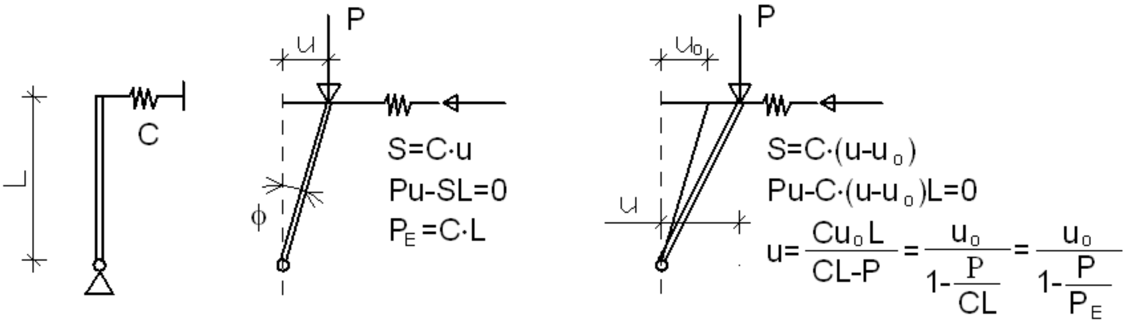
Stabilitás vizsgálatok

Klasszikus vizsgálat (perfekt) Modern vizsgálat (imperfekt)



Southwell─vonal:

$$u\left(1-\frac{P}{P_E}\right)=u_o$$

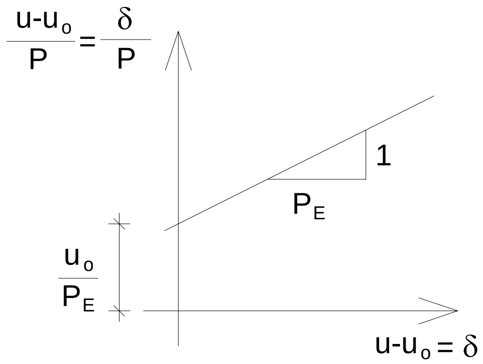
$$u\left(P_E-P\right)=u_o\cdot P_E$$

$$\left(u-u_o\right)P_E=u\cdot P$$

$$\frac{u-u_o}{P}=\frac{u}{P_E}-\frac{u_o}{P_E}+\frac{u_o}{P_E}$$

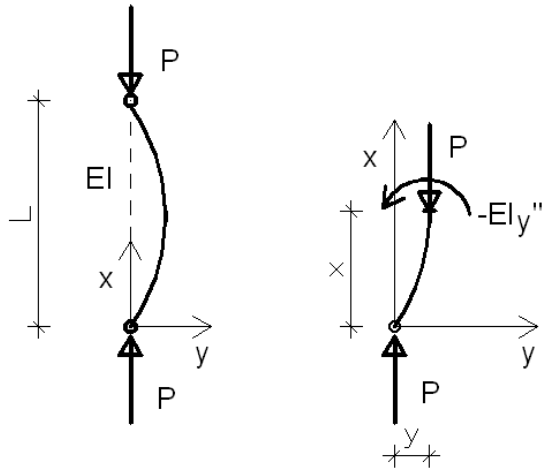
$$\frac{u-u_o}{P}=\frac{u-u_o}{P_E}+\frac{u_o}{P_E}$$

$$\frac{\delta}{P}=\frac{\delta}{P_E}+\frac{u_o}{P_E}$$



Nyomott oszlop kihajlása

Perfekt modell:



$$EIy''+Py=0$$

$$k^2=\frac{P}{EI}$$

$$y''+k^2y=0$$

$$y=A\sin kx+B\cos kx$$

$$x=0;\quad y=0\rightarrow B=0$$

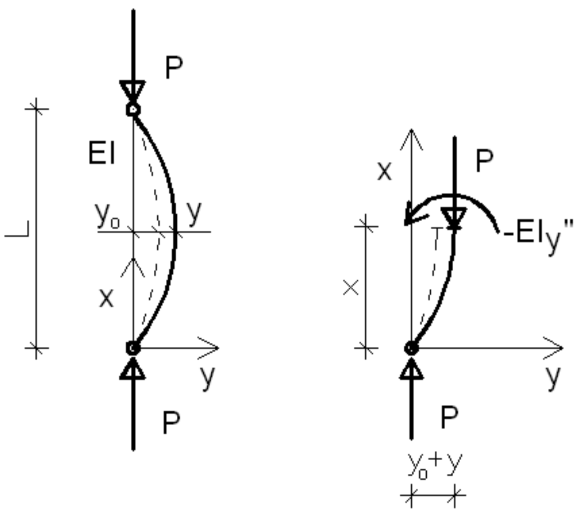
$$x=L;\quad y=0\rightarrow A\sin kL=0$$

$$A=0\,(\text{triviális})$$

$$\sin kL=0\,(\text{kritikus})\;kL=n\pi$$

$$P_{kr}=\frac{n^2\pi^2\,EI}{L^2};\;P_E=\frac{\pi^2\,EI}{L^2}$$

Imperfekt modell



$$y_o=e_0\sin\frac{\pi x}{L}$$

$$EIy''+P(y_0+y)=0$$

$$k^2=\frac{P}{EI}$$

$$y''+k^2y=-k^2e_0\sin\frac{\pi x}{L};\;y=y_h+y_p$$

$$\left[C\left(k^2-\frac{\pi^2}{L^2}\right)+k^2e_0\right]\sin\frac{\pi x}{L}+$$

$$+D\left(k^2-\frac{\pi^2}{L^2}\right)\cdot\cos\frac{\pi x}{L}=0$$

$$D=0$$

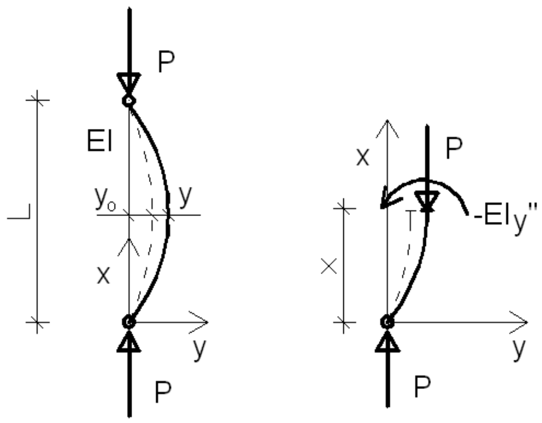
$$C=\frac{e_0}{\frac{\pi^2}{k^2L^2}-1}=e_0\frac{\alpha}{1-\alpha};\;\alpha=\frac{P}{P_E}$$

$$y=e_0\frac{\alpha}{1-\alpha}\sin\frac{\pi x}{L}$$

$$y_T=y_0+y=\left(1+\frac{\alpha}{1-\alpha}\right)e_0\sin\frac{\pi x}{L}=e_0\frac{1}{1-\alpha}\sin\frac{\pi x}{L}$$

középső km. eltolódása: $e=e_0\dfrac{1}{1-\dfrac{P}{P_E}}$

A kihajlási csökkentő tényező: Nyomott rúd



$$y = e_0 \sin \frac{\pi x}{L}; \qquad e = e_0 \frac{1}{1 - \frac{P}{P_E}}$$

Határállapot:

$$\frac{P}{P_y} + \frac{M}{M_y} \le 1; \; M = Pe = Pe_0 \frac{1}{1 - \frac{P}{P_E}}$$

$$\frac{P}{P_y} + \frac{Pe_0}{\left(1 - \frac{P}{P_E}\right) M_y} = 1$$

$$P_y = A \cdot f_y; \; M_y = W \cdot f_y$$

$$P_E = \frac{\pi^2 EI}{L^2} = \frac{\pi^2 EA}{\lambda^2}; \; \lambda = \bar{\lambda} \cdot \lambda_E = \bar{\lambda}\pi \sqrt{\frac{E}{f_y}}$$

$$\frac{P}{P_y} + \frac{Pe_o}{\left(1 - \frac{P}{P_y} \cdot \frac{P_y}{P_E}\right) M_y} = 1;$$

$$\frac{P_y}{P_E} = \bar{\lambda}^2; \; \eta = e_0 \frac{A}{W}; \; \chi = \frac{P}{P_y}$$

$$\frac{P}{P_y} + \frac{\frac{P}{P_y}}{1 - \frac{P}{P_y} \bar{\lambda}^2} \eta = 1; \qquad \chi + \frac{\chi}{1 - \chi \cdot \bar{\lambda}^2} \eta = 1; \qquad (1 - \chi)\left(1 - \chi \bar{\lambda}^2\right) = \eta \chi$$

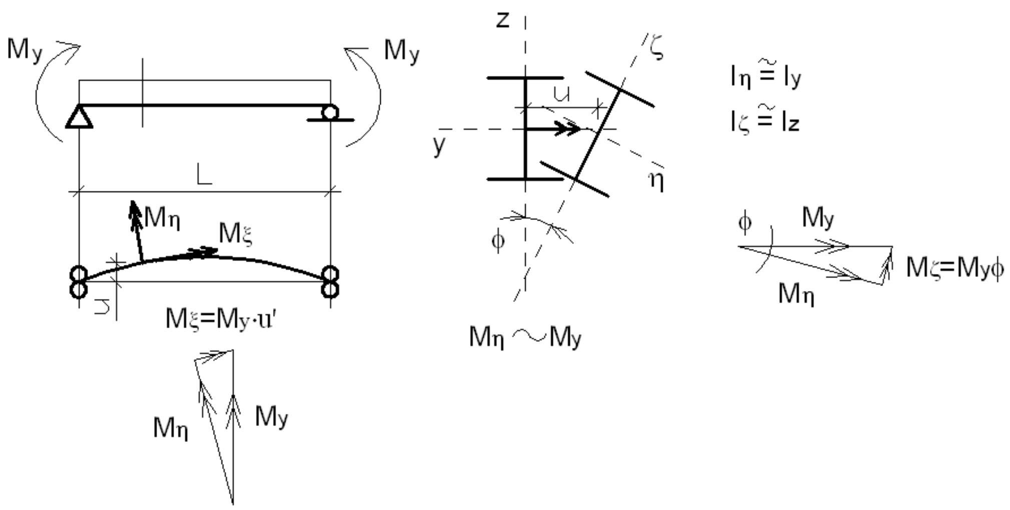
$$\bar{\lambda}^2 \cdot \chi^2 - \left(1 + \eta + \bar{\lambda}^2\right)\chi + 1 = 0$$

$$\chi = \frac{\left(1 + \eta + \bar{\lambda}^2\right) - \sqrt{\left(1 + \eta + \bar{\lambda}^2\right)^2 - 4\bar{\lambda}^2}}{2\bar{\lambda}^2}; \qquad \eta = \alpha\left(\bar{\lambda} - \bar{\lambda}_0\right)$$

$$\Phi = 0{,}5\left[1 + \alpha\left(\bar{\lambda} - \bar{\lambda}_0\right) + \bar{\lambda}^2\right]; \qquad \bar{\lambda} = \sqrt{\frac{A \cdot f_y}{P_E}}; \qquad P_E = N_{cr}$$

$$\chi = \frac{1}{\Phi + \sqrt{\Phi^2 - \bar{\lambda}^2}}$$

Gerenda kifordulás: Perfekt modell



$$-EI_z u'' = M_y \varphi \qquad\Big|\qquad EI_z u^{IV} + M_y \varphi'' = 0$$

$$GI_t \varphi' - EI_\omega \varphi''' = M_y u' \qquad\Big|\qquad M_y u'' - GI_t \varphi'' + EI_\omega \varphi^{IV} = 0$$

$$u = A \sin\frac{\pi x}{L} \qquad \varphi = B \sin\frac{\pi x}{L}$$

$$\begin{vmatrix} +\frac{\pi^4}{L^4} EI_z & -\frac{\pi^2}{L^2} M_y \\ -\frac{\pi^2}{L^2} M_y & +\frac{\pi^2}{L^2} GI_t + \frac{\pi^4}{L^4} EI_\omega \end{vmatrix} \begin{vmatrix} A \\ B \end{vmatrix} = 0 \qquad \det \begin{vmatrix} \frac{\pi^2}{L^2} EI_z & -M_y \\ -M_y & GI_t + \frac{\pi^2}{L^2} EI_\omega \end{vmatrix} = 0$$

$$\frac{\pi^2}{L^2} EI_z \left(GI_t + \frac{\pi^2}{L^2} EI_\omega \right) - M_y^2 = 0; \qquad M_{y,cr} = \sqrt{\frac{\pi^2 EI_z}{L^2} \left(GI_t + \frac{\pi^2}{L^2} EI_\omega \right)}$$

$$M_{y,cr} = \frac{\pi^2 EI_z}{L^2} \left[\frac{L^2 GI_t}{\pi^2 EI_z} + \frac{I_\omega}{I_z} \right]^{\frac{1}{2}}$$

$$M_{y,cr} = \frac{\pi}{L} \sqrt{EI_z \cdot GI_t} \cdot \sqrt{1 + \frac{\pi^2 EI_\omega}{L^2 GI_t}}$$

Gerenda kifordulás: Imperfekt modell

$$EI_z u^{IV} + M_y(\varphi_0'' + \varphi'') = 0$$
$$M_y(u_0'' + u'') - GI_t\varphi'' + EI_\omega\varphi^{IV} = 0$$

$$P_z \cdot A - M_y \cdot C = M_y \cdot C_0$$
$$-M_y \cdot A + B_\omega \cdot C = M_y \cdot A_0$$

$$A = \frac{1}{D}\left(M_y \cdot B_\omega \cdot C_0 + M_y^{\ 2} \cdot A_0\right)$$
$$C = \frac{1}{D}\left(M_y \cdot P_z \cdot A_0 + M_y^{\ 2} \cdot C_0\right)$$
$$D = P_z \cdot B_\omega - M_y^{\ 2} = M_{y,cr}^2 - M_y^{\ 2}$$

$$u_0 = A_0 \cdot \sin\frac{\pi x}{L}; u = A \cdot \sin\frac{\pi x}{L}$$
$$\varphi_0 = C_0 \cdot \sin\frac{\pi x}{L}; \varphi = C \cdot \sin\frac{\pi x}{L}$$
$$P_z = \frac{\pi^2 EI_z}{L^2}; B_\omega = GI_t + \frac{\pi^2 EI_\omega}{L^2}; P_{E\omega} = \frac{1}{i_0^{\ 2}} \cdot B_\omega$$

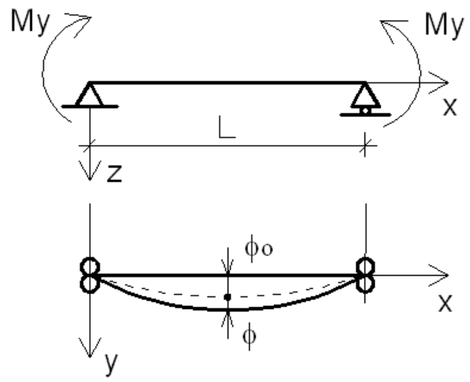
A_0=0 esetén:

$$C = \frac{1}{D}C_0 M_y^{\ 2} = \frac{M_y^{\ 2}}{M_{y,cr}^2 - M_y^{\ 2}}C_0 = \frac{\left(\frac{M_y}{M_{y,cr}}\right)^2}{1 - \left(\frac{M_y}{M_{y,cr}}\right)^2} \cdot C_0$$

Teljes elfordulás:

$$C_1 = C + C_0 = \frac{\left(\frac{M_y}{M_{y,cr}}\right)^2}{1 - \left(\frac{M_y}{M_{y,cr}}\right)^2} \cdot C_0 + C_0 = \frac{1}{1 - \left(\frac{M_y}{M_{y,cr}}\right)^2} \cdot C_0$$

A kifordulást csökkentő tényező: Hajlított gerenda



Kezdeti imperfekció:

$$\varphi = \varphi_0 \cdot \frac{1}{1 - \dfrac{M_y^{\,2}}{M_{y,cr}^2}}$$

Hajlítás síkjára merőleges nyomaték:

$$M_z = M_y \cdot \varphi$$

Határállapot:

$$\frac{M_y}{W_y} + \frac{M_z}{W_z} \le f_y$$

$$\frac{M_y}{W_y} + \frac{M_y}{W_y} \cdot \frac{\varphi_0 \cdot W_y}{W_z} \cdot \frac{1}{1 - \left(\dfrac{M_y}{M_{y,cr}} \right)^2} = f_y$$

$$M_{el,y} = W_y \cdot f_y$$

$$\frac{M_y}{M_{el,y}} = \chi_{LT}; \qquad \frac{M_{el,y}}{M_{y,cr}} = \bar{\lambda}_{LT}^{\,2}$$

$$\eta = \varphi_0 \cdot \frac{W_y}{W_z}$$

$$\chi_{LT} + \frac{\chi_{LT}}{1 - \chi_{LT}^{\,2} \cdot \bar{\lambda}_{LT}^{\,4}} \eta = 1$$

$$(1 - \chi_{LT})(1 - \chi_{LT}^{\,2} \cdot \bar{\lambda}_{LT}^{\,4}) = \eta \chi_{LT}$$

Rondal [1987]: $\eta = \alpha(\bar{\lambda} - \bar{\lambda}_0)$; $\bar{\lambda}_0 = 0{,}4$

hengerelt szelvény: $\alpha = 0{,}60$

hegesztett szelvény: $\alpha = 1{,}20$

Rondal [1987]: (Síkbeli kihajláshoz hasonlóan)

$$(1 - \chi_{LT})(1 - \chi_{LT} \cdot \bar{\lambda}_{LT}^{\,2}) = \eta \cdot \chi_{LT}$$

hengerelt szelvény: $\alpha = 0{,}32$

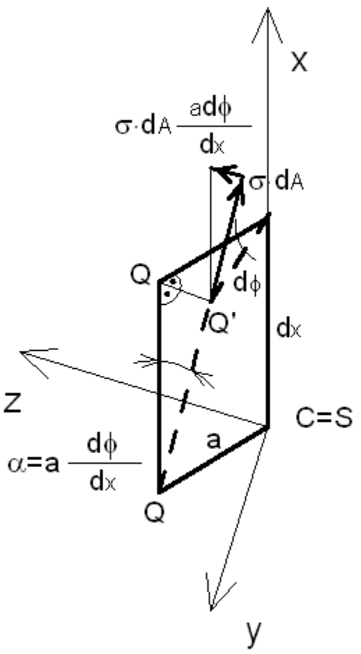
hegesztett szelvény: $\alpha = 0{,}78$

Nyomott─hajlított rúd instabilitása

Kritikus nyomaték: $i_0 = \sqrt{\dfrac{I_y + I_z}{A}}$; $P_{EZ} = \dfrac{\pi^2 EI_z}{L^2}$; $P_{E\omega} = \dfrac{1}{i_\omega^2}\left(GI_t + \dfrac{\pi^2 EI_\omega}{L^2}\right)$

$$M_{y,cr} = \sqrt{\frac{\pi^2 EI_z}{L^2}\left(GI_t + \frac{\pi^2 EI_\omega}{L^2}\right)}; \quad M_{y,cr}{}^2 = P_{Ez} \cdot P_{E\omega} \cdot i_\omega{}^2$$

Rúdtengely irányú nyomóerőből keletkező csavarónyomaték:



$$a^2 = y^2 + z^2$$

$$dM_T = a(\sigma dA)\left(a\frac{d\varphi}{dx}\right)$$

$$M_T = \frac{d\varphi}{dx}\sigma\int a^2 dA = \frac{d\varphi}{dx}\frac{P}{A}\left(I_y + I_z\right) = \frac{d\varphi}{dx}P \cdot i_\omega{}^2$$

$$EI_z u'' + Pu + M_y\varphi = 0 \quad \Big| \quad EI_z u^{IV} + Pu'' + M_y\varphi'' = 0$$

$$M_y u' + Pi_\omega{}^2 \cdot \varphi' - GI_t\varphi' + EI_\omega\varphi^{III} = 0 \quad \Big| \quad M_y u'' + Pi_\omega{}^2\varphi'' - GI_t\varphi'' + EI_\omega\varphi^{IV} = 0$$

$$u = A \cdot \sin\frac{\pi x}{L}; \qquad \varphi = B \cdot \sin\frac{\pi x}{L}$$

$$\begin{vmatrix} +\dfrac{\pi^2 EI_z}{L^2} - P & -M_y \\ -M_y & -Pi_\omega{}^2 + GI_t + \dfrac{\pi^2 EI_\omega}{L^2} \end{vmatrix} \begin{vmatrix} A \\ B \end{vmatrix} = 0 \qquad \begin{vmatrix} (P_{Ez} - P) & -M_y \\ -M_y & i_\omega{}^2(P_{E\omega} - P) \end{vmatrix} = 0$$

$$M_y{}^2 = i_\omega{}^2(P_{EZ} - P)(P_{E\omega} - P) = i_\omega{}^2 \cdot P_{Ez} \cdot P_{E\omega}\left(1 - \frac{P}{P_{Ez}}\right)\left(1 - \frac{P}{P_{E\omega}}\right)$$

$$\frac{M_y{}^2}{M_{y,cr}^2} = \left(1 - \frac{P}{P_{Ez}}\right)\left(1 - \frac{P}{P_{E\omega}}\right)$$