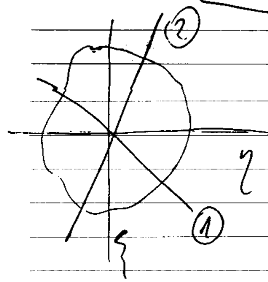
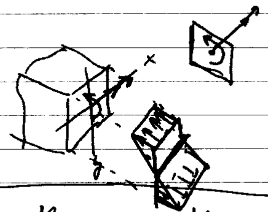
Tiszta egyenes hajlítás

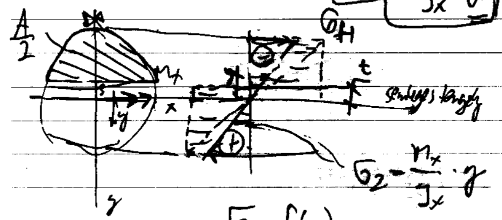


Egyenes: $M \equiv$ főirány, összetett hajlítás: $M \not\equiv$ főirány

Ferde: $M \not\equiv$ főirány



Egyenes hajlítás: $\boxed{\sigma_z = \frac{M_x}{J_x} \cdot y}$, $\boxed{\sigma_{z,max} = \frac{M_x}{J_x} \cdot y_{max} = \frac{M_x}{W_x} \le \sigma_H}$



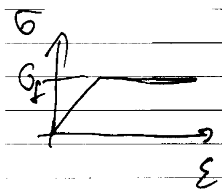
$\frac{J_x}{y_{max}} = K_x = W_x$

$K_x = \frac{M_x}{E J_x}$; $\rho = \frac{1}{K_x}$

görbület, görbületi sugár

$\sigma_z = \frac{M_x}{J_x} \cdot y$

$\sigma_z = f(y)$

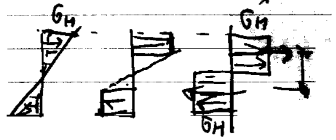


$$\boxed{\begin{aligned} M_{rug} &= \frac{J_x}{y_{max}} \cdot \sigma_H = W_x \cdot \sigma_H \\ M_{képl} &= 2 \cdot S_{x0} \cdot \sigma_H \end{aligned}}$$

képlékeny többlet-tartalék:

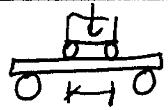
$$n = \left(\frac{M_{képl}}{M_{rug}} - 1\right) \cdot 100\%$$

$$M_{tart} = M_{képl} - M_{rug}$$

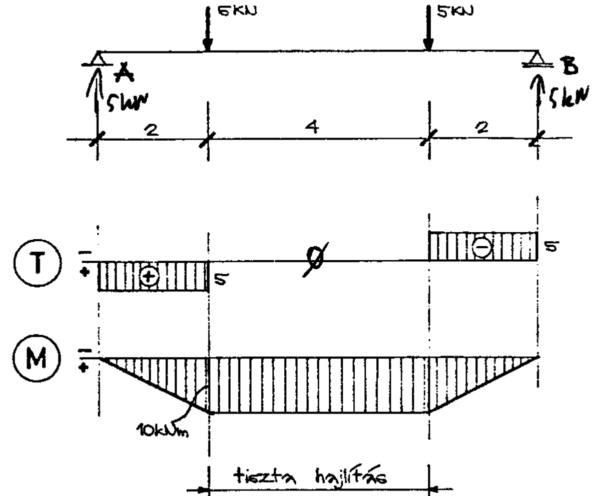


4.1. TISZTA, EGYENES HAJLÍTÁS – RUGALM. ÁLL.

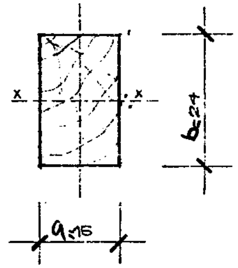
Tiszta hajlítás



1. Ellenőrizzük az alábbi tartót hajlításra!



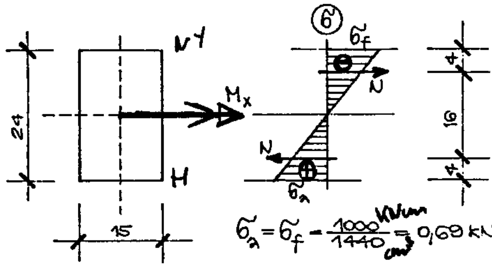
$A = B = 5$ kN (↑)



$\sigma_H = 1{,}4$ kN/cm² (II.o. fenyő)

Ellenőrizzük a keresztmetszetet!

$J_x = \frac{a \cdot b^3}{12}$



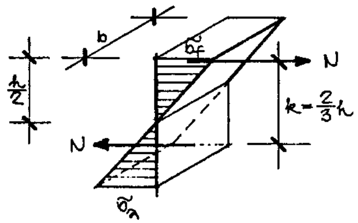
$\sigma = \frac{M}{J_x} y = \frac{M}{K_x}$

$K_{xa} = K_{xf} = \frac{a \cdot b^2}{6} = \frac{15 \cdot 24^2}{6}$

$K_{xa} = K_{xf} = 1440$ cm³

$\sigma_a = \sigma_f = \frac{1000 \text{ kNcm}}{1440 \text{ cm}^3} = 0{,}69$ kN/cm² $< \sigma_H = 1{,}4$ kN/cm² ✓

Feszültségi ábra alapján:



$N = \sigma \cdot b \cdot \frac{h}{2} \cdot \frac{1}{2} = \sigma \frac{b \cdot h}{4}$

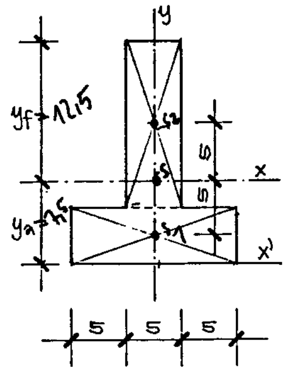
$\sigma_f = \sigma_a = \sigma$

$M = N \cdot k = \sigma \frac{b \cdot h}{4} \cdot \frac{2}{3} h$

$M = \sigma \frac{b \cdot h^2}{6} = \sigma \cdot K_x$

$K_x = W_x = \frac{J_x}{y_{max}} = \frac{\frac{a \cdot b^3}{12}}{\frac{b}{2}} = \frac{a \cdot b^2}{6}$

4.2. TISZTA, EGYENES HAJLÍTÁS – RUGALM. ÁLL.



$M_{HR} = ?$

$\sigma_+ = 1,1\ kN/cm^2$ (II.o. puhafa)

$$\sigma_H = \frac{M_{HR}}{J_x} \cdot y_{max} \rightarrow M_{HR} = \frac{\sigma_H \cdot J_x}{y_{max}}$$

$S_{x'} = 15 \cdot 5 \cdot 2,5 + 15 \cdot 5 \cdot 12,5 - 75(2,5 + 12,5) = \underline{1125\ cm^2}$

$y_a = \frac{S_{x'}}{A} = \frac{1125}{2 \cdot 15 \cdot 5} = \underline{7,5\ cm}$ $\quad y_f = 20 - 7,5 = \underline{12,5\ cm}$

$J_x = \frac{15 \cdot 5^3}{12} + 15 \cdot 5 \cdot 5^2 + \frac{5 \cdot 15^3}{12} + 15 \cdot 5 \cdot 5^2 =$

$= 156,25 + 1875 + 1406,25 + 1875 = \underline{5312,5\ cm^4}$ ✓

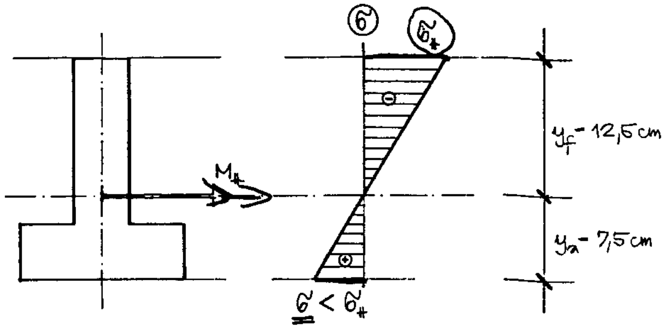
$M_{HR} = \sigma_H \cdot K_{min}$

$K_f = \frac{J_x}{y_f} = \frac{5312,5}{12,5} = \boxed{425\ cm^3}$

$K_a = \frac{J_x}{y_a} = \frac{5312,5}{7,5} = 708\ cm^3$

$$\sigma_H = \frac{M_{HR}}{K_{min}}$$

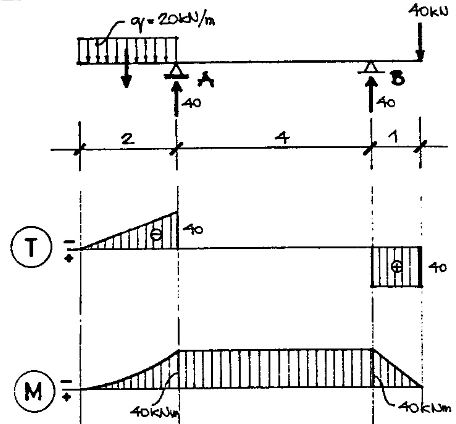
$K_{xm} = K_f = 425\ cm^3$

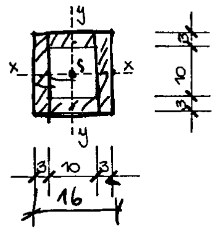


$M_{HR} = K_f \cdot \sigma_+ = 425 \cdot 1,1 = 467,5\ kNcm = \underline{4,68\ kNm}$

$$M_{HR} = \frac{\sigma_H \cdot J_x}{y_{max}} = \frac{1,1 \cdot 5312,5}{12,5} = \underline{467,5\ kNcm} = \underline{4,68\ kNm}$$

4.3. TISZTA, EGYENES HAJLÍTÁS – RUGALM. ÁLL.





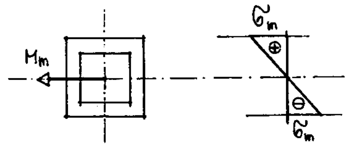
$\sigma_+ = 20\ kN/cm^2$

A keresztmetszeti adatok:

$J_x = \frac{16^4}{12} - \frac{10^4}{12} = 5461,33 - 833,33 = 4628\ cm^4$

$K_f = K_a = K_x = \frac{J_x}{y} = \frac{4628}{8} = 578,5\ cm^3$

Feszültségek meghatározása:



$\sigma_m = \frac{M}{K} = \frac{M}{W}$

$\sigma_m = \frac{4000}{578,5} = 6,91\ kN/cm^2$

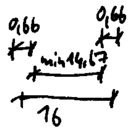
$\sigma_m = 6,91\ kN/cm^2 < 20\ kN/cm^2 = \sigma_+$ ✓ megf.

16x16 $\quad t_{min} = ?$

$$\sigma_H \geq \frac{M}{J_x} \cdot y_{max} = 20\ kN/cm^2 \rightarrow J_x \geq \frac{M}{\sigma_H} \cdot y_{max}$$

$$J_{x\,min} = \frac{4000}{20} \cdot 8 = \underline{1600\ cm^4} = \frac{16^4}{12} - \frac{a^4}{12}$$

$$\frac{a^4}{12} = \frac{16^4}{12} - 1600 \rightarrow a = \sqrt[4]{16^4 - 12 \cdot 1600} = \underline{14,67\ cm}$$

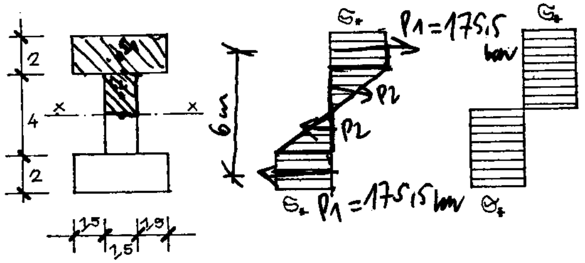


$\boxed{t_{alk} = 1\ cm = 10\ mm}$

4.4. TISZTA, EGYENES HAJLÍTÁS – KÉPLÉKENY ÁLL.

1.) Határozzuk meg a határnyomatékot
- ha az övek képlékeny, a gerinc rugalmas állapotban van
- ha a teljes keresztmetszet képlékeny állapotban van

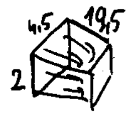
$\sigma_H = 19{,}5\ kN/cm^2$



a.) $M_H = M_{HR}' + M_{HR}''$

$M_{HR}' = \sigma_H \cdot K_x'$ $\quad K_x' = \frac{1{,}5 \cdot 4^2}{6} = 4\ cm^3$

$M_{HR}' = 19{,}5 \cdot 4 = 78\ kNcm = 0{,}78\ kNm$



$M_{HK}'' = \sigma_x \cdot 2 \cdot S_x''$ $\quad S_x'' = 2 \cdot 4{,}5 \cdot 3 = 27\ cm^3$

$M_{HK}'' = 19{,}5 \cdot 2 \cdot 27 = 1053\ kNcm = 10{,}53\ kNm$

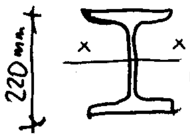
$M_H = 0{,}78 + 10{,}53 = 11{,}31\ kNm$

b.) $M_{HK} = \sigma_H \cdot 2 \cdot S_x$ $\quad S_x$ – a félkeresztmetszet statikai nyomatéka az x' tengelyre

$M_{HK} = 19{,}5 \cdot 2 \cdot 30 = 1170\ kNcm$ $\quad S_x = 2 \cdot 4{,}5 \cdot 3 + 1{,}5 \cdot 2{,}0 \cdot 1{,}0 = 30\ cm^3$

$M_{HK} = 11{,}7\ kNm$

2.) Mekkora az I 220-as szelvényből készült darupálya képlékeny tartaléka?



$K_x = 278\ cm^3$ ✓

$(\sigma_H = 2000\ kp/cm^2)$ $\quad$ SI-ben: $\sigma_H = 20\ kN/cm^2$ ✓

$S_x = 162\ cm^3$ ✓ (a félszelvény statikai nyomatéka az x – x tengelyre.)

Az I 220-as szelvény határnyomatéka:

$M_{HR} = 2 \cdot \cancel{10^5} \cdot 2{,}78 \cdot \cancel{10^{-4}} = 5{,}56 \cdot 10 = 55{,}6\ kNm$

$20 \cdot 278 = 5560\ kNcm = 55{,}6\ kNm$

Képlékeny határállapotban:

$M_t = \sigma_H \cdot 2 \cdot S_x = 2 \cdot 10^5 \cdot 2 \cdot 1{,}62 \cdot 10^{-4} = 64{,}8\ kNm$

$20 \cdot 2 \cdot 162 = 6480\ kNcm = 64{,}8\ kNm$

A képlékeny tartalék:

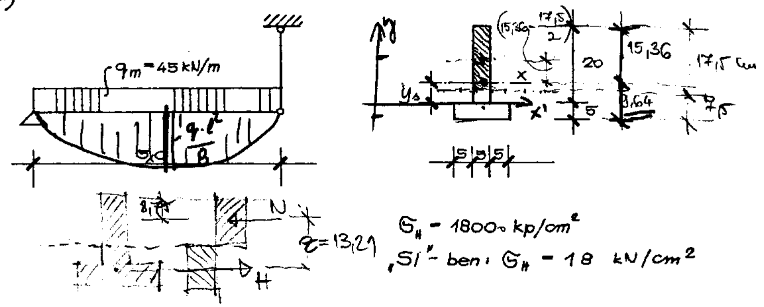
$M_t - M_H = 64{,}8 - 55{,}6 = 9{,}2\ kNm$

$\frac{A}{2} = 87{,}5\ cm^2$

$87{,}5 = 5 \cdot m$

$m = 17{,}5\ cm$

3.)



$\sigma_H = 1800\ kp/cm^2$

SI-ben: $\sigma_H = 18\ kN/cm^2$

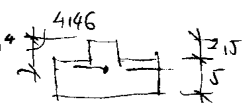
Mekkora a vázolt gerendatartó maximális hajlítóigénybevétele?
A képlékeny tartalék hány %-a van kihasználva?

$M_m = \frac{q \cdot l^2}{8} = \frac{45 \cdot 5^2}{8} = 140{,}6\ kNm$

Keresztmetszeti jellemzők:

$y_s = \frac{20 \cdot 5 \cdot 10 - 15 \cdot 5 \cdot 2{,}5}{20 \cdot 5 + 15 \cdot 5} = 4{,}64\ cm$ $\quad A = 175\ cm^2$

$J_x = \frac{15 \cdot 5^3}{3} + \frac{5 \cdot 20^3}{3} - 175 \cdot 4{,}64^2 = 1{,}019 \cdot 10^4\ cm^4$



$J_x = \frac{5 \cdot 20^3}{12} + 5 \cdot 20 \cdot 5{,}36^2 + \frac{15 \cdot 5^3}{12} + 15 \cdot 5 \cdot 7{,}14^2 = 10190\ cm^4$

$e_a = 5{,}0 + 4{,}64 = 9{,}64$ cm

$e_f = 20{,}0 - 4{,}64 = 15{,}36$ cm

$K_a = \dfrac{1019 \cdot 10^4}{9{,}64} = 1057{,}05\ cm^3 = 1{,}057 \cdot 10^{-3}\ m^3$

$K_f = \dfrac{1019 \cdot 10^4}{15{,}36} = 663{,}41\ cm^3 = 6{,}63 \cdot 10^{-4}\ m^3$

$M_{HR} = \sigma_H \cdot K_{min} = 1{,}8 \cdot 10^5 \cdot 6{,}63 \cdot 10^{-4} = 119{,}3$ kNm $< M_m = 140{,}6$

$18 \cdot 663{,}41 = 11930$ kNcm $= \underline{\underline{119{,}3\ kNm}} < 140{,}6$

Képlékeny határnyomaték számítása

a félterület $A' = \dfrac{175}{2} = 87{,}5\ cm^2$

a magasság $m' = \dfrac{87{,}5}{5} = 17{,}5$ cm

a statikai nyomaték :

$$S_x' = 87{,}5\left(15{,}36 - \frac{17{,}5}{2}\right) = 578{,}4\ cm^3 = 5{,}78 \cdot 10^{-4}\ m^3$$

$M_{HK} = 1{,}8 \cdot 10^5 \cdot 2 \cdot 5{,}78 \cdot 10^{-4} = 208{,}1$ kNm $- 18 \cdot 87{,}5 \cdot 13{,}21 = \underline{\underline{20805{,}7\ kNcm}}$

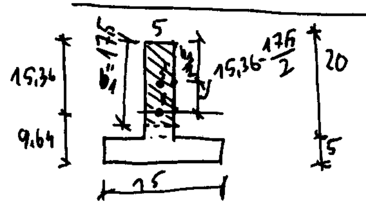
Képlékeny tartalék :

$M_{KT} = M_{HK} - M_{HR} = 208{,}1 - 119{,}3 = 88{,}8$ kNm

$M_{Km} = M_m - M_{HR} = 140{,}6 - 119{,}3 = 21{,}3$ kNm

A kihasznált képlékeny tartalék % - ban :

$\dfrac{21{,}3}{88{,}8} \cdot 100 = \underline{23{,}99\ \%}$



$A = 175\ cm^2$

$\dfrac{A}{2} = \underline{\underline{87{,}5\ cm^2}} = b_1 \cdot 5$

$b_1 = \underline{\underline{17{,}5\ cm}}$

$M_{HK} = \sigma_H \cdot 2 \cdot S_0$

$S_{0x}' = \underbrace{17{,}5 \cdot 5}_{87{,}5} \cdot \left(15{,}36 - \dfrac{17{,}5}{2}\right) = \underline{\underline{578{,}4\ cm^3}}$

$M_{HK} = 18 \cdot 2 \cdot 578{,}4 = \underline{\underline{20822{,}4\ kNcm}} =$
$= \underline{208{,}22\ kNm}$

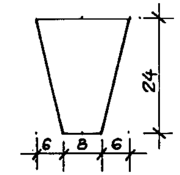
$M_{KT} = 208{,}22 - 119{,}3 = \underline{\underline{88{,}92\ kNm}}$

$M_{Km} = 140{,}6 - 119{,}3 = 21{,}3$ kNm

$\dfrac{21{,}3}{88{,}92} \cdot 100\% = \boxed{23{,}95\ \%}$

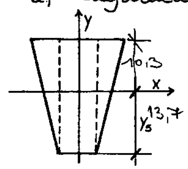
4.5. TISZTA, EGYENES HAJLÍTÁS – KÉPL. ÁLL.

Példa Határozzák meg a keresztmetszet rugalmas és képlékeny határnyomatékát, és az ezekhez tartozó feszültségi ábrát!



$\sigma_H = 15\ N/mm^2 = 1{,}5\ kN/cm^2$

a, Rugalmas feszültség elosztás esetén



$$y_s = \frac{8 \cdot 24 \cdot 12 + 2 \cdot 6 \cdot 24 \cdot \frac{1}{2} \cdot 16}{192 + 144} = \frac{4608}{336} = 13{,}7\ cm$$

$$J_x = \left[\frac{8 \cdot 24^3}{3} + \frac{12 \cdot 24^3}{4}\right] - 336 \cdot 13{,}7^2 = 15\,274\ cm^4$$

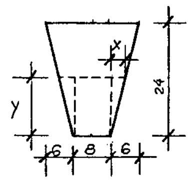
$$K_{x,a} = \frac{J_x}{y_s} = \frac{15\,274}{13{,}7} = 1115\ cm^3$$

$M_{HR} = 1{,}5 \cdot 1115 = 1672{,}5$ kNcm $= 16{,}72$ kNm

b, Képlékeny feszültség elosztás esetén

$\sigma = \sigma_H$

területfelező vonal helye



$F = 336\ cm^2$ $\qquad \dfrac{F}{2} = 168\ cm^2$

$168 = 8 \cdot y + x \cdot y$

$x : y = 6 : 24$

$x = 6 \cdot \dfrac{y}{24} = \dfrac{y}{4}$ $\qquad x = \dfrac{6y}{24} = \dfrac{y}{4}$

$168 = 8y + \dfrac{y}{4} \cdot y$

$0{,}25\, y^2 + 8y - 168 = 0$

$y^2 + 32y - 672 = 0$

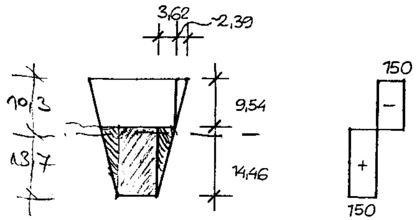
$y_{1,2} = \dfrac{-32 \pm \sqrt{1024 + 4 \cdot 2688}}{2}$

$y = 14{,}46$ cm

$$168 = 8y + \frac{y^2}{4}$$

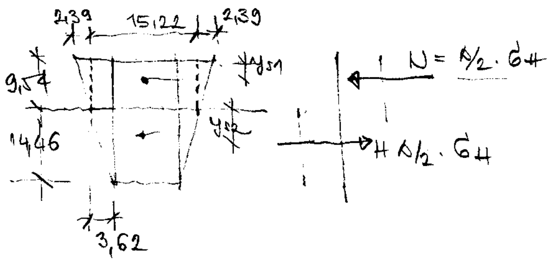
$$y^2 + 32y - 672 = 0$$

$y_1 = 14{,}46$ cm (y_2 negatív)



$$M_{HK} = 1{,}5\left[(8+2\times 3{,}62)\times 9{,}54 \times \frac{9{,}54}{2} + 2{,}39 \times 9{,}54 \times \frac{2}{3} \times 9{,}54 + 8\times 14{,}46 \times \frac{14{,}46}{2} + 3{,}62 \times 14{,}46 \times \frac{14{,}46}{2}\right] =$$

$$= 1{,}5\,[\underbrace{693{,}5 + 145 + 836{,}5 + 253}_{1928}] = 2892 \text{ kNcm} = 28{,}92 \text{ kNm}$$



$$z = 6{,}48 + 4{,}99 = 11{,}47$$

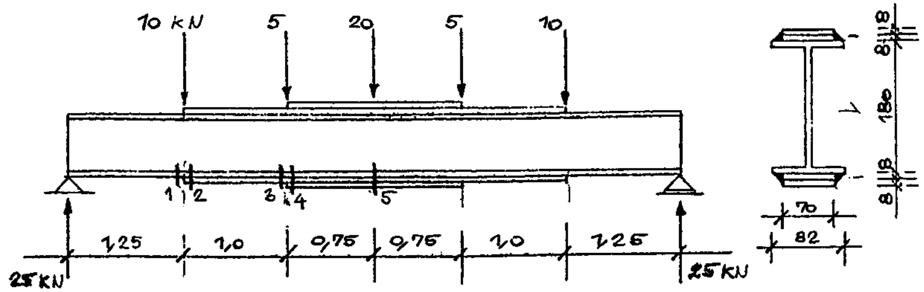
$$y_{s1} = \frac{2 \cdot \frac{2{,}39 \cdot 9{,}54}{2} \cdot \frac{9{,}54}{3} + \frac{15{,}22 \cdot 9{,}54 \cdot 9{,}54}{2}}{168} = 4{,}55$$

$$y_{s2} = \frac{2 \cdot \frac{3{,}62 \cdot 14{,}46}{2} \cdot \frac{14{,}46}{3} + \frac{8 \cdot 14{,}46 \cdot 14{,}46}{2}}{168} = 6{,}48$$

$$M_{HK} = 11{,}47 \cdot 168 \cdot 1{,}5 = 28{,}90 \text{ kNm}$$

4.6 Változó keresztmetszetű tartó hajlítása

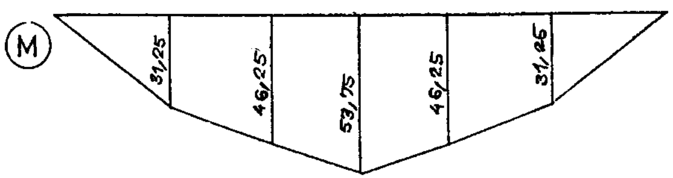
Ellenőrizzük az alábbi I tartót



I-180 $J_{x1} = 1450$ cm^4

$K_{x1} = 161$ cm^3

$\sigma_H = 20$ kN/cm^2



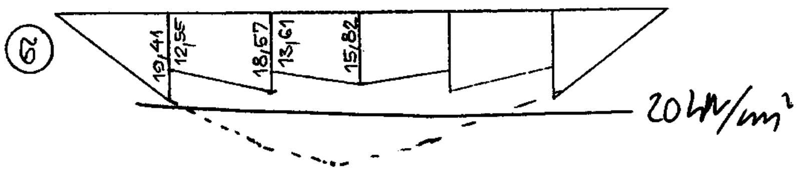
$$J_{x2} = 1450 + 2 \cdot \overset{t_1}{9{,}4}^2 \cdot \overset{A_1}{\overbrace{0{,}8 \cdot 7}} = 2440 \text{ cm}^4 \qquad K_{x2} = \frac{2440}{9{,}8} = 249 \text{ cm}^3$$

$$J_{x3} = 1450 + 2 \cdot \overset{t_2}{9{,}8}^2 \cdot \overset{A_2}{\overbrace{1{,}6 \cdot 7}} = 3601 \text{ cm}^4 \qquad K_{x3} = \frac{3601}{10{,}6} = 339{,}7 \text{ cm}^3$$

$$\sigma_1 = \frac{M_{1,2}}{K_{x1}} = \frac{3125}{161} = 19{,}41 \frac{\text{kN}}{\text{cm}^2} \quad ; \quad \sigma_3 = \frac{M_{3,4}}{K_{x2}} = \frac{4625}{249} = 18{,}57 \frac{\text{kN}}{\text{cm}^2}$$

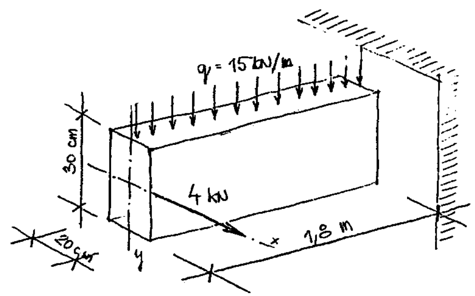
$$\sigma_2 = \frac{M_{1,2}}{K_{x2}} = \frac{3125}{249} = 12{,}55 \frac{\text{kN}}{\text{cm}^2} \quad ; \quad \sigma_4 = \frac{M_{3,4}}{K_{x2}} = \frac{4625}{339{,}7} = 13{,}61 \frac{\text{kN}}{\text{cm}^2}$$

σ – ábra a tartó hossza mentén: $\sigma_5 = \frac{M_5}{K_{x3}} = \frac{5375}{339{,}7} = 15{,}82 \frac{\text{kN}}{\text{cm}^2}$



5.1 Ferde hajlítás

Ellenőrizze a konzolt hajlításra! $\sigma_H = 1{,}2\ \mathrm{kN/cm^2}$



$$M_x = \frac{15\cdot 1{,}8^2}{2} = 24{,}3\ \mathrm{kNm}$$

$$M_y = 4\cdot 1{,}8 = 7{,}2\ \mathrm{kNm}$$

$K_x = 3000\ \mathrm{cm^3}$ $\quad J_x = K_x\cdot y_{max} = \frac{20\cdot 30^3}{12}$

$K_y = 2\,000\ \mathrm{cm^3}$

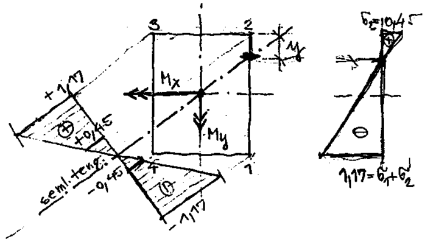
$$M_x \Rightarrow \frac{2430}{3000} = 0{,}81\ \frac{\mathrm{kN}}{\mathrm{cm^2}}$$

$$M_y \Rightarrow \frac{720}{2000} = 0{,}36\ \frac{\mathrm{kN}}{\mathrm{cm^2}}$$

a.) befogási keresztmetszet:

$$\sigma = \pm\frac{M_x}{K_x} \pm \frac{M_y}{K_y}$$

$$\sigma = \pm\frac{M_x}{J_x}\cdot u_1 \pm \frac{M_y}{J_y}\cdot y_2$$



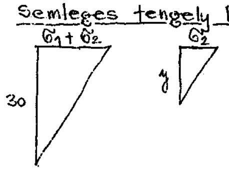
$\sigma_1 = -0{,}81 - 0{,}36 = -1{,}17\ \frac{\mathrm{kN}}{\mathrm{cm^2}} < 1{,}2$ ✓

$\sigma_2 = +0{,}81 - 0{,}36 = +0{,}45\ \frac{\mathrm{kN}}{\mathrm{cm^2}}$

$\sigma_3 = +0{,}81 + 0{,}36 = +1{,}17\ \frac{\mathrm{kN}}{\mathrm{cm^2}}$

$\sigma_4 = -0{,}81 + 0{,}36 = -0{,}45\ \frac{\mathrm{kN}}{\mathrm{cm^2}}$

Semleges tengely helye:

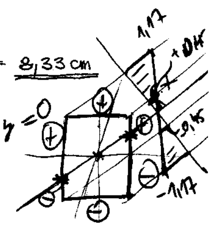


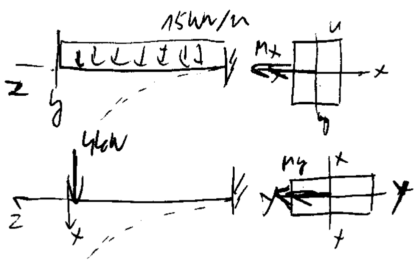
$$y : 30 = \sigma_2 : (\sigma_1 + \sigma_2)$$

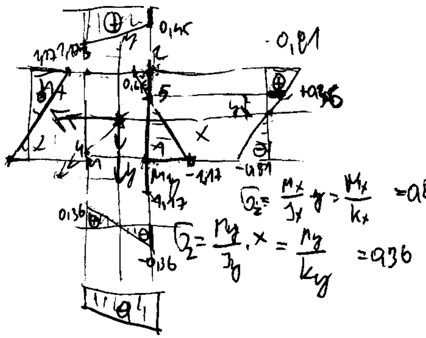
$$y = \frac{30\cdot 0{,}45}{1{,}17 + 0{,}45} = \frac{13{,}5}{1{,}62} = 8{,}33\ \mathrm{cm}$$

$$\sigma_5 = 0{,}36 + \frac{M_x}{J_x}\cdot y = 0{,}36 - \frac{2430}{45000}\,y = 0$$

$$y = 0{,}36\cdot\frac{45000}{2430} = 6{,}67\ \mathrm{cm}$$





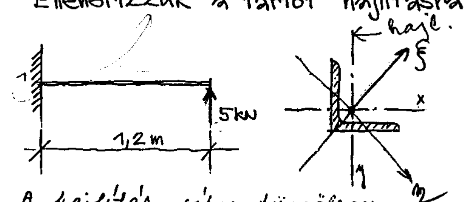


$$\sigma_2 = \frac{M_x}{J_x}\cdot y = \frac{M_x}{K_x} = 0{,}81$$

$$\sigma_2 = \frac{M_y}{J_y}\cdot x = \frac{M_y}{K_y} = 0{,}36$$

5. 2. Ferdehajlítás

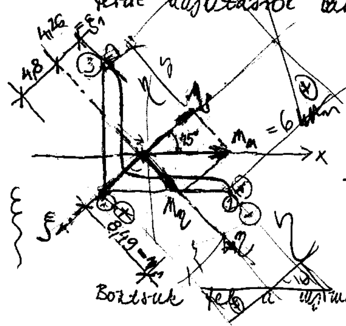
Ellenőrizzük a tartót hajlításra!



L 120·120·12

$\sigma_H = 20\ \mathrm{kN/cm^2}$

A hajlítás síkja függőleges, az L szögacél főtengelyei 45°-os szöget zárnak be a vízszintes tengellyel, így ferde hajlításról van szó.

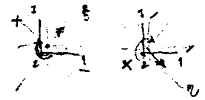


Keresztmetszeti adatok:

$J_\xi = J_{max} = 584\ \mathrm{cm^4}$

$J_\eta = J_{min} = 152\ \mathrm{cm^4}$

(táblázatból)



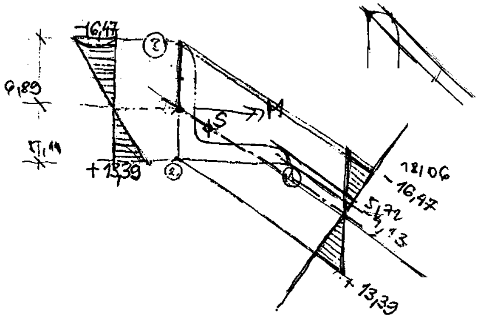
Bontsuk fel a nyomatékvektort főtengely irányú összetevőkre:

$$M_\xi = M_\eta = M_x\cdot 0{,}7071 = 6\cdot 0{,}7071 = 4{,}243\ \mathrm{kNm}$$

$$\sigma_1 = +\frac{M_\xi}{J_\xi}\cdot\eta_1 - \frac{M_\eta}{J_\eta}\cdot\xi_1 = \underbrace{\frac{424{,}3}{584}\cdot 8{,}49}_{6{,}17} - \underbrace{\frac{424{,}3}{152}\cdot 3{,}69}_{-11{,}89} = -5{,}72\ \frac{\mathrm{kN}}{\mathrm{cm^2}}$$

$$\sigma_2 = +\frac{M_\eta}{J_\eta}\cdot\xi_2 = \frac{424{,}3}{152}\cdot 4{,}8 = +13{,}39\ \frac{\mathrm{kN}}{\mathrm{cm^2}}$$

$$\sigma_3 = -\frac{M_\xi}{J_\xi}\cdot\eta_3 - \frac{M_\eta}{J_\eta}\cdot\xi_3 = -\underbrace{\frac{424{,}3}{584}\cdot 8{,}49}_{-6{,}17} - \underbrace{\frac{424{,}3}{152}\cdot 3{,}69}_{-11{,}89} = -18{,}05\ \frac{\mathrm{kN}}{\mathrm{cm^2}}$$



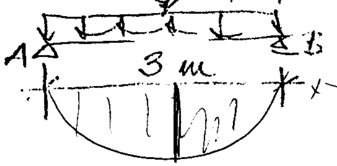
$\sigma_{max} = \sigma_3 < \sigma_H$ megfelel

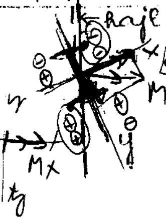
(A kerekítésnél a keresztmetszet egy részén valamivel nagyobb a feszültség, mint a σ_3 értéke, de az arányokból érezhető, hogy σ_H-t még nem éri el.)

5. FERDE HAJLÍTÁS

5.2.

$q = 4\ kN/m$





$\sigma_H = 20\ kN/cm^2$

I 160

$K_x = 117\ cm^3$

$K_y = 14{,}8\ cm^3$

MEGFELEL-E A SZELVÉNY? (KORACHA. FEDŐLAPÚ BEFOGÁSSZELVÉNY TÁBLÁZAT)

$$\sigma_M = \frac{M_x}{K_x} + \frac{M_y}{K_y} = \frac{M_H \cos}{}\ \ \cos 18° = 0{,}9511 \qquad \sin 18° = 0{,}3090$$

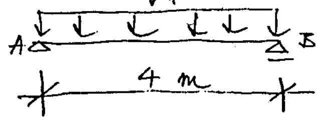
$$M_H = \frac{4 \cdot 3^2}{8} = 4{,}5\ kNm = \underline{450\ kNcm}$$

$$\sigma_M = \frac{428}{117} + \frac{139{,}1}{14{,}8} = 13{,}06\ kN/cm < \sigma_H = 20\frac{kN}{cm^2}\ \checkmark$$

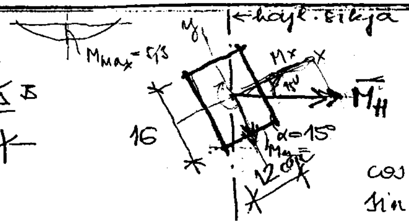
MEGFELEL

---

5. 3. $q_H = ?$



$\sigma_H = 1{,}35\ \frac{kN}{cm^2}$



$M_m = \frac{q \cdot l^2}{8}$

$K_x = \frac{12 \cdot 16^2}{6} = 512$

$K_y = \frac{16 \cdot 12^2}{6} = 384$

$\cos 15° = 0{,}9659$

$\sin 15° = 0{,}2588$

a) Határozzák meg a tartó határteherbírását rugalmas állapotban

b) Jelöljék be a semleges tengelyt és rajzolják meg a σ ábrát

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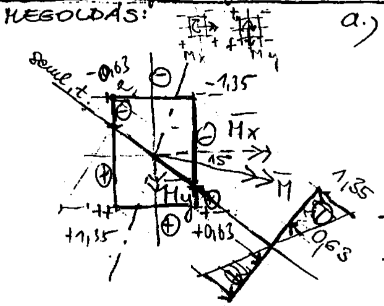
MEGOLDÁS:



a.) $K_x = \frac{12 \cdot 16^2}{6} = 512\ cm^3$

$K_y = \frac{16 \cdot 12^2}{6} = 384\ cm^3$

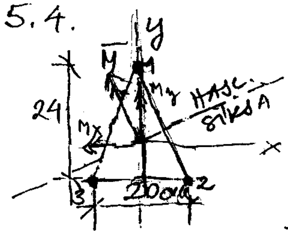
$$\sigma_H = \frac{M_x}{K_x} + \frac{M_y}{K_y} = \frac{M \cdot \cos\alpha}{K_x} + \frac{M \cdot \sin\alpha}{K_y}$$

$$M = \frac{\sigma_H \cdot K_x \cdot K_y}{K_y \cdot \cos\alpha + K_x \cdot \sin\alpha} = 527\ kNcm$$

$$\boxed{q_H} = \frac{8M}{l^2} = \frac{8 \cdot 5{,}27}{4^2} = \boxed{2{,}635}\ \frac{kN}{m}$$

b.) $\sigma_2 = -\frac{527 \cdot 0{,}9659}{512} + \frac{527 \cdot 0{,}2588}{384} = -0{,}99 + 0{,}36 = -0{,}63\frac{kN}{cm^2}$

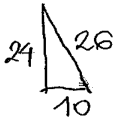
5.4.



– M_H = ? (rugalmasan)

– SEML. TENGELY; σ ábra

$\sigma_H = 1{,}4\ kN/cm^2$



$\frac{10}{26} = 0{,}3846$

$\frac{24}{26} = 0{,}9231$

$$I_x = \frac{20 \cdot 24^3}{36} = 7680\ cm^4 \qquad K_{xf} = \frac{7680}{16} = 480\ cm^3$$

$$I_y = 2 \cdot \frac{24 \cdot 10^3}{12} = 4000\ cm^4 \qquad K_{xa} = \frac{7680}{8} = 960\ cm^3$$

$$K_y = \frac{4000}{10} = 400\ cm^3$$

$\sigma_2 = +\sigma_{max} = \sigma_H$

$\sigma_3 = -\sigma_{max}$

ha $\sigma_H = \sigma_1$

$$1{,}4 = \frac{0{,}3846 M}{480} \Rightarrow M = 1747\ kNcm$$

ha $\sigma_H = \sigma_3$

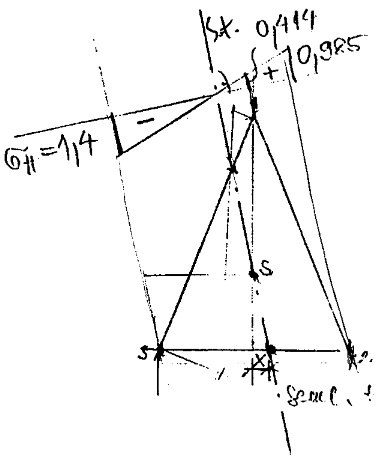
$$1{,}4 = \frac{0{,}3846 M}{960} + \frac{0{,}9231 M}{400} \Rightarrow M = 516{,}91\ kNcm\ \checkmark$$

$\boxed{M_H = 5{,}17\ kNm}$

$$\sigma_1 = \frac{198{,}8}{480} = \oplus 0{,}414\ \frac{kN}{cm^2}\ \oplus$$

$$\sigma_3 = -\sigma_H = -1{,}40\ \frac{kN}{cm^2}\ \ominus$$

$$\sigma_2 = -\frac{198{,}8}{960} + \frac{477}{400} = 0{,}985\frac{kN}{cm^2}$$



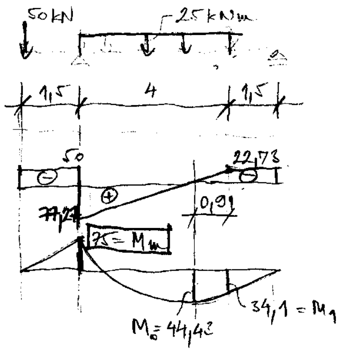
seml. t. másik pontja

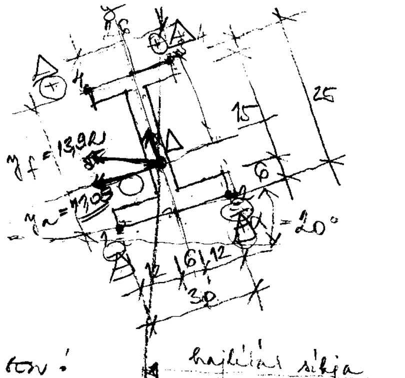
ha $y = 8\ cm$

$$\sigma = \frac{M_x}{I_x} \cdot y + \frac{M_y}{I_y} \cdot x = 0 \rightarrow ból$$

$$\frac{198{,}8}{7680} \cdot 8 = \frac{477}{4000} \cdot x$$

$x = 1{,}73\ cm$





A Mmax km. tőv: keresztmetszetnek σ ábrát is

$$y_a = \frac{\overset{540}{30\cdot 6\cdot 3} + \overset{1215}{6\cdot 15\cdot 13{,}5} + \overset{2392}{4\cdot 26\cdot 23}}{\underbrace{180 + 90 + 104}_{374}} =$$

$$J_x = \underset{540}{\frac{30\cdot 6^3}{12}} + \underset{11751{,}55}{180\cdot 8{,}08^2} + \underset{1687{,}5}{\frac{6\cdot 15^3}{12}} + \underset{527{,}07}{90\cdot 2{,}42^2} + \underset{138{,}66}{\frac{26\cdot 4^3}{12}} + \underset{14776{,}98}{104\cdot 11{,}92^2} :$$

$$J_x = 29\,421{,}76\ \text{cm}^4$$

$$J_y = \underset{13500}{\frac{6\cdot 30^3}{12}} + \underset{270}{\frac{15\cdot 6^3}{12}} + \underset{5858{,}66}{\frac{4\cdot 26^3}{12}} = 19\,628{,}66\ \text{cm}^4$$

$$M_x = \cos 20^\circ \cdot 75 = 70{,}47\ \text{kNm}$$

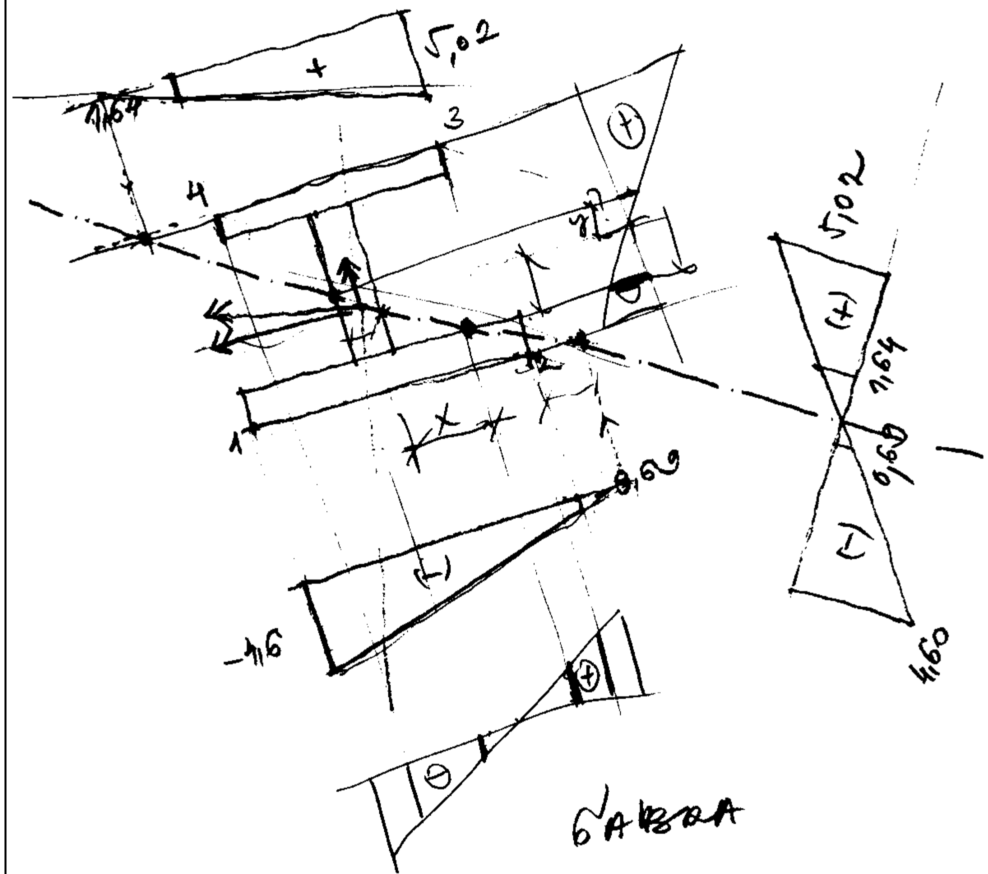
$$M_y = \sin 20^\circ \cdot 75 = 25{,}65\ \text{kNm}$$

$$\sigma_1 = -\overset{-2{,}65}{\frac{7047}{29421{,}76}\cdot 11{,}08} - \overset{-1{,}96}{\frac{2565}{19628{,}66}\cdot 15} = -4{,}60\ \text{kN/cm}^2$$

$$\sigma_2 = -\overset{-2{,}65}{\frac{7047}{29421{,}76}\cdot 11{,}08} + \overset{+1{,}96}{\frac{2565}{19628{,}66}\cdot 15} = -0{,}69\ \text{kN/cm}^2$$

$$\sigma_3 = +\overset{+3{,}33}{\frac{7047}{29421{,}76}\cdot 13{,}92} + \overset{+1{,}69}{\frac{2565}{19628{,}66}\cdot 13} = +5{,}02$$

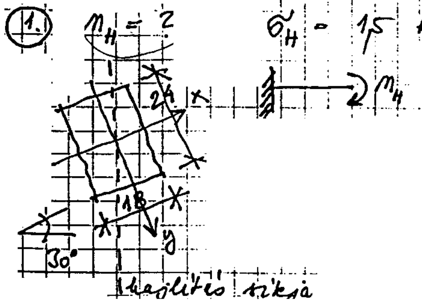
$$\sigma_4 = +\overset{+3{,}33}{\frac{7047}{29421{,}76}\cdot 13{,}92} - \overset{-1{,}69}{\frac{2565}{19628{,}66}\cdot 13} = +1{,}64$$



σ ÁBRA

FERDE HAJLÍTÁS

(1.) $M_H = ?$ $\sigma_H = 1,5\ kN/cm^2$



$\sigma_{max} = \sigma_H = \frac{M_x}{K_x} + \frac{M_y}{K_y} = \frac{0,866M}{1728} + \frac{0,5M}{1296}$

$1,5 = 0,0005M + 0,0004M$

$1,5 = 0,0009M$

$M_x = M \cdot \cos 30° = 0,866\ M$

$M_y = M \cdot \sin 30° = 0,500\ M$

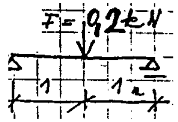
$K_x = \frac{18 \cdot 24^2}{6} = 1728\ cm^3$

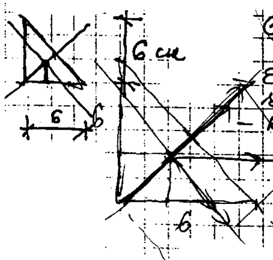
$K_y = \frac{24 \cdot 18^2}{6} = 1296\ cm^3$

$M = \frac{1,5}{0,0009} = \cancel{1666,6\ kNcm \sim 16,7\ kNm}$

$\boxed{M_H = 16,70\ kNm}$

(2.) $F = 9,2\ kN$

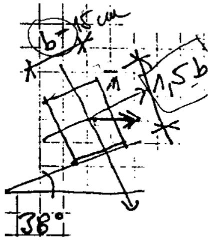




$\sigma_H = 2\ kN/cm^2$

Ell. fesz. összeadásával!
Rajzoljunk σ-ábrát!

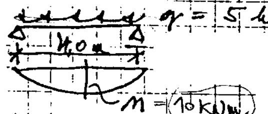
(3.)



($M = 10\ kNm$)

$b = ?$ (egész cm-re kerekítve)

$\sigma_H = 1,2\ kN/cm^2$



$\frac{q l^2}{8}$

$M_x = M \cdot \cos 38° = 10 \cdot 0,788 = 7,88\ kNm = 788\ kNcm$

$M_y = M \cdot \sin 38° = 10 \cdot 0,616 = 6,16\ kNm = 616\ kNcm$

$K = \frac{b \cdot h^2}{6}$

$b = 16$

$h = 24$

$K_x = \frac{b \cdot (1,5b)^2}{6} = \frac{2,25 b^3}{6}$

$K_y = \frac{1,5b \cdot b^2}{6} = \frac{1,5 b^3}{6}$

$\sigma_H = \sigma_H = 1,2 = \frac{788 \cdot 6}{b \cdot (1,5b)^2} + \frac{616 \cdot 6}{1,5b \cdot b^2} = \frac{2101,3}{b^3} + \frac{2464}{b^3}$

(4728 / 2,25 b³; 3696 / 1,5 b³)

$1,2 b^3 = 4565,3$

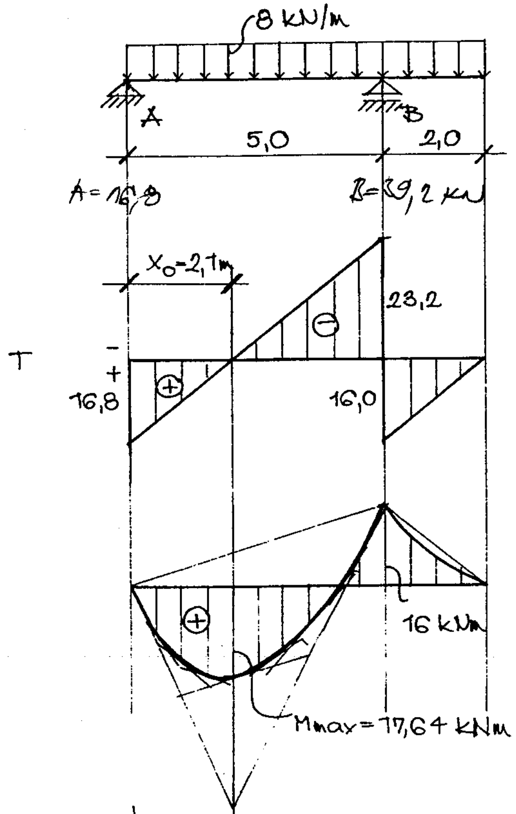
$b^3 = 3804,42$

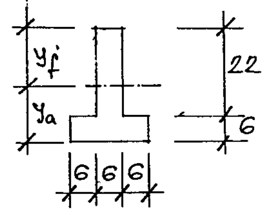
$b = \sqrt[3]{3804,42} = 15,6\ cm \sim \boxed{16\ cm}$

$h = 1,5b$ $h = \boxed{24\ cm}$

6.1. ÖSSZETETT HAJLÍTÁS - NYÍRÓFESZÜLTSÉGEK

ÖSSZETETT HAJLÍTÁS, HAJLÍTÁSBÓL SZÁRMAZÓ, NYÍRÓFESZÜLTSÉGEK





$\sigma_H = 200\ kN/cm^2$

1. REAKCIÓK:

$B = \frac{8 \cdot 7 \cdot 3,5}{5} = 39,20\ kN$

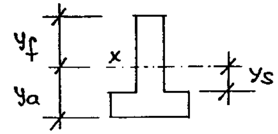
$A = 8 \cdot 7 - 39,20 = 16,80\ kN$

$x_0 = \frac{16,8}{8} = 2,10\ m$

$M_{max} = 16,8 \cdot 2,1 - 8 \cdot 2,1 \cdot 1,05 = 17,64\ kNm$

$\frac{q l^2}{8} = \frac{8 \cdot 5^2}{8} = 25,0\ kNm$

2. KERESZTMETSZETI JELLEMZŐK:



$$y_s = \frac{22\cdot 6\cdot 11 - 18\cdot 6\cdot 3}{6\,(22+18)} = 4{,}70\ \text{cm}$$

$y_f = 17{,}30\ \text{cm}$ $\quad y_a = 10{,}70\ \text{cm}$

$$J_x = \frac{6\cdot 22^3}{12} + 6\cdot 22\cdot 6{,}3^2 + \frac{18\cdot 6^3}{12} + 6\cdot 18\cdot 7{,}7^2 =$$

$$= 5324 + 5239 + 324 + 6403 \rightarrow \underline{J_x = 17290\ \text{cm}^4}$$

$$K_f = \frac{17290}{17{,}30} = 999{,}42\ \text{cm}^3$$

$$K_a = \frac{17290}{10{,}7} = 1615{,}9\ \text{cm}^3$$

3. „σ" FESZÜLTSÉGEK MEGHATÁROZÁSA:

$M^{(+)}_{max}$ – BÓL

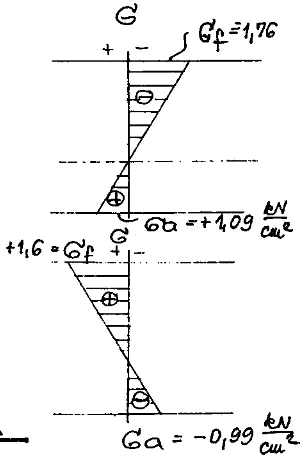
$$\sigma_a = \frac{1764}{1616} = 1{,}092\ \text{kN/cm}^2 < \sigma_H$$

$$\sigma = \frac{-1764}{999} = -1{,}764\ \text{kN/cm}^2 < \sigma_H$$

$M^{(-)}_{max}$ – BÓL

$$\sigma_a = \frac{-1600}{1616} = -0{,}99\ \text{kN/cm}^2 < \sigma_H$$

$$\sigma_f = \frac{1600}{999} = +1{,}6\ \text{kN/cm}^2 < \sigma_H$$

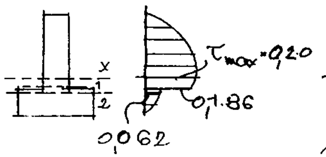


4. „τ" FESZÜLTSÉGEK MEGHATÁROZÁSA

$T_{MAX} = 23{,}20$ kN HELYÉN

$$\tau = \frac{T\cdot S}{b\cdot J_x} \qquad S_x = 6\cdot 17{,}3\cdot 8{,}65 = 897{,}87\ \text{cm}^3$$

$$\tau_{max} = \frac{23{,}2 \cdot 897{,}87}{6 \cdot 17290} = 0{,}20\ \frac{kN}{cm^2}$$



$$S_2 = S_1 = 18\cdot 6\cdot 7{,}7 = 831{,}6\ \text{cm}^3$$

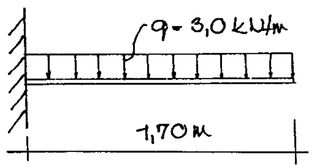
$$\tau_1 = \frac{23{,}2 \cdot 831{,}6}{6\cdot 17290} = 0{,}186\ \frac{kN}{cm^2}$$

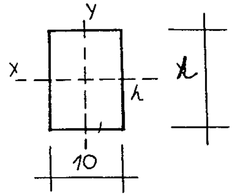
$$\tau_2 = \frac{23{,}2 \cdot 831{,}6}{18\cdot 17290} = 0{,}062\ \frac{kN}{cm^2}$$

6.2. ÖSSZETETT HAJLÍTÁS – NYÍRÓFESZÜLTSÉGEK

MILYEN MAGAS LEGYEN A NÉGYSZÖG KM-Ű FAGERENDA, HOGY $\sigma_H = 12\ \text{kN/mm}^2$ FESZÜLTSÉG ESETÉN MEGFELELJEN?

$\sigma_H = 1{,}2\ \text{kN/cm}^2$ $\quad \tau_H = 0{,}13\ \text{kN/cm}^2$





MÉRTÉKADÓ IGÉNYBEVÉTEL

$$M_{max} = \frac{q\,\ell^2}{2} = \frac{3{,}0\cdot 1{,}7^2}{2} = 4{,}34\ \text{kNm}$$

$$\sigma = \frac{M}{K_x} \rightarrow K_{x\,szüks} = \frac{M_{max}}{\sigma_H} = \frac{434}{1{,}2} = 361{,}67$$

$$K_x = \frac{J_x}{e_{max}} = \frac{10\cdot h^3}{12}\cdot\frac{1}{h/2} = 1{,}67\,h^2\ (\text{cm}^3)$$

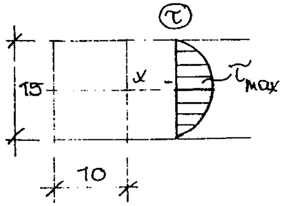
$$1{,}67\,h^2_{szüks} = \frac{434}{1{,}2}$$

$$h_{szüks} = \sqrt{\frac{434}{1{,}2\cdot 1{,}67}} = \sqrt{\frac{434}{2{,}00}} = \sqrt{217} = 14{,}72\ \text{cm}$$

$$14{,}7 \approx \underline{15\ \text{cm}}$$

MINIMUM 15 CM MAGAS KONZOL SZÜKSÉGES.

ELLENŐRIZZÜK A MEGTERVEZETT KM-T NYÍRÁSRA!



$$T_{max} = q\cdot\ell = 3{,}0\cdot 1{,}7 = 5{,}10\ \text{kN}$$

$$\tau_{max} = \frac{S\cdot T}{b\cdot J_x}$$

$$S = \frac{10\cdot 7{,}5^2}{2} = 281{,}25\ \text{cm}^3\ (281{,}25\cdot 10^{-6}\ \text{m}^3)$$

$$J_x = \frac{a\cdot b^3}{12} = \frac{10\cdot 15^3}{12} = 2812{,}5\ \text{cm}^4\ (2812{,}5\cdot 10^{-8}\ \text{m}^4)$$

$$\tau_{max} = \frac{281{,}25\cdot 5{,}1}{10\cdot 2812{,}5} = 0{,}051\ \text{kN/cm}^2 < \tau_H = 0{,}13\ \text{kN/cm}^2$$

A VÁLASZTOTT GERENDA NYÍRÁSRA IS MEGFELEL!

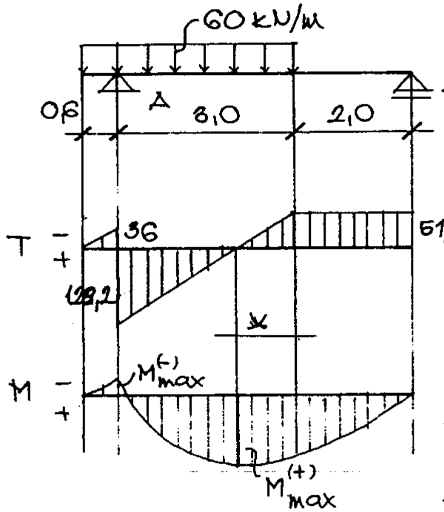
6.3. ÖSSZETETT HAJLÍTÁS – NYÍRÓFESZÜLTSÉGEK

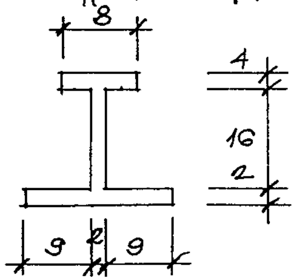
ÖSSZETETT HAJLÍTÁS, HAJLÍTÁSBÓL SZÁRMAZÓ NYÍRÓFESZÜLTSÉGEK

ELLENŐRIZZÜK A TARTÓT HAJLÍTÁSRA (RUGALMAS ÉS KÉPLÉKENY ALAPON) ILLETVE NYÍRÁSRA!

$\sigma_H = 1950\ kp/cm^2 = 19{,}5\ kN/cm^2$

$\tau_H = 1350\ kp/cm^2 = 13{,}5\ kN/cm^2$





1. REAKCIÓK

$$A = \frac{3{,}6 \cdot 60 \cdot 3{,}8}{6{,}0} = 164{,}16\ kN\ (\uparrow)$$

$$B = 3{,}6 \cdot 60 - 164{,}16 = 51{,}84\ kN\ (\uparrow)$$

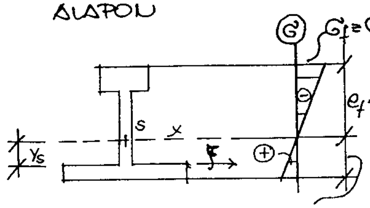
$$M_{max}^{(-)} = -60 \cdot 0{,}6 \cdot 0{,}3 = -10{,}8\ kNm$$

$$x = \frac{51{,}8}{60{,}0} = 0{,}863\ m$$

$$M_{max}^{(+)} = 51{,}8 \cdot 2{,}863 - 60 \cdot \frac{0{,}863^2}{2} = 126\ kNm$$

$M_M = 126\ kNm$; $T_M = 128{,}2\ kN$

2. ELLENŐRZÉS HAJLÍTÁSRA RUGALMAS ALAPON



$$y_s = \frac{8 \cdot 4 \cdot 18 + 2 \cdot 16 \cdot 8 - 20 \cdot 2 \cdot 1}{8 \cdot 4 + 2 \cdot 16 + 20 \cdot 2} = 7{,}62\ cm$$

$$J_x = J_\xi - F \cdot y_s^2 = \frac{1}{12} \cdot 8 \cdot 4^3 + 8 \cdot 4 \cdot 18^2 + \frac{1}{3} \cdot 2 \cdot 16^3 + \frac{1}{3} \cdot 20 \cdot 2^3 - 104 \cdot 7{,}62^2 =$$

$$J_x = 7160\ cm^4$$

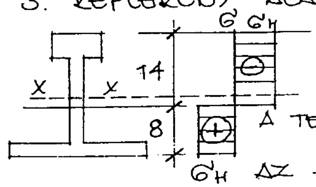
$$\boxed{M_{HR} = \sigma_H \cdot k_f}$$

$$M_{HR} = 195 \cdot \frac{7{,}16 \cdot 10^7}{12{,}38} = 112{,}78 \cdot 10^6\ Nmm = 112{,}78\ kNm$$

$M_{HR} < M_M = 126\ kNm$

RUGALMASAN A TARTÓ NEM FELEL MEG

3. KÉPLÉKENY ALAPON



$A = 104\ cm^2$

$A/2 = 52\ cm^2$

A TERÜLETFELEZŐ TENGELY HELYE AZ ALSÓ SZÁLTÓL 8 CM-RE VAN

$$M_{HK} = \sigma_H (|S_{NY}| + |S_H|) = 195 \cdot (800 \cdot 10^3) = 156 \cdot 10^6\ Nmm$$

$$= 156\ kNm > M_M = 126\ kNm$$

$$|S_{NY}| + |S_H| = 8 \cdot 4 \cdot 12 + 2 \cdot 10 \cdot 5 + 2 \cdot 6 \cdot 3 + 20 \cdot 2 \cdot 7 = 800\ cm^3$$

$$(= 8 \cdot 10^{-4}\ m^3)$$

KÉPLÉKENYEN MEGFELEL

4. ELLENŐRZÉS NYÍRÁSRA

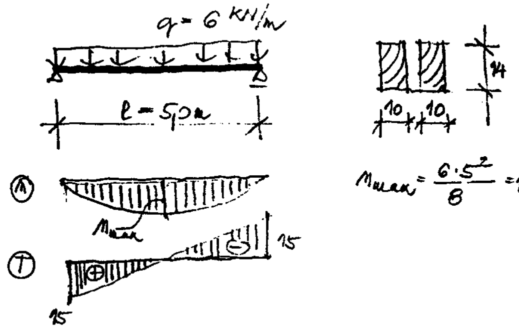
$$\tau_{MAX} = \frac{S\,T}{b\,J_x} = \frac{128{,}2 \cdot 4{,}02}{2 \cdot 7160} = 3{,}6\ kN/cm^2$$

A MAXIMUM HELYE A SÚLYPONTI TENGELY!

$$S = 8 \cdot 4 \cdot 10{,}38 + 2 \cdot 8{,}38 \cdot 4{,}19 = 4{,}02\ cm^3\ (4{,}02 \cdot 10^{-6}\ m^3)$$

$\tau_{max} < \tau_H = 13{,}5\ kN/cm^2$ MEGFELEL!

ÖSSZETETT HAJLÍTÁS



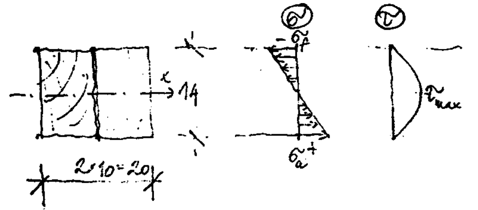
$\sigma_H = 1,3\ kN/cm^2$

$\tau_H = 0,17\ kN/cm^2$

$M_{max} = \frac{6 \cdot 5^2}{8} = 18,75\ kNm$

$\tau = 1,5 \frac{T}{b \cdot h}$

a)

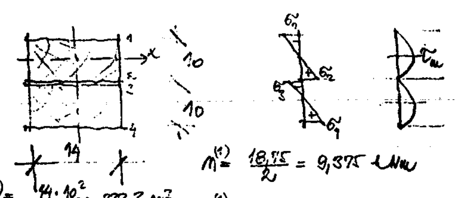


$\sigma_a = \sigma_f = \frac{M_x}{K_x} = \frac{1875}{653,3} = 2,87\ \frac{kN}{cm^2}$

$\tau_m = 1,5 \frac{15}{20 \cdot 14} = 0,08\ \frac{kN}{cm^2}$ ✓

$K_x = \frac{20 \cdot 14^2}{6} = 653,3\ cm^3$

b)



$\sigma_{1,4} = \frac{M^{(1)}}{K_x^{(1)}} = \frac{937,5}{233,3} = 4,01\ \frac{kN}{cm^2}$

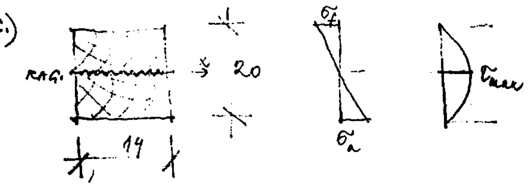
$\tau_m = 1,5 \frac{7,5}{10 \cdot 14} = 0,08\ \frac{kN}{cm^2}$

$K_x^{(1)} = \frac{14 \cdot 10^2}{6} = 233,3\ cm^3$

$M^{(1)} = \frac{18,75}{2} = 9,375\ kNm$

$T^{(1)} = 7,5\ kN$

c)



$\sigma_a = \sigma_f = \frac{1875}{933,3} = 2,01\ \frac{kN}{cm^2}$

$\tau_m = 1,5 \cdot \frac{15}{20 \cdot 14} = 0,08\ \frac{kN}{cm^2}$

$K_x = \frac{14 \cdot 20^2}{6} = 933,3\ cm^3$

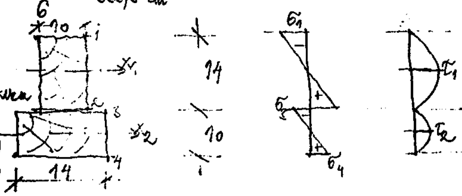
d)

$M_1 = 1241,55\ kNcm$

$M_2 = 633,45$

$T_1 = 9,93\ kN$

$T_2 = 5,07\ kN$



$I_{x_1} = \frac{10 \cdot 14^3}{12} = 2286,67$

$I_{x_2} = \frac{14 \cdot 10^3}{12} = 1166,67$

$\frac{M_1}{M_2} = \frac{T_1}{T_2} = 1,96$

$\boxed{I_{x_1} = 1,96\, I_{x_2}}$

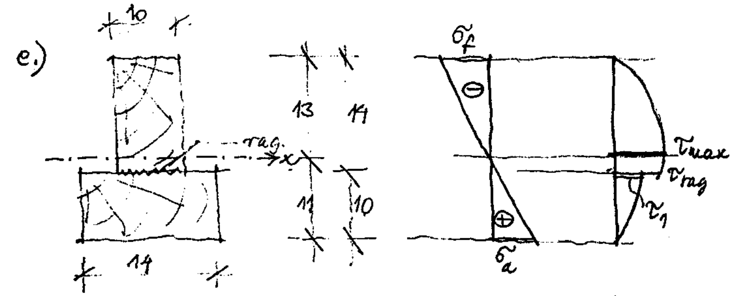
$\sigma_{1,2} = \frac{1241,55}{2286,67} \cdot 7 = 3,80\ \frac{kN}{cm^2}$

$\sigma_{3,4} = \frac{633,45}{1166,67} \cdot 5 = 2,71\ \frac{kN}{cm^2}$

$\tau_1 = 1,5 \cdot \frac{9,93}{140} = 0,$

$\tau_2 = 1,5 \cdot \frac{5,07}{140} = 0,$

e)



$I_x = 13\ 533\ cm^4$

$K_f = \frac{I_x}{13} = 1041\ cm^3$

$K_a = \frac{I_x}{11} = 1230\ cm^3$

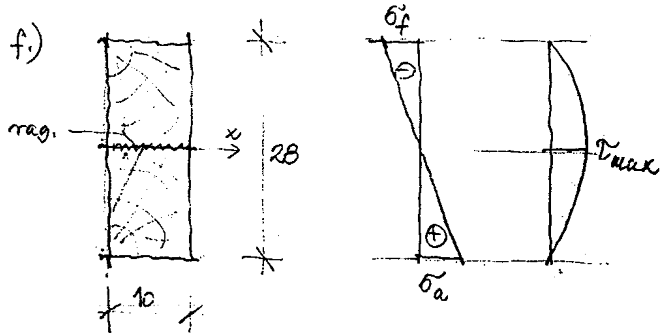
$\sigma_f = \frac{1875}{1041} = 1,8\ \frac{kN}{cm^2}$

$\sigma_a = \frac{1875}{1230} = 1,52\ \frac{kN}{cm^2}$

$\tau_{max} = \frac{13 \cdot 10 \cdot 6,5 \cdot 15}{10 \cdot 13533} = 0,094$

$\tau_r = \frac{10 \cdot 14 \cdot 6 \cdot 15}{10 \cdot 13533} = 0,09$

f)



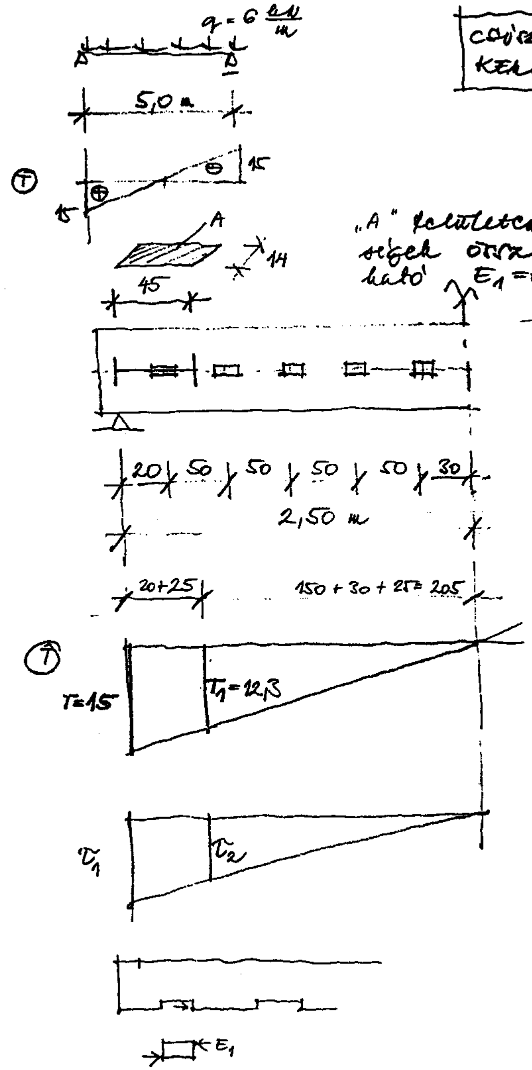
$K_x = \frac{10 \cdot 28^2}{6} = 1306,67\ cm^3$

$\sigma_{a,f} = \frac{1875}{1306,67} = 1,43\ \frac{kN}{cm^2}$

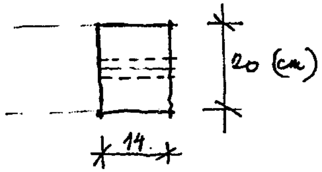
$\tau_{max} = 1,5 \cdot \frac{15}{280} = 0,08\ \frac{kN}{cm^2}$

(ugyanaz az kihajlás!)

CSÚSZTATÓERŐ AZ ELSŐ KEMÉNYFABETÉTEN



„A" felületen fellépő csúsztatófeszült-ségek összege adja az 1. betétre ható $E_1 = E_{max}$ csúsztatóerőt.



$$\frac{T_1}{15} = \frac{205}{250} \Rightarrow T_1 = 12{,}3 \text{ kN}$$

$$\tau = 1{,}5 \cdot \frac{T}{b \cdot h}$$

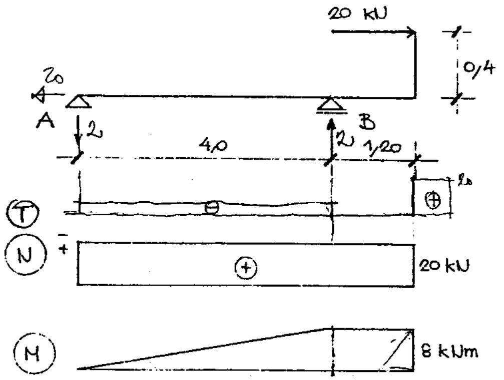
$$\tau_1 = \frac{3}{2} \cdot \frac{15}{280} = 0{,}080 \text{ kN/cm}^2$$

$$\tau_2 = \frac{3}{2} \cdot \frac{12{,}3}{280} = 0{,}066 \text{ —"—}$$

$$E_1 = 45 \cdot 14 \cdot \underset{0{,}073}{\frac{0{,}080 + 0{,}066}{2}} = \boxed{45{,}99} \text{ kN}$$

7.1. KÜLPONTOS HÚZÓ- ÉS NYOMÓIGÉNYBEVÉTEL

1) Milyen lesz a σ feszültségmegoszlás rugalmas állapotban az alábbi tartó „B" támasza fölött? I 120



táblázatból: $A = 14{,}2\ cm^2$

$K_x = 54{,}7\ cm^3$

$-\Sigma M_A = 0 = -48 + 0{,}4 \cdot 20$

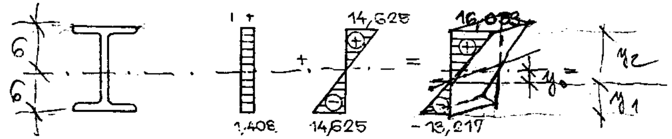
$\frac{8}{4} = 3 \Rightarrow A = 7$

$i_x^2 = \frac{328}{14{,}2} = 23{,}098$

$$\sigma = \frac{20}{14{,}2\ cm^2} \pm \frac{800}{54{,}7} = 1{,}408 \pm 14{,}625$$

$\sigma_1 = 16{,}033\ kN/cm^2$

$\sigma_2 = -13{,}217\ kN/cm^2$



$y_0 = 6 - y_1 = 0{,}58$

$(16{,}033 + 13{,}217) : 12 = 13{,}217 : y_1$

$\frac{158{,}604}{29{,}25} = y_1 = 5{,}42$

2.) Mekkora lehet az előbbi keresztmetszet alkalmazása esetén a 20 kN erő külpontossága, ha csak húzó-feszültségeket engedünk meg a tartóban?

E feltétel teljesüléséhez az erő döféspontjának a magidom határán kell lennie.

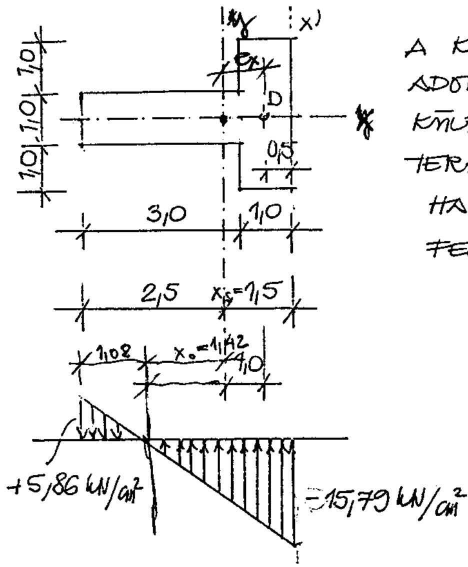
$J_x = 328\ cm^4$

$J_y = 21{,}5\ cm^4$

$A = 14{,}2\ cm^2$

7.2. KÜLPONTOS NYOMÁS (VAN HÚZÓSZIL.)

EGYSZERES KÜLPONTOSSÁG – RUGALMAS ÁLL.

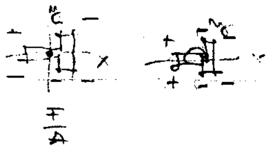


A KERESZTMETSZETET AZ ADOTT PONTBAN (MÉRTÉKADÓ KÜLPONTOSSÁG) 46 kN ERŐ TERHELI
HATÁROZZUK MEG A σ FESZÜLTSÉGI ÉRTÉKEKET!

$$i_y^2 = \frac{J_y}{A} = \frac{8{,}5}{6} = 1{,}42$$

$$x_0 = \frac{i_y^2}{e_x} = \frac{\frac{8{,}5}{6}}{1{,}0} = 1{,}42$$

KERESZTMETSZETI ADATOK:



$$A = 1 \cdot 3 + 3 \cdot 1 = 6\ cm^2$$

$$x_S = \frac{3 \cdot 0{,}5 + 3 \cdot 2{,}5}{6} = 1{,}50\ cm$$

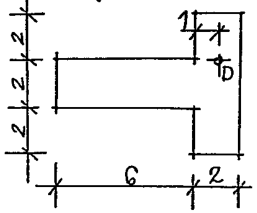
$$J_y = \frac{1 \cdot 3^3}{3} + \frac{3 \cdot 1^3}{3} - 6 \cdot 0{,}5^2 = 10 - 1{,}5 = 8{,}5\ cm^4$$

$$\sigma_j = -\frac{46}{6} - \frac{46 \cdot 1}{8{,}5} \cdot 1{,}5 = -7{,}67 - 8{,}12 = \underline{\underline{-15{,}79\ kN/cm^2}}$$

$$\sigma_b = -\frac{46}{6} + \frac{46 \cdot 1}{8{,}5} \cdot 2{,}5 = -7{,}67 + 13{,}53 = \underline{\underline{+5{,}86\ kN/cm^2}}$$

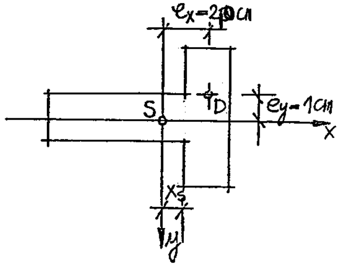
7.3. KÜLPONTOS NYOMÁS (VAN HÚZÓSZIL.)

KÉTSZERES KÜLPONTOSSÁG – RUGALMAS ÁLL.



A MEGJELÖLT „D" DÖFÉSPONTBAN $N_H = 110$ kN NYOMÓERŐ MŰKÖDIK.

HATÁROZZUK MEG A σ ÉRTÉKEKET!



KERESZTMETSZETI ADATOK.

$A = 2\cdot 6\cdot 2 = 24\ cm^2$

$X_S = \dfrac{2\cdot 6\cdot 3 - 2\cdot 6\cdot 1}{24} = 1{,}00$ cm

$J_x = \dfrac{6\cdot 2^3}{12} + \dfrac{2\cdot 6^3}{12} = 40\ cm^4$

$J_y = \dfrac{2\cdot 6^3}{3} + \dfrac{6\cdot 2^3}{3} - 24\cdot 1^2 =$

$= 136\ cm^4$

A SEMLEGES TENGELY ÉS A KOORDINÁTA TENGELYEK METSZÉSPONTJAI:

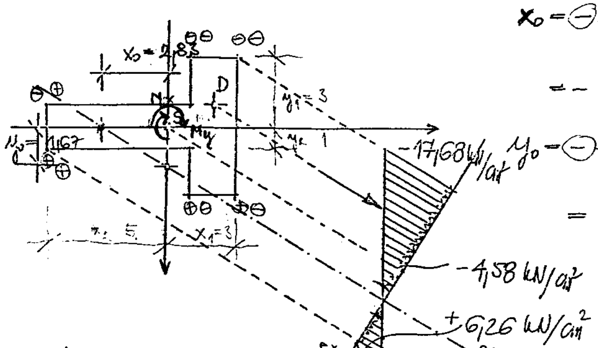
$i^2 = \dfrac{J}{A}$

$x_0 = \ominus \dfrac{i_y^2}{e_x} = -\dfrac{136}{2\cdot 24} =$

$= -2{,}83$ cm

$y_0 = \ominus \dfrac{i_x^2}{e_y} = -\dfrac{40}{24(-1)} =$

$= +1{,}67$ cm



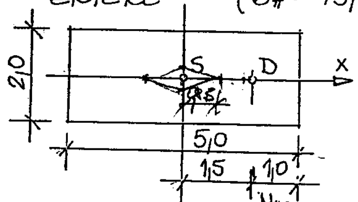
$-\sigma_{max} = -\dfrac{110}{24} - \dfrac{110\cdot 2}{136}\cdot 3 - \dfrac{110\cdot 1}{40}\cdot 3 = -4{,}58 - 4{,}58 - 8{,}25 = -17{,}6$

$+\sigma_{max} = -\dfrac{110}{24} + \dfrac{110\cdot 2}{136}\cdot 5 + \dfrac{110\cdot 1}{40}\cdot 1 = -4{,}58 + 8{,}09 + 2{,}75 = +6{,}26$

7.4. KÜLPONTOS NYOMÁS (VAN HÚZÓSZIL.)

MEKKORA A „D" DÖFÉSPONTBAN MŰKÖDŐ HÚZÓ ERŐ N_{HR} RUGALMAS ÉRTÉKE? ($\sigma_H = 19{,}5$ kN/cm²)

$\frac{b}{6} = 0{,}83$, $\frac{h}{6} = 0{,}33$

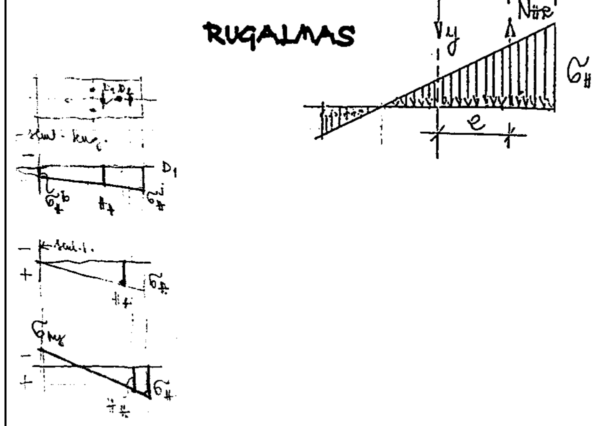


KERESZTMETSZETI ADATOK

$A = 2\cdot 5 = 10\ cm^2$

$K_y = \dfrac{2\cdot 5^2}{6} = 8{,}33\ cm^3$

RUGALMAS



$\sigma = \sigma_H = \dfrac{F}{A} + \dfrac{F\cdot e}{K_y} \rightarrow N_{HR}$

$N_{HR} = \dfrac{\sigma_H}{\dfrac{1}{A} + \dfrac{e}{K_y}} =$

$= \dfrac{19{,}5}{\dfrac{1}{10} + \dfrac{1{,}5}{8{,}33}} =$

$= 19{,}5/(0{,}1 + 0{,}18) = 69{,}6$

ELLENŐRIZZÜK A σ ÉRTÉKEKET A SZÉLSŐ SZÁLAKBAN

$\sigma_1 = +\dfrac{69{,}64}{10} + \dfrac{69{,}64\cdot 1{,}5}{8{,}33} = 6{,}96 + 12{,}54 = 19{,}5$ kN/cm²

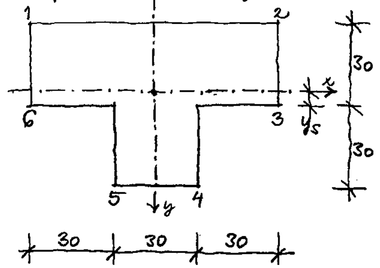
$\sigma_2 = +6{,}96 - 12{,}54 = -5{,}58$ kN/cm²

7.5. KÜLPONTOS IGÉNYBEVÉTEL

MAGIDOM MEGHATÁROZÁSA

→ síkidomot érintő, de nem metsző egyenesek antipólusait összekötő egyenesek határolják

$$X = -\frac{(i_y)^2}{x_0} \qquad y = -\frac{(i_x)^2}{y_0}$$



Határozzuk meg a keresztmetszet magidomának koordinátáit!

$y_s = 7{,}5$ cm

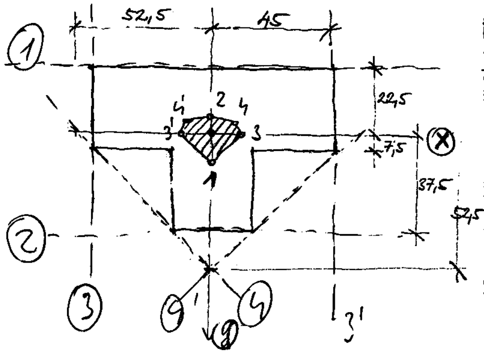
$$I_x = \frac{90\cdot 30^3}{3} + \frac{30\cdot 30^3}{3} - 3600\cdot 7{,}5^2 = 878\,000\ cm^4$$

$$I_y = \frac{30\cdot 90^3}{12} + \frac{30\cdot 30^3}{12} = 1\,890\,000\ cm^4$$

$$i_x^2 = \frac{878\,000}{3600} = 244\ cm^2 \qquad i_y^2 = \frac{1\,890\,000}{3600} = 525\ cm^2$$

$$\boxed{x_0\cdot\xi = -i_y^2} \qquad \boxed{y_0\cdot\eta = -i_x^2}$$

A keresztmetszetet burkoló egyeneseket sorra semleges tengelyeknek tekintve meghatározzuk a hozzájuk tartozó döféspontokat!

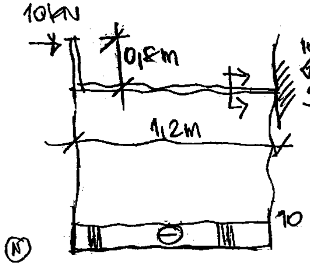


SEML. TENG.	MAGIDOM HATÁRPONTJA (x)	(y)
1	$x_1 = 0$	$y_1 = \frac{244}{22{,}5}$
2	$x_2 = 0$	$y_2 = \frac{244}{37{,}5}$
3	$x_3 = \frac{525}{45}$	$y_3 = 0$
4	$x_4 = \frac{525}{52{,}5}$	$y_4 = \frac{244}{52{,}5}$

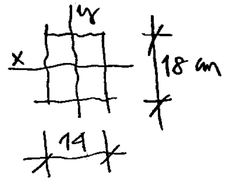
$$x_A = -\frac{(i_y)^2}{n} \qquad y_A = -\frac{(i_x)^2}{m}$$

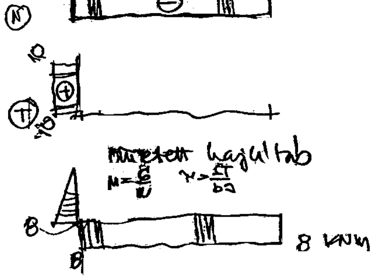
$$X = \frac{-(i_y)^2}{x_0} \qquad y = \frac{-(i_x)^2}{y_0}$$

$\sigma_H = 1{,}5$ kN/m² Megfelelő teherbírású-e a tartó vízszintes korona?



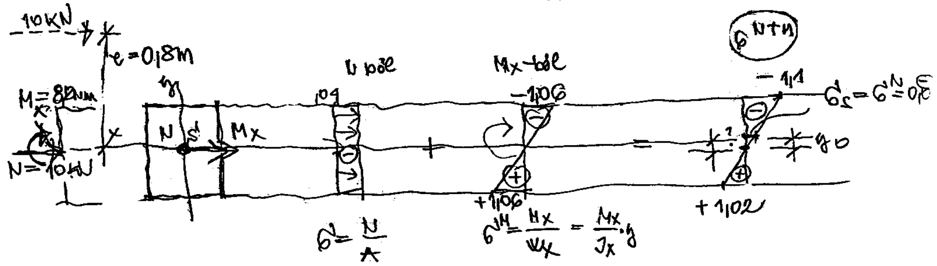
$10\cdot 0{,}8 = 8$ kNm





normálerő + hajlítás } ⇒ külpontos nyomás összetett igénybevétel

$N \Rightarrow \sigma$
$M \Rightarrow \sigma$ } $\sigma = ?$



$$\sigma^N = \frac{N}{A} \qquad \sigma^M = \frac{M_x}{W_x} = \frac{M_x}{I_x}\cdot y$$

$$-\sigma^N = \frac{10\ kN}{14\cdot 18\ cm^2} = 0{,}04\ kN/m^2$$

$$\pm\sigma^M = \frac{800\ kNcm}{\frac{14\cdot 18^2}{6}} = 1{,}06\ kN/m^2$$

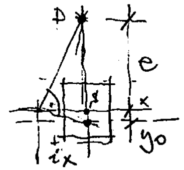
$$\pm\sigma = -\frac{N}{A} \pm \frac{M_x}{W_x}$$

előjeles összegzés

y_0 = semleges tengely helye

e = külpontosság mértéke

$$e = \frac{M_x}{N} = \frac{8{,}0\ kNm}{10\ kN} = 0{,}8\ m$$



$$\sigma = 0 = -\frac{N}{A} \pm \frac{M_x}{I_x}\cdot y_0 \quad \text{ahol } M_x = e\cdot N$$

$$0 = -\frac{N}{A} + \frac{N\cdot e}{I_x}\cdot y_0 \quad /:N$$

$$0 = -\frac{1}{A} + \frac{e\cdot y_0}{I_x} \qquad \frac{1}{A} = \frac{e\cdot y_0}{I_x} \qquad \frac{I_x}{A} = i_x^2 = e\cdot y_0 \qquad i_x^2 = e\cdot y_0$$