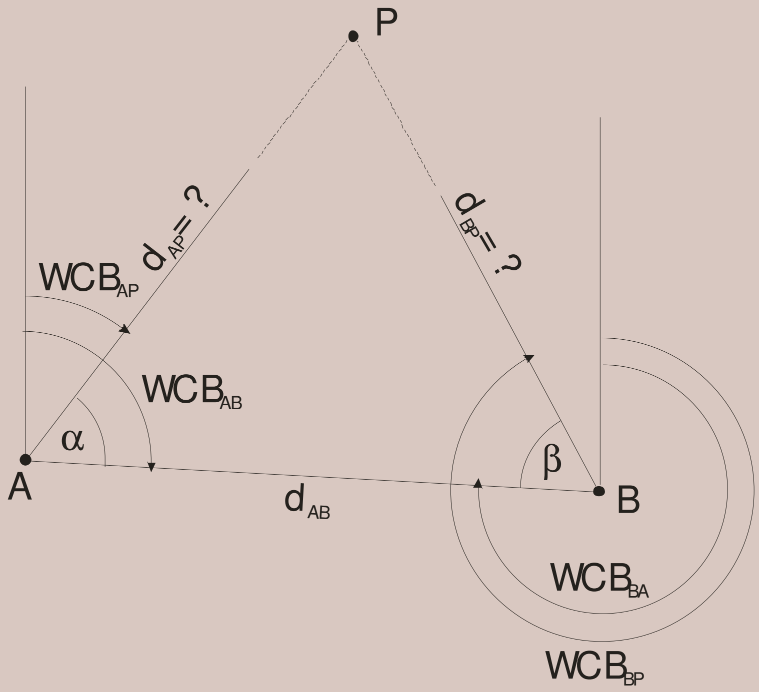
Quadrant	E_B-E_A	N_B-N_A	c
I.	+	+	0
II.	+	-	+180°
III.	-	-	+180°
IV.	-	+	+360°

Intersections

Aim: the coordinates of an unknown point should be computed. Measurements are taken from two different stations to the unknown point, and the so formed triangle should be solved.



Foresection with inner angles



A (E_A,N_A)
B (E_B,N_B) are known
α, β are observed

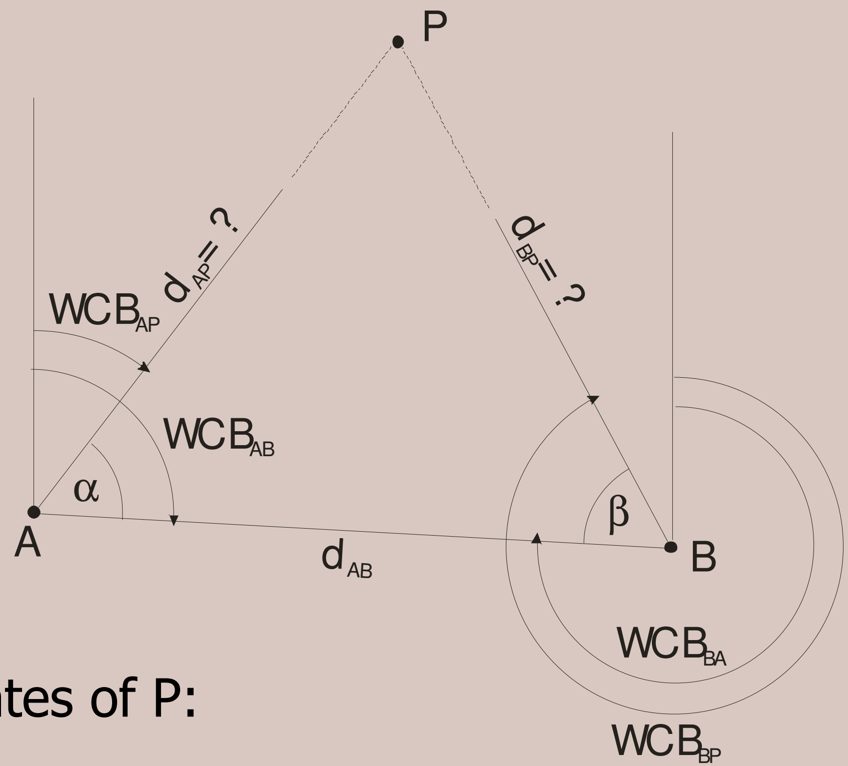
1. Compute WCB_{AB}, d_{AB} using the 2nd fundamental task of surveying.
2. Using the sine theorem compute d_{AP} and d_{BP}!

$$d_{AP} = d_{AB}\frac{\sin\beta}{\sin(\alpha+\beta)} \qquad d_{BP} = d_{AB}\frac{\sin\alpha}{\sin(\alpha+\beta)}$$

3. Compute WCB_{AP} and WCB_{BP}:

$$WCB_{AP} = WCB_{AB} - \alpha \qquad WCB_{BP} = WCB_{BA} + \beta$$

Foresection with inner angles



4. Compute the coordinates of P:

$$From\ A:$$

$$E_P^A = E_A + d_{AP} \sin WCB_{AP} \qquad N_P^A = N_A + d_{AP} \cos WCB_{AP},$$

$$From\ B:$$

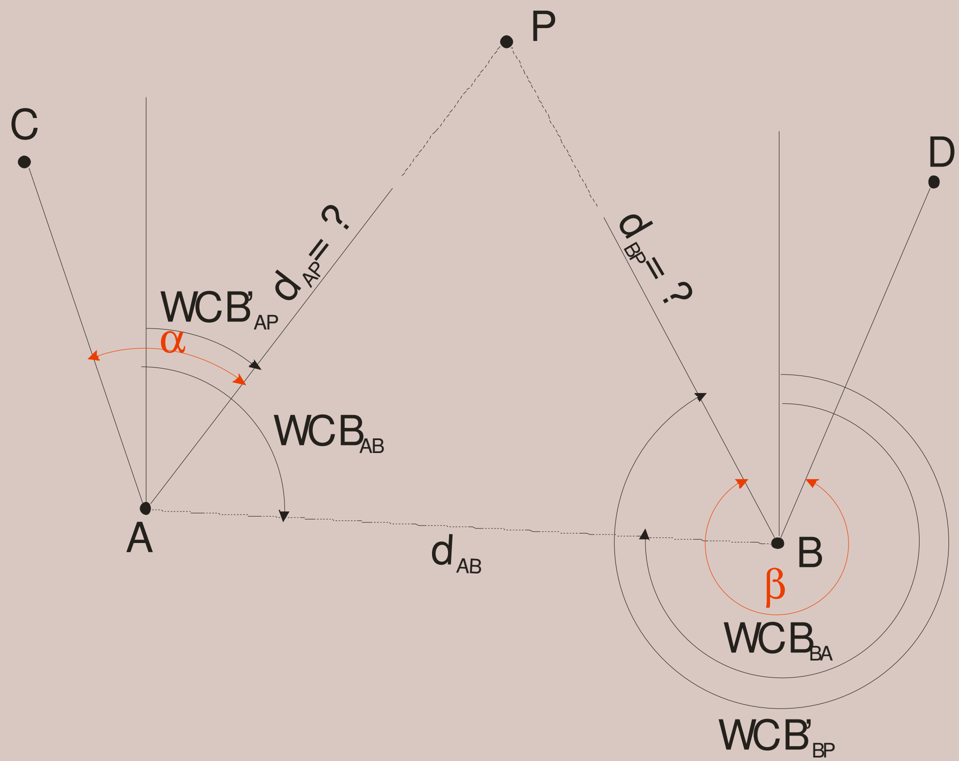
$$E_P^B = E_B + d_{BP} \sin WCB_{BP} \qquad N_P^B = N_B + d_{BP} \cos WCB_{BP},$$

$$\Downarrow$$

$$E_P = \frac{E_P^A + E_P^B}{2} \qquad N_P = \frac{N_P^A + N_P^B}{2}$$

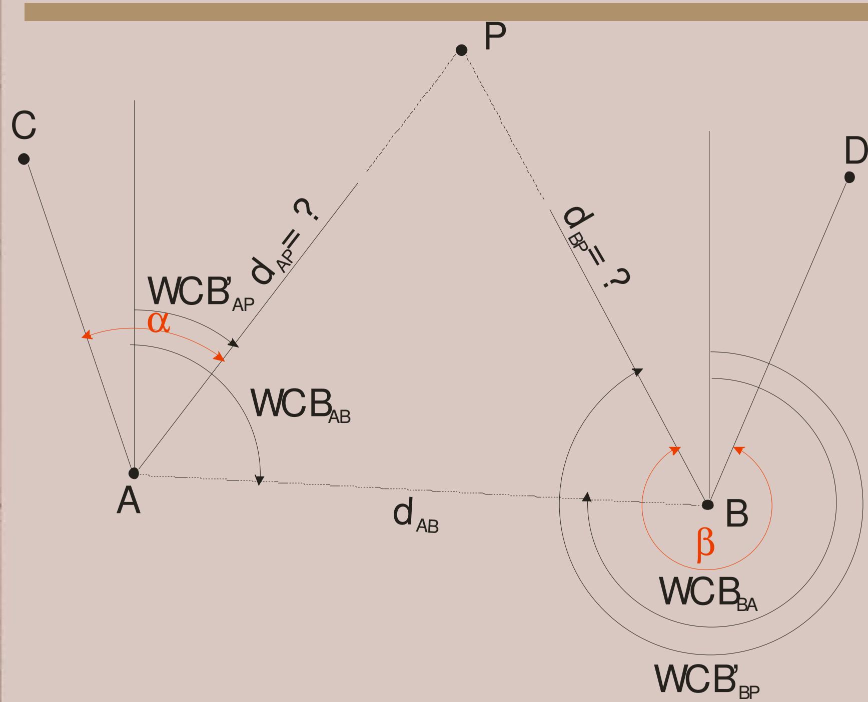
Foresection with WCBs

What happens, when B is not observable from A?



A,B,C and D are known points, α and β are measured.

Foresection with WCBs

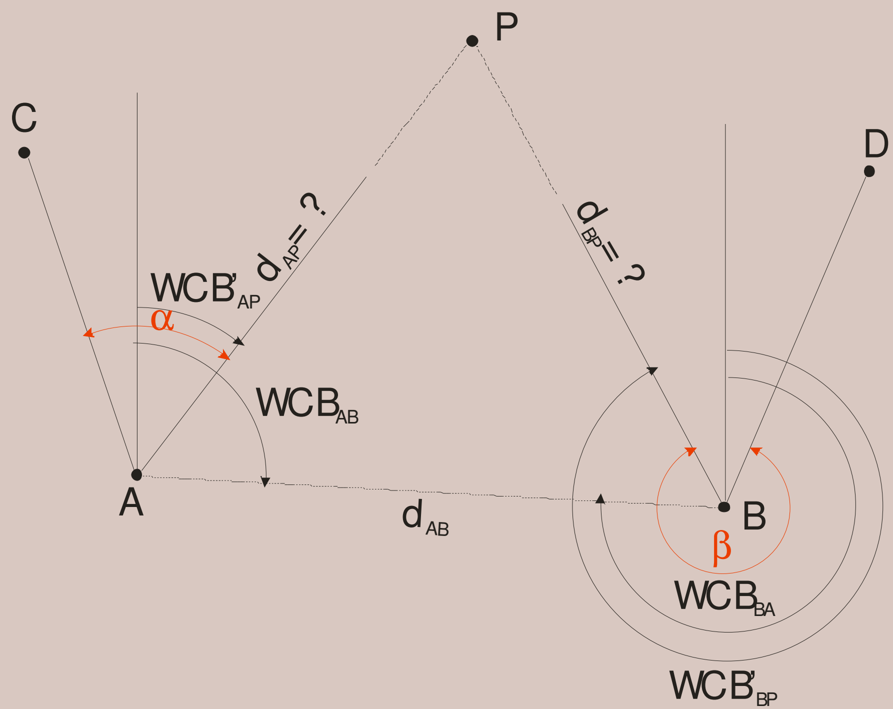


$$WCB'_{AP} = WCB_{AC} + \alpha$$
$$WCB'_{BP} = WCB_{BD} + \beta$$

The equations of the lines AP and BP:

$$N_1 = N_A + (E - E_A) \cdot \cot WCB_{AP}$$
$$N_2 = N_B + (E - E_B) \cdot \cot WCB_{BP}$$

Foresection with WCBs



Let's compute the intersection of the lines AP and BP:

$$N_1 = N_2$$

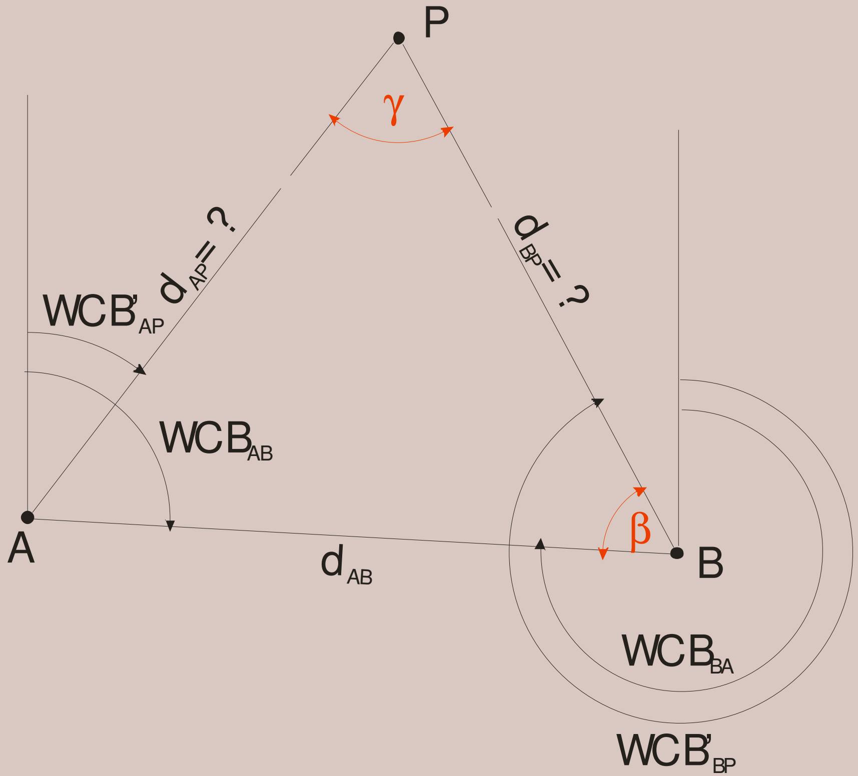
$$E\left(\cot WCB_{AP} - \cot WCB_{BP}\right) = N_B - N_A + E_A \cot WCB_{AP} - E_B \cot WCB_{BP}$$

$$E_P = \frac{N_B - N_A + E_A \cot WCB_{AP} - E_B \cot WCB_{BP}}{\cot WCB_{AP} - \cot WCB_{BP}}$$

$$N_P = N_A + \left(E_P - E_A\right)\cot WCB_{AP}$$

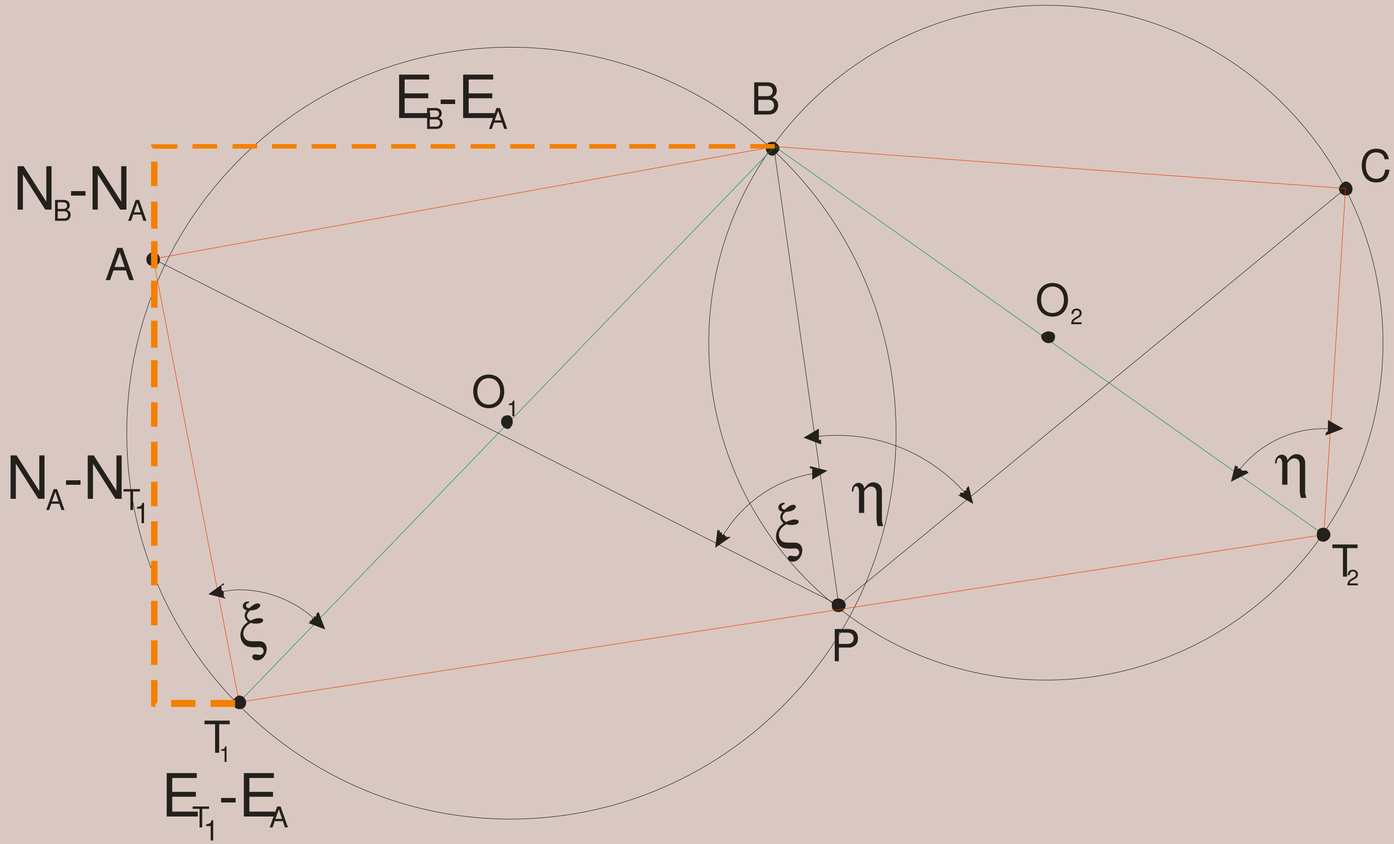
Different types of intersections

How can we use intersections, when A or B is not suitable for setting up the instrument:



α can be computed by $\alpha=180°-\gamma-\beta$. => Foresection.

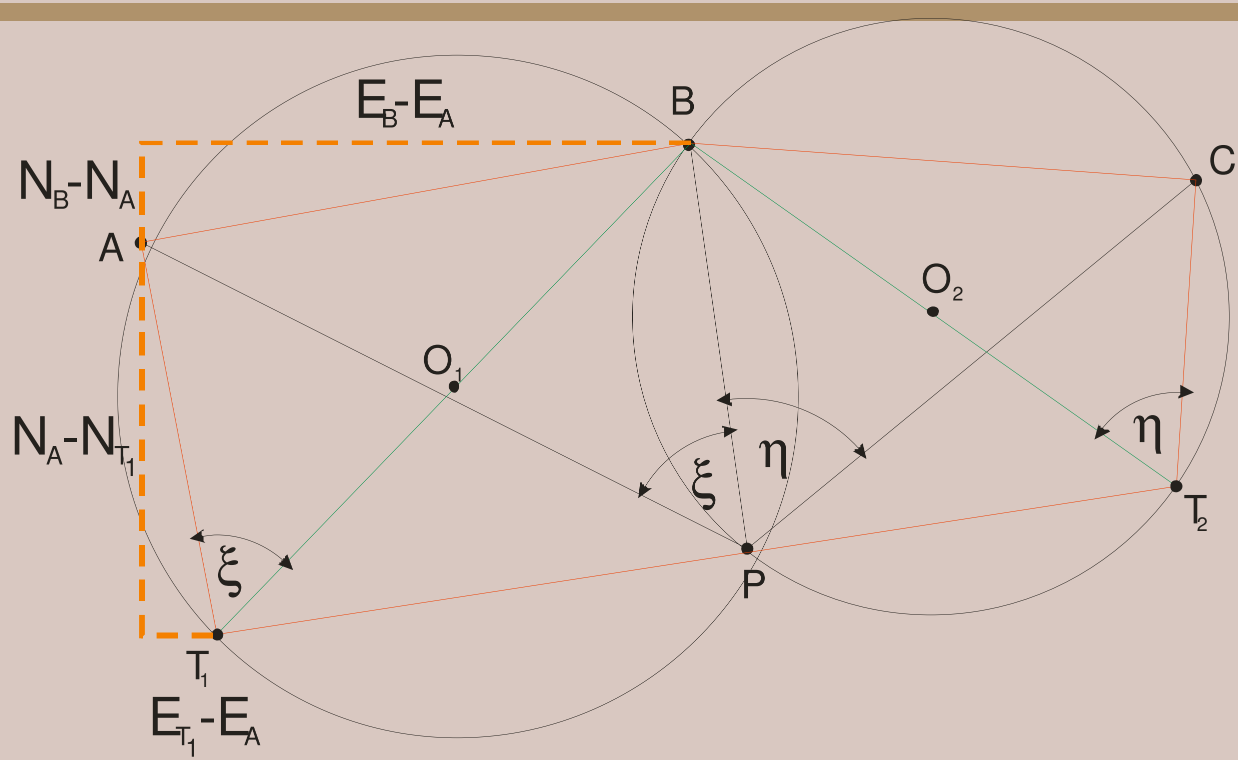
Resection



A,B,C are known control points
ξ and η are observed angles

Aim: compute the coordinates of P (the station)

Resection



Compute the coordinates of T_1 and T_2!

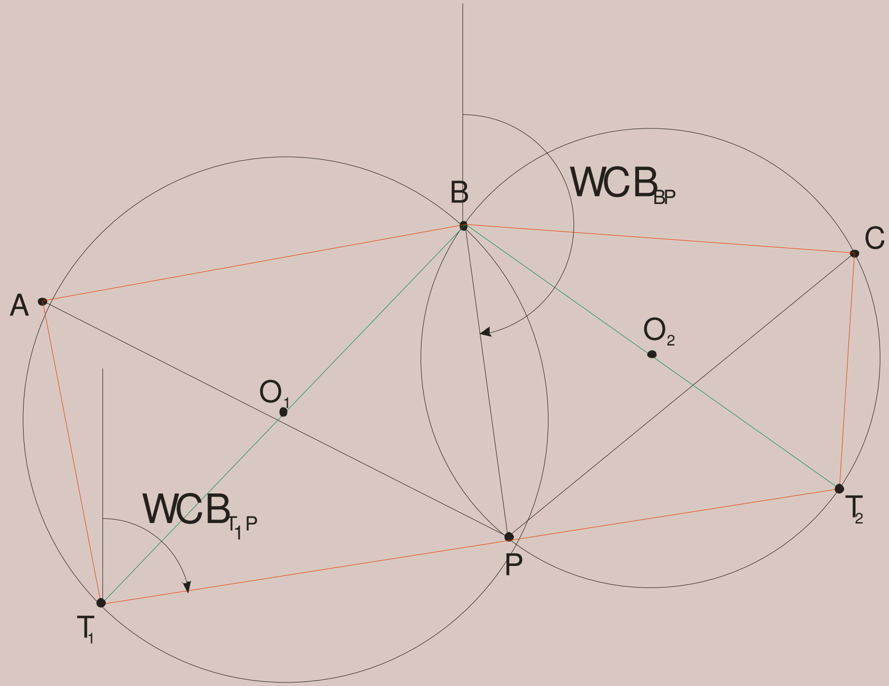
$$\frac{E_B - E_A}{N_A - N_{T_1}} = \frac{N_B - N_A}{E_{T_1} - E_A} = \cot\xi$$

$$\Downarrow$$

$$N_{T_1} = \frac{E_B - E_A - N_A \cot\xi}{\cot\xi},$$

$$E_{T_1} = \frac{N_B - N_A + E_A \cot\xi}{\cot\xi}.$$

Resection



Since T_1, P and T_2 are on a straight line:

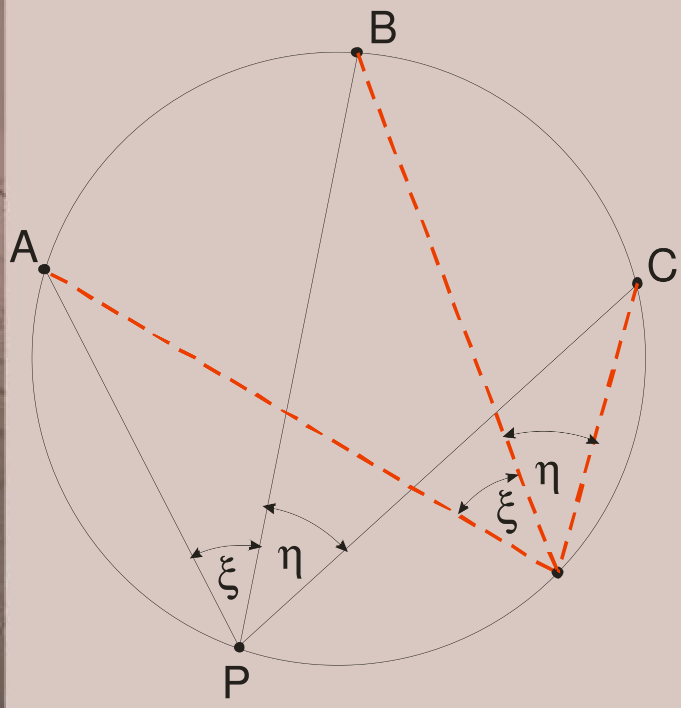
$$WCB_{T_1P} = WCB_{T_1T_2}$$

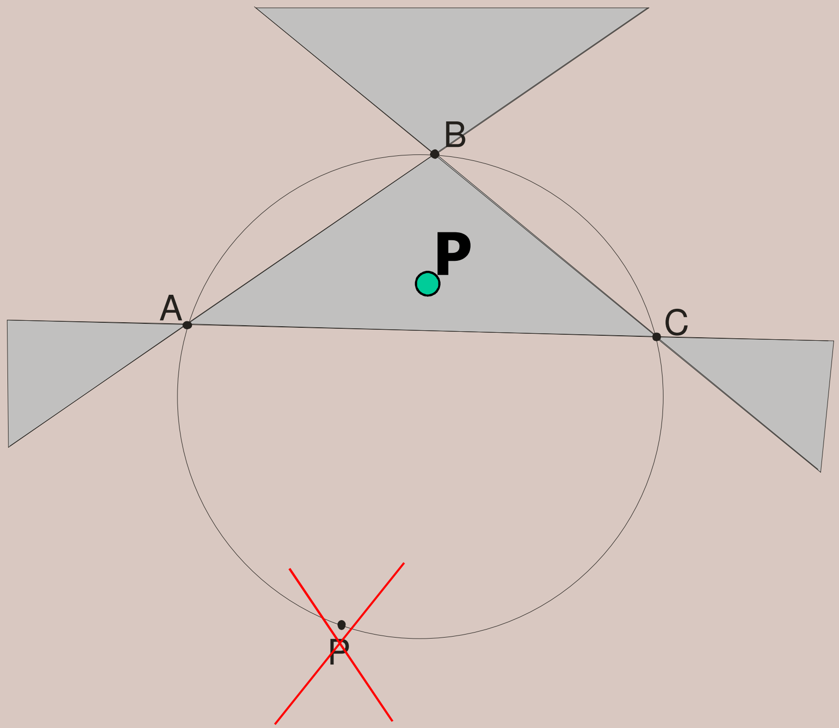
$$WCB_{BP} = WCB_{T_1T_2} + 90°$$

Foresection with WCBs

Resection – the dangerous circle

What happens, if all the four points are on one circumscribed circle?

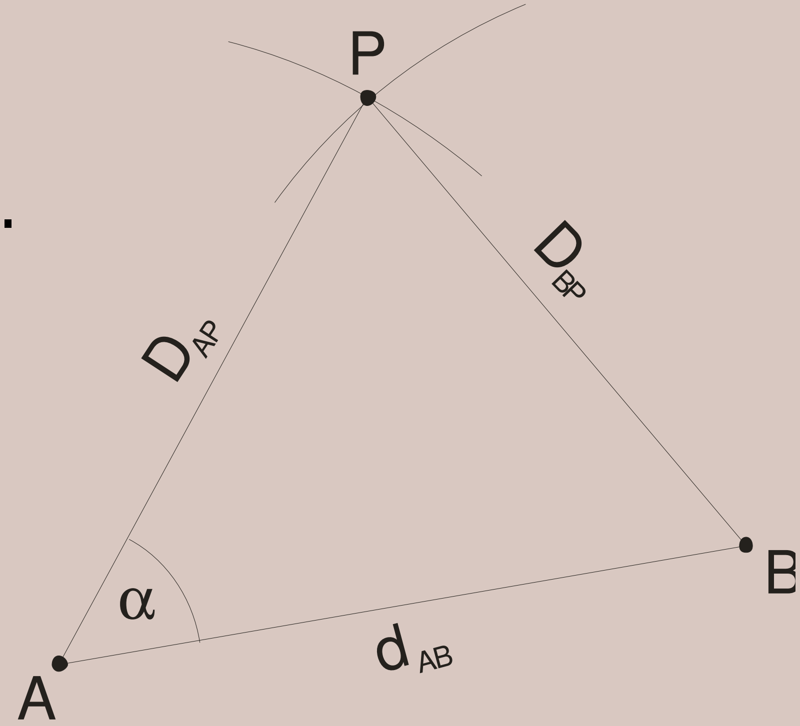




Arcsection

A, B are known control points,
D_{AP} and D_{BP} are measured.

Aim: compute the coordinates of P!



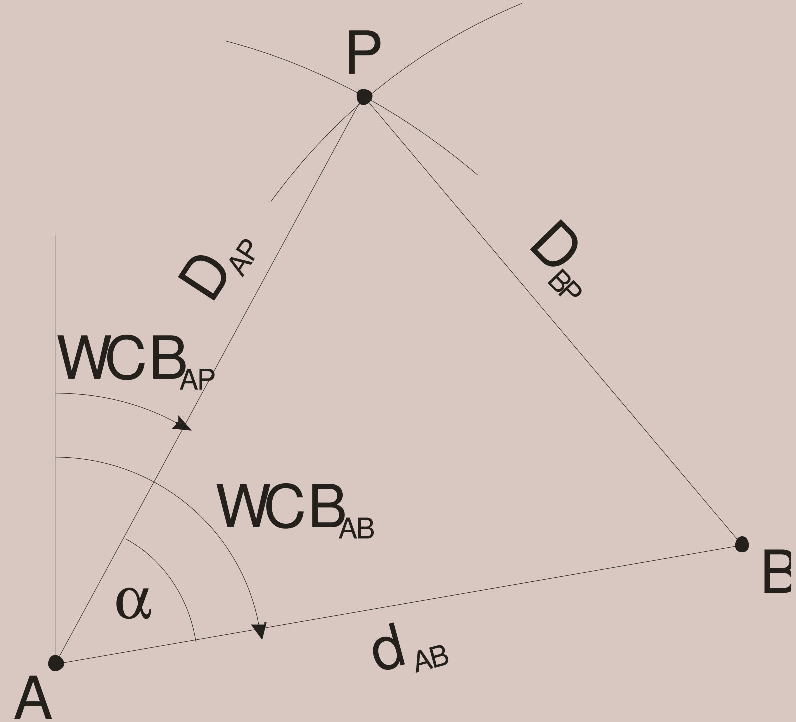
Using the cosine theorem, compute the angle α:

$$D_{BP}^2 = D_{AP}^2 + d_{AB}^2 - 2D_{AP}d_{AB}\cos\alpha$$

$$\Downarrow$$

$$\alpha = \arccos\frac{D_{AP}^2 + d_{AB}^2 - D_{BP}^2}{2D_{AP}d_{AB}}.$$

Arcsection



Compute WCB_{AB} from the coordinates of A and B,

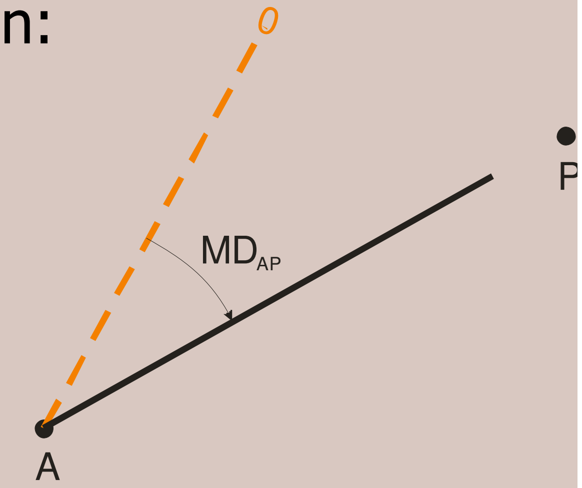
$$WCB_{AP} = WCB_{AB} - \alpha$$

1st fundamental task of surveying

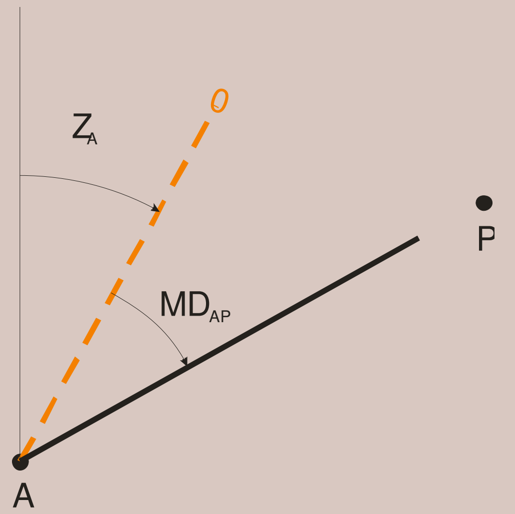
Orientation

How can the WCB be determined from observations?

Recall the definition of mean direction:



All the angular observations refer to the index of the horizontal circle, but they should refer to the Northing instead!



Orientation

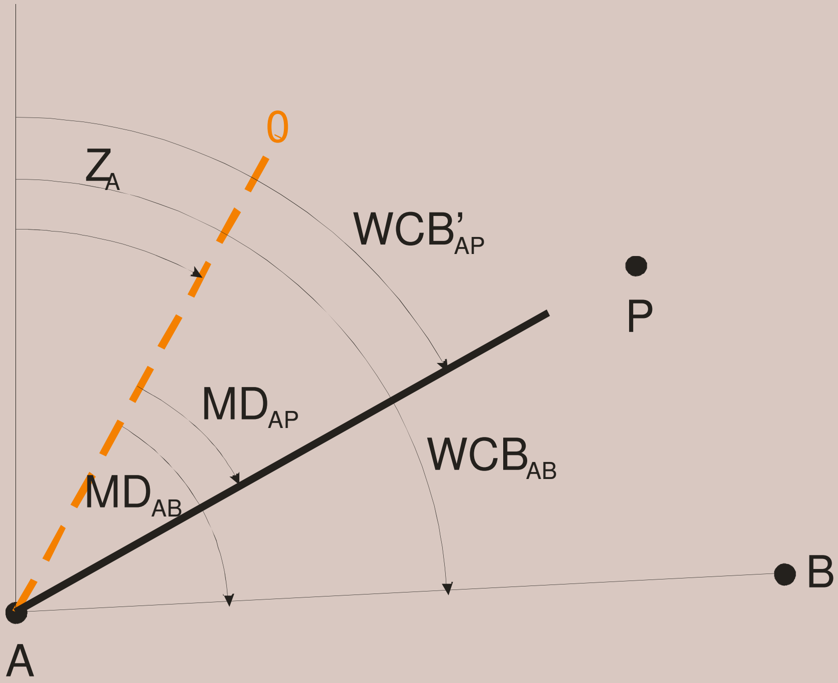
z_A – orientation angle

Orientation

How to find the orientation angle?

A,B are known points, MD_{AP} and MD_{AB} are observed.

Aim: Compute WCB'_{AP}



Compute the orientation angle:

$$z_A = WCB_{AB} - MD_{AB}$$

Computing the WCB'_{AP}:

$$WCB'_{AP} = z_A + MD_{AP}$$

Computing the mean orientation angle

In case of more orientations, as many orientation angles can be computed as many control points are sighted:

$$z_A^B = WCB_{AB} - MD_{AB}$$

$$z_A^C = WCB_{AC} - MD_{AC}$$

$$z_A^D = WCB_{AD} - MD_{AD}$$

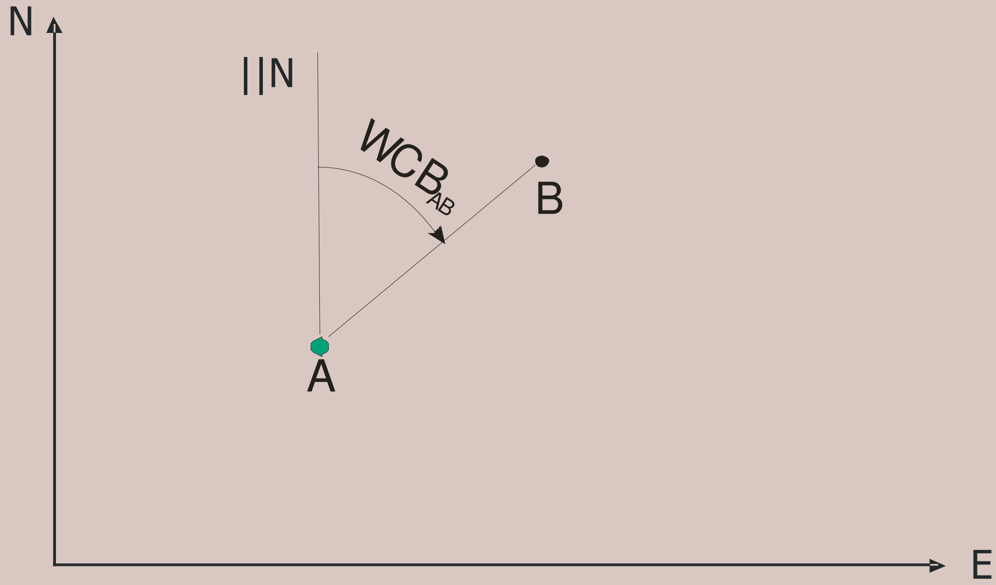
z_A^B, z_A^C and z_A^D are usually slightly different due observation and coordinate error.

However, the orientation angle is constant for a station and a set of observations.

Mean orientation angle:

$$z_A = \frac{z_A^B \cdot d_{AB} + z_A^C \cdot d_{AC} + z_A^D \cdot d_{AD}}{d_{AB} + d_{AC} + d_{AD}}$$

WCB vs provisional WCB



Whole circle bearing (WCB$_{AB}$): computed from coordinates, between two points, which coordinates are known.

Provisional whole circle bearing (WCB'$_{AB}$): an angular quantity, which is similar to the whole circle bearing. However it is computed from observations, by summing up the (mean) orientation angle and the mean direction.

Thank You for Your Attention!