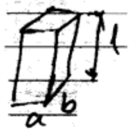
Központosan nyomott karcsú szerkezetek

Zömök rudak: $\sigma = \frac{-F}{A}$ $\frac{kN}{cm^2}$ (−)

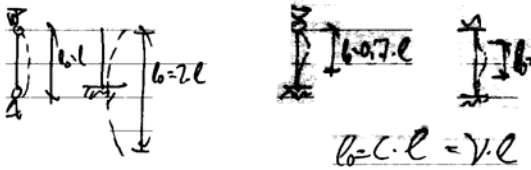


$l/_{a,b} \leq 5$

Karcsú szerkezet: $l \gg a,b$

$F_{krit} = \frac{\pi^2 E J}{l_0^2}$ Euler $\sigma_{krit} = \frac{\pi^2 E J_{min}}{l_0^2 A}$; $\frac{J_{min}}{A} = i_{min}^2$ [cm²]

$\lambda_{max} = \frac{l_0}{i_{min}}$ karcsúsági tényező $\rightarrow \sigma_{krit} = \frac{\pi^2 E}{\lambda_{max}^2}$, $\sigma_{krit} \leq \sigma_A$



→ félhullám hossz

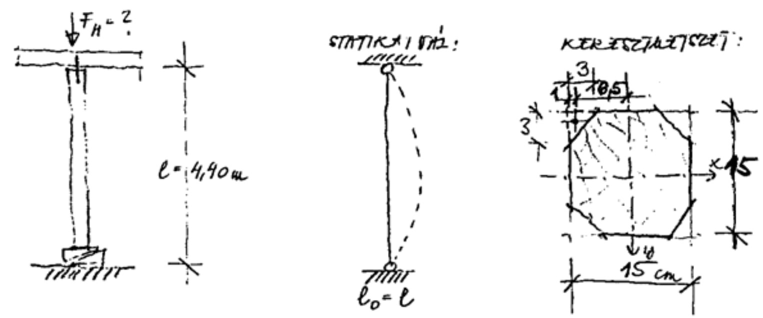
$l_0 = C \cdot l = \nu \cdot l$

Lépések:
- l_x, l_y
- i_x, i_y
- $\rightarrow \lambda_x, \lambda_y \rightarrow \lambda_{max}$
- $\rightarrow \varphi \rightarrow N_H$

méretezés: $N_H = \varphi \cdot A \cdot \sigma_{HnyH}$ $\varphi \leq 1$ $\varphi_T = f(\lambda)$

$\lambda_x = \frac{l_{0x}}{i_x}$; $\lambda_y = \frac{l_{0y}}{i_y}$

2/1 | KÖZPONTOS NYOMÁS (FA)



F. 56. II. OSZT. FENYŐFA

$\sigma_{Hny} = 1{,}425$ kN/cm² (módosítótényezőkkel felszorzott érték!)

$E = 1200$ kN/cm²

a.) $F_H = ?$ (szabvány szerint)

$A = 15^2 - 4 \cdot 3^2 \cdot \frac{1}{2} = 207$ cm²

$J_{min} = \frac{15^4}{12} - 4\left(\frac{3^4}{36} + 4{,}5 \cdot 6{,}5^2\right) = 3450$ cm⁴ (bármely súlyponti tengelyre azonos az inercia érték!)

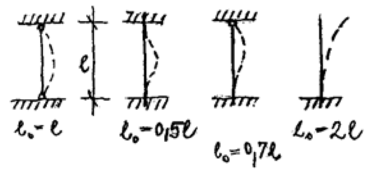
$i_{min} = \sqrt{\frac{J_{min}}{A}} = \sqrt{\frac{3450}{207}} = 4{,}09$ cm

$\lambda_{max} = \frac{l_0}{i_{min}} = \frac{440}{4{,}09} = 107{,}6$

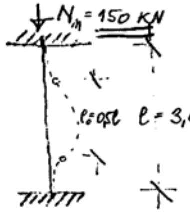
λ	φ
105	0,286
110	0,265

a.) $\boxed{\varphi} = 0{,}286 - \frac{0{,}286 - 0{,}265}{5} \cdot 2{,}6 = \boxed{0{,}275}$

$\boxed{F_H} = \varphi \cdot A \cdot \sigma_{Hny} = 0{,}275 \cdot 207 \cdot 1{,}425 = \boxed{81{,}12 \text{ kN}}$



2/2. KÖZPONTOS NYOMÁS (FA)



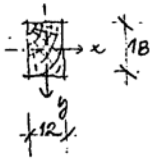
$5 : 0,045 = 1,96 : X$

K 68 II. TÖLGYFA

$\sigma_{Hny}'' = 1,73$ kN/cm²

$E = 1400$ kN/cm²

Ellenőrizzük a fa oszlopot! (ábra szerint)



$A = 12 \cdot 18 = 216$ cm²

$I_{min} = I_y = \frac{18 \cdot 12^3}{12} = 2592$ cm⁴

$i_y = \sqrt{\frac{I_y}{A}} = 3,46$ cm

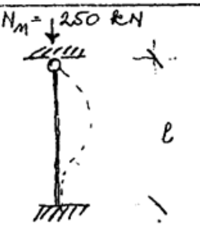
$\lambda_{max} = \frac{l_0}{i_y} = \frac{180}{3,46} = 52,02$

λ	φ
50	0,694
55	0,649

$\varphi = 0,694 - \frac{0,694 - 0,649}{5} \cdot 2,02 = 0,676$

$\boxed{F_H} = \varphi \cdot A \cdot \sigma_{Hny} = 0,676 \cdot 216 \cdot 1,73 = \boxed{252,6 \text{ kN}} > N_M = 150$ kN

megfelel ✓



F 62 I. oszt. FENYŐFA

$\sigma_{Hny}'' = 1,84$ kN/cm²

$l_{max} = ?$ (szabv. szerint)

$l_0 = 0,7 \; l$ $\qquad I_x = \frac{2 \cdot 12 \cdot 30^3}{36} = 18.000$ cm⁴

$I_{min} = I_y = 2 \cdot \frac{30 \cdot 12^3}{12} = \underline{8640 \text{ cm}^4}$

$i_y = \sqrt{\frac{8640 \cdot 2}{24 \cdot 30}} = 4,89$ cm

$A = 360$ cm²

$N_M = F_H = \varphi \cdot A \cdot \sigma_{Hny}$

$\varphi_{min} = \frac{N_M}{A \cdot \sigma_{Hny}} = \frac{250}{360 \cdot 1,84} = 0,377$

$\boxed{\lambda_{max} = 88,28}$

λ	φ
85	0,398
90	0,366

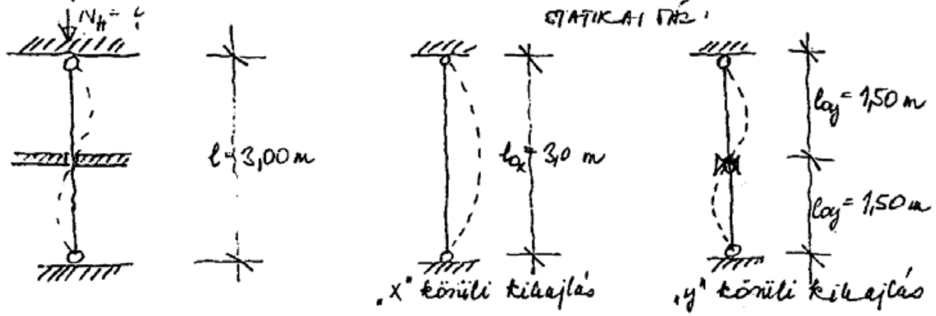
$\frac{0,032}{5} = 0,0064$

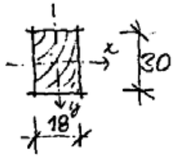
$0,398 - 0,377 = 0,021 : 0,0064 = 3,28 + 85$

$\lambda = \frac{l_0}{i_y} \rightarrow l_0 = \underline{88,28} \cdot 4,89 = \underline{\underline{431,69 \text{ cm}}}$

$l_0 = 0,7 \cdot l_{max} \rightarrow \quad l_{max} = \frac{l_0}{0,7} = \frac{431,69}{0,7} = \underline{\underline{616,7 \text{ cm}}}$

2/4. KÖZPONTOS NYOMÁS (FA)





F 56 II. o. FENYŐ

$\sigma_{Hny}'' = 1,43$ kN/cm²

$F_H = ?$ (szabv. szerint)

$A = 18 \cdot 30 = 540$ cm²

$I_x = \frac{18 \cdot 30^3}{12} = 40\,500$ cm⁴

$i_x = \sqrt{\frac{I_x}{A}} = 8,66$ cm

$\lambda_x = \frac{l_{ox}}{i_x} = \frac{300}{8,66} = \boxed{34,64 = \lambda_{max}}$

$I_y = \frac{30 \cdot 18^3}{12} = 14580$ cm⁴

$i_y = \sqrt{\frac{I_y}{A}} = 5,19$ cm

$\lambda_y = \frac{l_{oy}}{i_y} = \frac{150}{5,19} = 28,9$

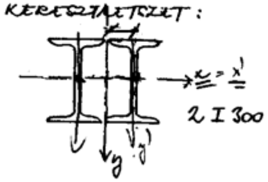
λ_{max} →	φ
30	0,844
35	0,811

$\varphi = 0,844 - \frac{0,844 - 0,811}{5} \cdot 4,64 = 0,813$

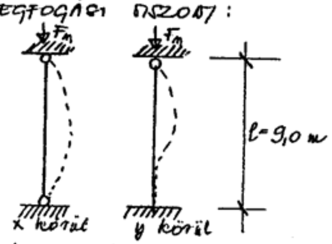
$\boxed{F_H} = 0,813 \cdot 540 \cdot 1,43 = \boxed{628 \text{ kN}}$

2/5 | KÖZPONTOS NYOMÁS (ACÉL)

KERESZTMETSZET:



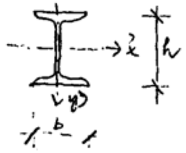
MEGFOGÁSI VISZONY:



$F_m = 1200$ kN

a.) Ellenőrizzük az acélpillért! ($\sigma_H = \underline{24\ kN/cm^2}$)

1 db I szelvény szükséges adatai:



$h = 300$ mm $\qquad I'_x = 9800\ cm^4 \qquad \boxed{i'_x = 11,9\ cm}$

$b = 125$ mm $\qquad I'_y = 451\ cm^4 \qquad i'_y = 2,56\ cm$

$A' = 69,1\ cm^2$

$$F_H = \varphi \cdot A \cdot \sigma_H$$

x:
$$\lambda_x = \frac{l_{ox}}{i_x} = \frac{900}{11,9} = 75,63$$
$$l_{ox} = l = 900\ cm$$
$$i_x = \sqrt{\frac{I_x}{A}} = \sqrt{\frac{\cancel{2} \cdot I'_x}{\cancel{2} \cdot A'}} = i'_x = \underline{\underline{11,9\ cm}}$$

/$I_x = 19600\ cm^4$/

$90 \rightarrow 0,545$
$95 \rightarrow 0,509$

$\varphi = \underline{\underline{0,521}}$

y:
$$\lambda_y = \frac{l_{oy}}{i_y} = \frac{630}{6,75} = \boxed{93,3} = \lambda_{max}$$
$$l_{oy} = 0,7\ l = 630\ cm$$
$$i_y = \sqrt{\frac{I_y}{A}} = \sqrt{\frac{\cancel{2} \cdot \left[451 + 69,1 \cdot \left(\frac{12,5}{2}\right)^2\right]}{\cancel{2} \cdot 69,1}} = \underline{\underline{6,75\ cm}}$$

$\lambda_{max} \rightarrow$

$$F_H = 0,521 \cdot \underbrace{2 \cdot 69,1}_{} \cdot 24 = \boxed{1728,05\ kN} > F_m = \underline{1200\ kN}\ \text{megfelel} \checkmark$$

b.) Mekkora az oszlop Euler szerinti kritikus ereje? ($\sigma_{aH} = 29,5\ kN/cm^2$)

A kritikus erőre vonatk. összefüggés akkor érvényes, ha

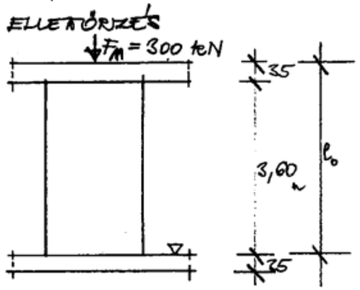
$$\sigma_{KR} \leq \sigma_{aH} \qquad \sigma_{KR} = \frac{\pi^2 \cdot E}{\lambda_y^2} = \frac{3,14^2 \cdot 20600}{93,3^2} = 23,36 < 29,5\ kN/cm^2$$

$$F_{KR} = \frac{\pi^2 \cdot E \cdot I_y}{l_{oy}^2} = \frac{3,14^2 \cdot 20600 \cdot 6300,4}{630^2} = 3227,4\ kN$$

/$F_{aH} = 2 \cdot 69,1 \cdot 29,5 = 4077\ kN$/

2/6 | KÖZPONTOS NYOMÁS (FALAZOTT SZERKEZET)

a.) ELLENŐRZÉS



Két végén rögzített megtámasztású téglapillér

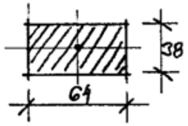
ANYAGA: kisméretű tégla, II. oszt. falazat

T 140 ($\sigma_{at} = 1,4\ kN/cm^2$)

H 25 ($\sigma_{ah} = 0,10\ kN/cm^2$)

$\sigma_f = 0,13\ kN/cm^2$

$\sigma_{fH} = \overset{(pillér)}{0,85} \cdot \overset{(II.o)}{1} \cdot 0,13 = \boxed{0,11\ kN/cm^2}$



$A = 64 \cdot 38 = \boxed{2432\ cm^2}$

$l_0 = 3,60 + 0,35 = \boxed{3,95\ m}$

$$\varphi = 0,88 - \frac{l_0}{150h} - 2\left(\frac{l_0}{50h}\right)^2$$

$$\varphi = 0,88 - \frac{395}{150 \cdot 38} - 2\left(\frac{395}{50 \cdot 38}\right)^2 = \boxed{0,724}$$

$$F_H = \varphi \cdot A \cdot \sigma_{fH} = 0,724 \cdot 2432 \cdot 0,11 = 193,68\ kN < F_m = 300 \quad \text{nem felel meg}$$

b.) TERVEZÉS

Határozzuk meg a falazat minőségét az F_m erőhöz I. oszt. falazat esetében!

$$\sigma_{fH\,szüks} = \frac{F_m}{\varphi \cdot A} = \frac{300}{0,724 \cdot 2432} = 0,17\ kN/cm^2$$

$$\sigma_{fH} = 0,85 \cdot 1,15 \cdot \sigma_f \quad \rightarrow \quad \sigma_f = \frac{0,17}{0,85 \cdot 1,15} = 0,174\ kN/cm^2$$

$\downarrow$

$\boxed{TF\ 18}$ tömör tégla
I. o. falazat

Ennek megfelel: pl. pillértégla ($\sigma_{af} = 2\ kN/cm^2$)

$\sigma_{ah} = 0,30\ kN/cm^2$ szil. habarcs

így $\underline{\sigma_f = 0,21\ kN/cm^2}$

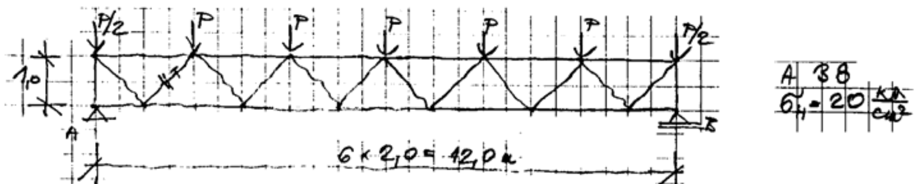
vagy: nagyszilárdságú tömör tégla

$\sigma_{af} = 1,4\ kN/cm^2$

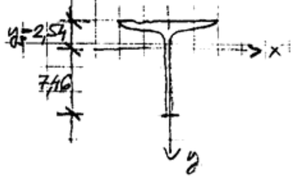
$\sigma_{ah} = 0,50\ kN/cm^2$ szil. habarcs

így $\underline{\sigma_f = 0,18\ kN/cm^2}$

2/7 KÖZPONTOS NYOMÁS (ACÉL)



A 38
$\sigma_H = 20 \frac{kN}{cm^2}$



$\frac{1}{2}$ I 200

$\boxed{P_{max} = ?}$ a jelölt rúd teherbírása alapján

$A_I = 33,5\ cm^2$ (tábl.)

$A_{\frac{1}{2}I} = 16,75\ cm^2$

$y_s = \frac{S_x}{A_{\frac{1}{2}}} = \frac{125}{16,75} = 7,46\ cm$

$I_x = \frac{2140}{2} - 16,75 \cdot 7,46^2 = 137,8\ cm^4$

$i_x = \sqrt{\frac{I_x}{A_{\frac{1}{2}}}} = \sqrt{\frac{137,8}{16,75}} = 2,87\ cm$

$i_y = 1,87\ cm$ (tábl.)

$l = 1,414\ m$

$l_{ox} = 0,8\ l = 113,1\ cm$ (rácsos tartó működése)

$l_{oy} = l = 141,4\ cm$ (r. t. működése ...)

$\lambda_x = \frac{113,1}{2,87} = 39,4$

$\lambda_y = \frac{141,4}{1,87} = 75,6 = \lambda_{max}$

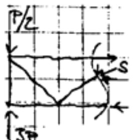
75 → 0,658
80 → 0,625

$\varphi = 0,654$

$N_H = \varphi \cdot A_{\frac{1}{2}} \cdot \sigma_H = 0,654 \cdot 16,75 \cdot 20 = 219,1\ kN$

$N_m = S = 2,5\ P \cdot \sqrt{2}$

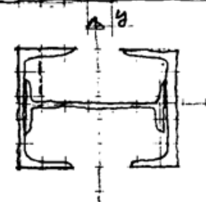
$P = \frac{N_H}{2,5\sqrt{2}} = \frac{219,1}{2,5\sqrt{2}} = 61,98\ kN$



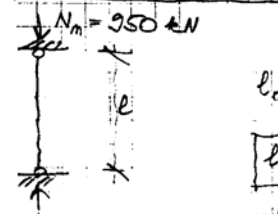
$S_y = 2,5P$

$S = 2,5P \cdot \sqrt{2}$

2/8 KÖZPONTOS NYOMÁS (ACÉL)



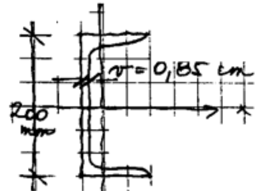
2 U 200
I 160



$l_{ox} = l_{oy} = l$

$\boxed{l_{max} = ?}$

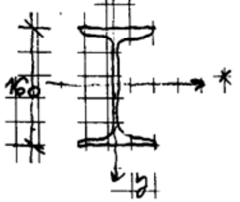
$\sigma_H' = 24\ kN/cm^2$ (A.45 ál. any.)



$I_x = 1910\ cm^4$

$I_y = 148\ cm^4$

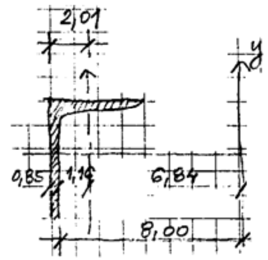
$A = 32,2\ cm^2$



$I_x = 935\ cm^4$

$I_y = 54,7\ cm^4$

$A = 22,8\ cm^2$



$A = 22,8 + 2 \cdot 32,2 = 87,2\ cm^2$

$I_x = 54,7 + 2 \cdot 1910 = 3874,7\ cm^4$

$I_y = 935 + 2\,(148 + 32,2 \cdot 6,84^2) = 4244\ cm^4$

$i_x = \sqrt{\frac{3874,7}{87,2}} = 6,67\ cm = i_{min}$

$i_y = \sqrt{\frac{4244}{87,2}} = 6,97\ cm$

φ	λ
0,425	115
0,457	110

$\lambda_{max} = \frac{l_o}{i_{min}}$

$l_o = \lambda_{max} \cdot i_{min}$

$l_o = 110,47 \cdot 6,67 = 736,8\ cm = 7,4\ m$

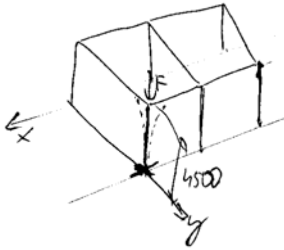
$N_m = \varphi \cdot A \cdot \sigma_H'$

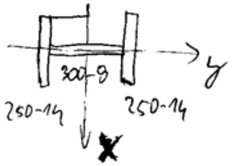
$\varphi = \frac{950}{87,2 \cdot 24} = \boxed{0,454}$

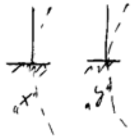
$\lambda = 110,47$

Nyomott rúdak (acél)

$F_{max} = ?$







$\nu_x = \nu_y = 2$

$\sigma_H = 20 \frac{kN}{cm^2}$

$A = 30 \cdot 0{,}8 + 2 \cdot 25 \cdot 1{,}4 = \underline{\underline{94\ cm^2}}$

$I_x = \left(\frac{25 \cdot 1{,}4^3}{12} + 25 \cdot 1{,}4 \cdot 15{,}7^2\right) \cdot 2 + \frac{0{,}8 \cdot 30^3}{12} = \underline{\underline{19066\ cm^4}}$

$I_y = \frac{1{,}4 \cdot 25^3}{2} \cdot 2 + \frac{30 \cdot 0{,}8^3}{12} = \underline{\underline{3647\ cm^4}}$

$i_x = \sqrt{\frac{19066}{94}} = \underline{\underline{14{,}24\ cm}};$

$i_y = \sqrt{\frac{3647}{94}} = \underline{\underline{6{,}23\ cm}}$

$l_{ox} = l_{oy} = \nu \cdot l = 2 \cdot 450 = \underline{\underline{900 cm}}$

$\lambda_{max} = \frac{l_0}{i_{min}} = \frac{900}{6{,}23} = \underline{\underline{144{,}5}} \rightarrow$

λ	φ
140	0,313
145	0,296

$\rightarrow \varphi = 0{,}313 + \frac{0{,}296 - 0{,}313}{5} \cdot 4{,}5 = \underline{\underline{0{,}2977}}$

$F_{max} = F_{KH} = \varphi \cdot A \cdot \sigma_H = 0{,}2977 \cdot 94 \cdot 20 = \underline{\underline{559{,}67\ kN}}$

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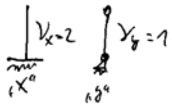
b.)



$\lambda_x = \frac{l_{ox}}{i_x} = \frac{\nu_x \cdot l}{i_x} = \frac{2 \cdot 450}{14{,}24} = \underline{\underline{63{,}2}}$

$\lambda_y = \frac{l_{oy}}{i_y} = \frac{\nu_y \cdot l}{i_y} = \frac{1 \cdot 450}{6{,}23} = \underline{\underline{72{,}2}}$

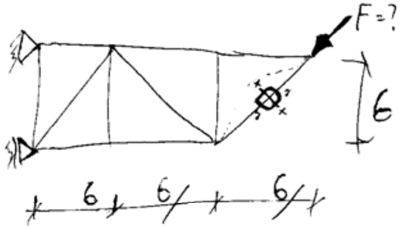
$\lambda_{max} = \underline{\underline{72{,}2}} \rightarrow$

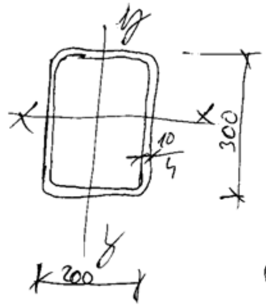
λ	φ
70	0,692
75	0,658

$\varphi = 0{,}692 + \frac{0{,}658 - 0{,}692}{75 - 70} \cdot (72{,}2 - 70) = \underline{\underline{0{,}677}}$

$F_{KH} = 0{,}677 \cdot 94 \cdot 20 = \underline{\underline{1272{,}7\ kN}}$

Nyomott rudak (acél)





$A = 93{,}4\ cm^2$

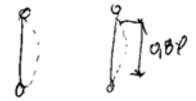
$I_x = 11507\ cm^4$

$I_y = 6144\ cm^4$

$i_x = 11\ cm$

$i_y = 8{,}11\ cm$

$\sigma_H = 24 \frac{kN}{cm^2}$



$\nu_x = 1 \quad \nu_y = 0{,}8$

$\lambda_x = \frac{\nu_x \cdot l}{i_x} = \frac{\sqrt{2}\,600}{11} = \underline{\underline{77{,}1}}$

$\lambda_y = \frac{\nu_y \cdot l}{i_y} = \frac{0{,}8 \cdot \sqrt{2}\,600}{8{,}11/1{,}67} = \underline{\underline{83{,}7}}$

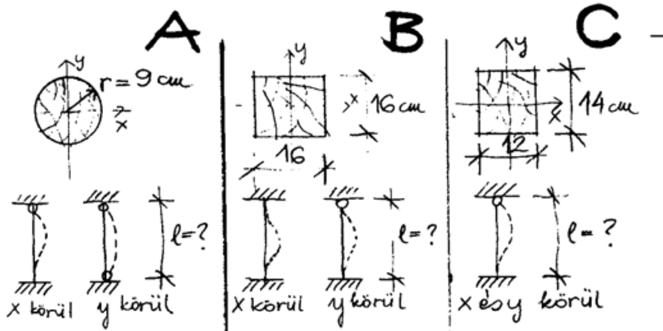
$\lambda_{max} = 83{,}7 \rightarrow$

λ	φ
80	0,691
85	0,649

$\varphi = 0{,}691 + \frac{0{,}649 - 0{,}691}{5} \cdot 3{,}7 = \underline{\underline{0{,}6599}}$

$F_{KH} = \varphi \cdot A \cdot \sigma_H = 0{,}6599 \cdot 93{,}4 \cdot 24 = \underline{\underline{1479\ kN}}$

MEKKORA LEHET AZ OSZLOP MAX. HOSSZA? (SZABV. SZERINT)



A	B	C
$N_M = 280$ kN	$N_M = 330$ kN	$N_M = 235$ kN
F56 I. o. fenyő	F56 II. o. fenyő	F62 I. o. fenyő
$\sigma_{Hny} = 2{,}1$ kN/cm²	$\sigma_{Hny} = 1{,}81$ kN/cm²	$\sigma_{Hny} = 2{,}33\ \frac{kN}{cm^2}$

λ	φ
35	0,811
40	0,775
45	0,736
50	0,694
55	0,649
60	0,604
65	0,558
70	0,514
75	0,472
80	0,434
85	0,398
90	0,366
95	0,336

A

$i = \frac{r}{2} = 4{,}5$ cm $\quad A = 254{,}47\ cm^2$

$\varphi = \frac{N}{\sigma \cdot A} = \frac{280}{2{,}1 \cdot 254{,}47} = 0{,}524$

λ	φ
65	0,558
70	0,514

$\lambda = 68{,}86$

$\ell_0 = \lambda \cdot i = 68{,}86 \cdot 4{,}5$

$\ell_0 = 309{,}88$ cm $\quad \ell = \ell_0 = 3{,}09$ m

B

$i = \frac{16}{\sqrt{12}} = 4{,}62$ cm

$\varphi = \frac{330}{1{,}81 \cdot 256} = 0{,}712 \rightarrow \lambda = 47{,}86$

$\ell_0 = 47{,}86 \cdot 4{,}62 = 221{,}11$ cm

$\ell_{0\,max} = 0{,}7 \cdot \ell \quad \ell = 315{,}87$ cm

$\ell = 3{,}15$ m

C

$i_{min} = \frac{12}{\sqrt{12}} = 3{,}46$ cm

$\varphi = \frac{235}{2{,}33 \cdot 168} = 0{,}600 \rightarrow \lambda = 60{,}43$

$\ell_0 = 60{,}43 \cdot 3{,}46 = 209{,}09$ cm

$\ell_0 = 0{,}7 \cdot \ell \qquad \ell = 298{,}69$ cm

$\ell = 2{,}98$ m