VIRTUÁLIS ELMOZDULÁSOK TÉTELE (VIRT. MUNKA TÉTELE)

EGY STATIKAILAG LEHETSÉGES ERŐRENDSZER BÁRMELY VIRTUÁLIS ELMOZDULÁSRENDSZEREN VÉGZETT MUNKÁJA ZÉRUS

[→ EGYENSÚLYBAN LÉVŐ MECHANIKAI RENDSZEREN VÉGZETT VIRTUÁLIS MUNKÁK ÖSSZEGE ZÉRUS]

$$\delta W = \delta W_e + \delta W_b = 0$$

EZ MINDEN EGYENSÚLYBAN LÉVŐ ERŐRENDSZERNEK FELTÉTELE.

A TÉTEL BÁRMILYEN ANYAGÚ SZILÁRD TESTRE ÉRVÉNYES.

A TÉTEL ERŐ JELLEGŰ MENNYISÉGEK SZÁMÍTÁSÁRA ALKALMAS

MEREV TESTEKRE VONATKOZTATVA $\delta W = \delta W_e = 0$

↳ VÉGES SZÁMÚ ELMOZDULÁSPARAMÉTER ⇒ ANYAGI EGYENSÚLYI EGYENL.

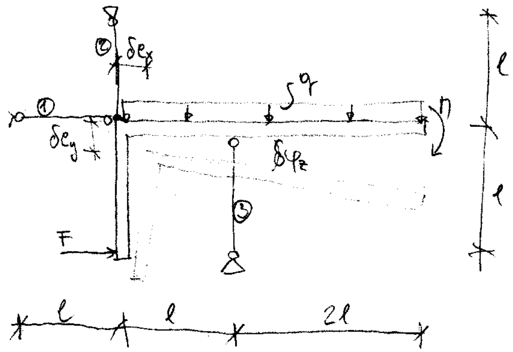
SZIL. TEST.

↳ VÉGES SZÁMÚ ELMOZD. FÜGGVÉNY ⇒ ANYAGI EGYENSÚLYI DIFF. EGY

A TÉTEL STAT. HATÁROZOTT SZERK. ERŐJATÉKÁNAK MEGÁLLAPÍTÁSÁRA A MEREV TESTRE VONATKOZÓ ALAKJÁBAN HASZNÁLHATÓ

STAT. HATÁROZATLAN ESETBEN AZ ANYAGEGYENLETEKET IS KELL HASZNÁLNI! (⇒ POTENCIÁLIS ENERGIA STACIONARITÁSI TÉTELE ... ... KÉSŐBB)

FELADAT : MEREV TESTBŐL ÁLLÓ TARTÓ REAKCIÓERŐI



A VIRT. ELMOZDULÁSRENDSZER PARAMÉTEREI

δe_x ; δe_y ; $\delta\varphi_z$

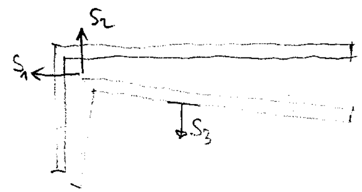
$$\delta u(x,y) = \delta e_x - \delta\varphi_z \cdot y = \delta u(\delta e_x, \delta\varphi_z)$$

$$\delta v(x,y) = \delta e_y + \delta\varphi_z \cdot x = \delta v(\delta e_y, \delta\varphi_z)$$

A VIRT. ELMOZD. RENDSZER GEOM. LEHETSÉGES:

$\Delta \ell_1 = \delta e_x$, $\Delta \ell_2 = \delta e_y$; $\Delta \ell_3 = \delta e_y + \ell \cdot \delta\varphi_z$

↳ KOMPATIBILITÁSI FELTÉTELEK



A VIRT. ELMOZD. TÉTELE:

$$\delta W = q \cdot 3 \cdot \ell \cdot \left(\delta e_y + \tfrac{3}{2}\ell \cdot \delta\varphi_z\right) + F \cdot (\delta e_x - \ell \delta\varphi_z)$$
$$+ M \cdot \delta\varphi_z - S_1 \cdot \delta e_x - S_2 \cdot \delta e_y + S_3 \cdot (\delta e_y + \ell \delta\varphi_z)$$

ÁTRENDEZVE A VIRT. ELMOZD. PARAM. SZERINT

$$\delta W = \delta e_x (F - S_1) + \delta e_y \cdot (q \cdot 3 \cdot \ell - S_2 + S_3)$$
$$+ \delta\varphi_z \left(q \cdot 3\ell \cdot \tfrac{3}{2}\ell - \ell \cdot F + M + S_3 \cdot \ell\right) = 0$$

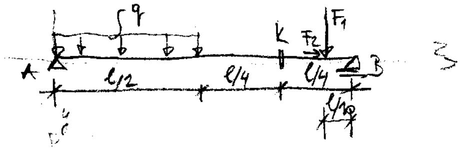
EZ CSAK AKKOR ZÉRUS, ~~HA~~ A 3 TAG KÜLÖN KÜLÖN AZ

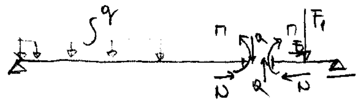
$$F - S_1 = 0$$
$$q \cdot 3\ell - S_2 + S_3 = 0$$
$$q \cdot \tfrac{9}{2}\ell^2 - F \cdot \ell + M + S_3 \cdot \ell = 0$$

⇓

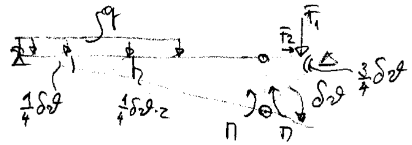
$$S_1 = F$$
$$S_3 = F - q \tfrac{9}{2}\ell - \frac{M}{\ell}$$
$$S_2 = F - q \cdot \tfrac{3}{2}\ell - \frac{M}{\ell}$$

KÉTTÁMASZÚ TARTÓ IGÉNYBEVÉTELEI:





Π



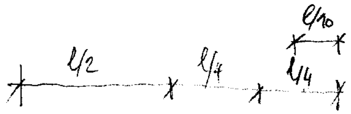
$\Rightarrow$ A BELSŐ MUNKA:

$$\int_0^{\frac{l}{2}} q\cdot\frac{1}{4}\delta\vartheta z\cdot dz + F_1\cdot\frac{3}{4}\delta\vartheta\cdot\frac{l}{10} - \Pi\cdot\delta\vartheta = 0$$

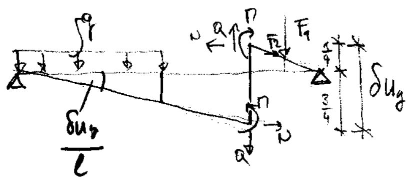
AZ INTEGRÁLÁST ELVÉGEZVE ÉS $\delta\vartheta$ KIEMELVE:

$$\left(\frac{q\cdot l^2}{32} + F_1\cdot\frac{3l}{40} - \Pi\right)\cdot\delta\vartheta = 0$$

$$\Rightarrow \boxed{\Pi = \frac{q\cdot l^2}{32} + F_1\cdot\frac{3l}{40}}$$



T



$\Rightarrow$ A BELSŐ MUNKA:

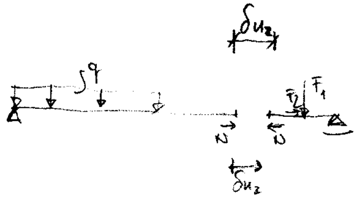
$$\int_0^{\frac{l}{2}} q\cdot\frac{\delta u_y}{l}\cdot z\cdot dz + Q\cdot\delta u_y - F_1\cdot\frac{1}{10}\cdot\delta u_y = 0$$

AZ INTEGRÁLÁST ELVÉGEZVE ÉS δu_y KIEMELVE:

$$\left(\frac{q\cdot l}{8} + Q - \frac{F_1}{10}\right)\delta u_y = 0$$

$$\Rightarrow \boxed{Q = \frac{F_1}{10} - \frac{q\cdot l}{8}}$$

N



$\Rightarrow$ A BELSŐ MUNKA

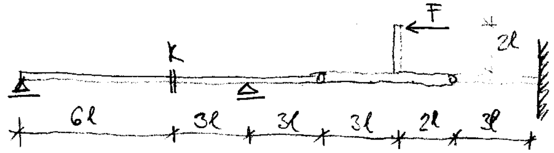
$$F_2\cdot\delta u_2 - N\cdot\delta u_2 = 0$$

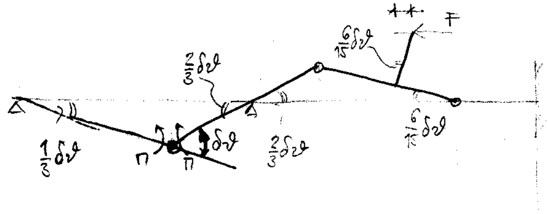
$$(F_2 - N)\,\delta u_2 = 0$$

$$\boxed{N = F_2}$$

F-8.1

Π_K





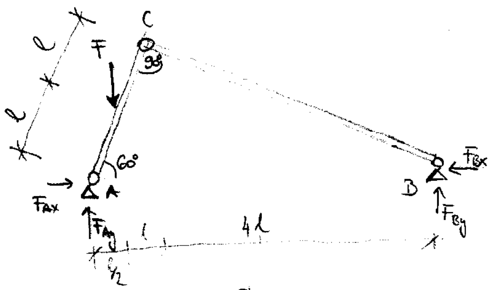
A BELSŐ MUNKA:

$$-\Pi_K\cdot\delta\vartheta - F\cdot\frac{6}{15}\delta\vartheta\cdot 2l = 0$$

$$\left(-\Pi - F\cdot\frac{6}{15}\cdot 2l\right)\delta\vartheta = 0$$

$$\Pi_K = -F\cdot\frac{12}{15}\cdot l = \underline{-\frac{4}{5}F\cdot l}$$

F-8.2

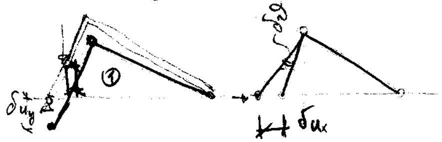


① $-F_{Ay}\cdot\delta u_y + F\cdot\frac{\delta u_y}{4l}\cdot\frac{7}{2}l = 0$

$$\left(-F_{Ay} + \frac{7}{8}F\right)\cdot\delta u_y = 0$$

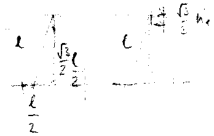
$$\boxed{F_{Ay} = \frac{7}{8}F}$$

② $F_{Ax}\cdot\delta u_x - F\cdot\frac{\sqrt{3}}{2}\delta u_x$



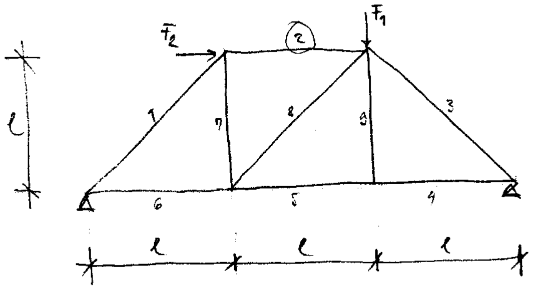
$l^2 - \frac{l^2}{4} = \frac{3}{4}l^2$

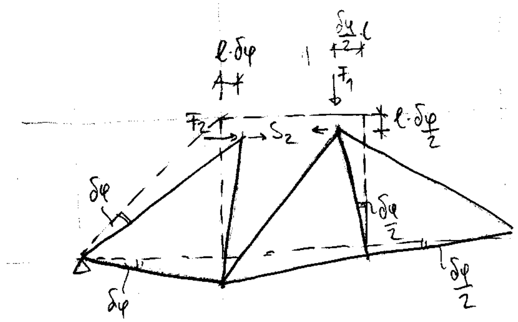
$\frac{\sqrt{3}}{2}l$



$\delta u_x = l\cdot\delta\vartheta$

$\boxed{S_2}$





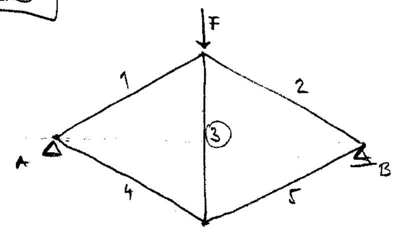
$$F_1 \cdot l \cdot \frac{\delta\varphi}{2} + F_2 \cdot l \cdot \delta\varphi - S_2\left(l \cdot \frac{\delta\varphi}{2} + l \cdot \delta\varphi\right) = 0$$

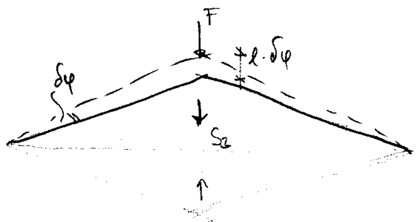
$$\delta\varphi\left(F_1 \cdot \frac{l}{2} + F_2 \cdot l - S_2 \cdot \frac{3}{2} l\right) = 0$$

$$\boxed{F_1 \cdot \frac{1}{3} + F_2 \cdot \frac{2}{3} = S_2}$$

$\boxed{F-8.3}$

$\boxed{S_3}$





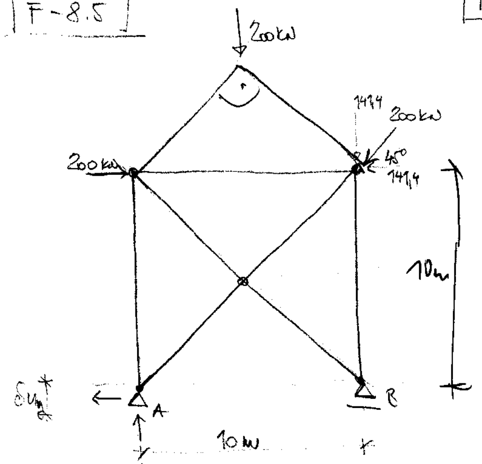
$$F \cdot l \cdot \delta\varphi + S_3 \cdot 2 \cdot l \cdot \delta\varphi = 0$$

$$(F \cdot l + S_3 \cdot 2l)\,\delta\varphi = 0$$

$$S_3 = -\frac{F}{2}$$

$\boxed{F-8.5}$

$\boxed{F_{Ax};\,F_{Ay}}$



$$-F_{Ay} \cdot 10\delta\varphi - 200 \cdot 10\delta\varphi + 200 \cdot 5\delta\varphi + 141{,}4 \cdot 10\delta\varphi = 0$$

$$\left(-F_{Ay} \cdot 10 - 200 \cdot 10 + 200 \cdot 5 + 141{,}4 \cdot 10\right)\delta\varphi = 0$$

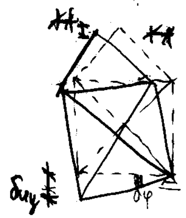
$$F_{Ay} = \frac{-200 \cdot 10 + 200 \cdot 5 + 141{,}4 \cdot 10}{10} = \underline{\underline{+41{,}4\ \text{kN}}}$$

$$F_{Ax} \cdot \delta u_x + 200 \cdot \delta u_x - 141{,}4 \cdot \delta u_x = 0$$

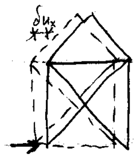
$$\left(F_{Ax} + 200 - 141{,}4\right) \cdot \delta u_x = 0$$

$$F_{Ax} = 141{,}4 - 200 = \underline{\underline{-58{,}6\ \text{kN}}}$$

AZ IRÁNYA A FELTÉTELEZETTEL ELLENTÉTES!



$\delta u_y = 10\,\delta\varphi$



VIRTUÁLIS ERŐK TÉTELE → (VIRT. KIEG. MUNKA TÉTELE)

EGY GEOMETRIAILAG LEHETSÉGES ELMOZDULÁSRENDSZERNEK BÁRMELY VIRTUÁLIS ERŐRENDSZEREN VÉGZETT MUNKÁJA ZÉRUS
↳ (KIEGÉSZÍTŐ MUNKA)

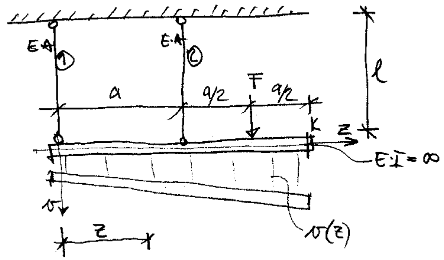
$$\delta\bar{W} = \delta\bar{W}_k + \delta\bar{W}_b = 0$$

BÁRMELY ANYAGÚ SZILÁRD TESTRE ÉS TARTÓRA ÉRVÉNYES, AMELY KIS ELMOZDULÁSOKAT VÉGEZ.

A TÉTEL ELMOZDULÁS JELLEGŰ MENNYISÉGEK SZÁMÍTÁSÁRA ALKALMAS. ANNYI FÜGGETLEN GEOM. EGYENLET, AHÁNY EGYMÁSTÓL FÜGGETLEN VIRTUÁLIS ERŐPARAMÉTER FÜGGVÉNYÉBEN FEJEZHETŐ KI A TEST EGYENSÚLYI ÁLLAPOTA.

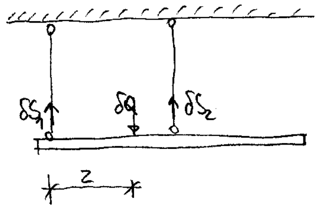
TARTÓK STATIKÁJA

1. STATIKAILAG HATÁROZOTTAN KAPCSOLT MEREV TEST ELMOZDULÁSAI



HATÁROZZUK MEG A SZERKEZET $v(z)$ ELMOZDULÁSAIT AZ ADOTT TEHER HATÁSÁRA

$v(z)$: ELTOLÓDÁSFÜGGVÉNY

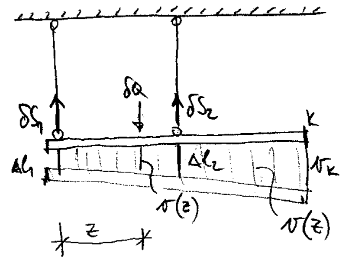


δQ : BÁRMELY ‚z' KOORD PONTBAN MŰKÖDŐ VIRTUÁLIS ERŐ.

$\delta Q;\ \delta S_1;\ \delta S_2$: STATIKAILAG LEHETSÉGES EGYENSÚLYI RENDSZER

$$\delta Q - \delta S_1 - \delta S_2 = 0$$
$$\delta Q \cdot z - \delta S_2 \cdot a = 0$$

$$\delta S_2 = \delta Q \cdot \frac{z}{a} \quad ; \quad \delta S_1 = \delta Q - \delta S_2 = \delta Q\left(\frac{a-z}{a}\right)$$



$$\delta\bar{W}_k = \delta Q \cdot v(z)$$

$$\delta\bar{W}_b = -\delta S_1 \cdot \Delta l_1 - \delta S_2 \cdot \Delta l_2 = -\delta Q\left(\frac{a-z}{a}\right)\cdot \Delta l_1 - \delta Q \cdot \frac{z}{a}\cdot \Delta l_2$$

$$\delta\bar{W} = \delta Q \cdot v(z) - \delta Q\left(\frac{a-z}{a}\right)\Delta l_1 - \delta Q \cdot \frac{z}{a}\cdot \Delta l_2 = 0$$

$$\delta Q\left(v(z) - \frac{a-z}{a}\Delta l_1 - \frac{z}{a}\cdot \Delta l_2\right) = 0$$

$$\boxed{v(z) = \frac{a-z}{a}\cdot \Delta l_1 + \frac{z}{a}\cdot \Delta l_2}$$

EZ A VIRT. ERŐK TÉTELÉNEK MEGFELELŐ GEOMETRIAI EGYENLET.

AZ ELTOLÓDÁSOK ÉRTÉKEIT CSAK Δl_1 ÉS Δl_2 ISMERETÉBEN TUDJUK MEGADNI

$$\Delta l_1 = \frac{S_1 \cdot l}{E \cdot A} \qquad \Delta l_2 = \frac{S_2 \cdot l}{E \cdot A}$$

AZ ERŐK $(S_1; S_2)$ AZ F KÜLSŐ ERŐBŐL SZÁMÍTHATÓ!

$$S_1 = -\frac{1}{2}F \qquad S_2 = \frac{3}{2}F$$

EZEKET BEHELYETTESÍTVE A $v(z)$ EGYENLETBE

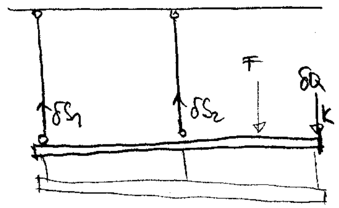
$$v(z) = \frac{F \cdot l}{E \cdot A}\left(2\frac{z}{a} - \frac{1}{2}\right)$$

EBBE BÁRMELY z-T HELYETTESÍTHETÜNK

$z = 2a$

$$v_K = \boxed{\frac{7}{2}\,\frac{F \cdot l}{E \cdot A}}$$

GYAKRAN NEM A TELJES ELMOZDULÁSFÜGGVÉNYT KELL MEGHATÁROZNI, HANEM CSAK EGY KIJELÖLT KERESZTMETSZET ELTOLÓDÁSA KELL.



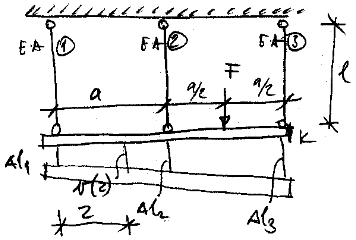
$\delta S_1 = -\delta Q \qquad \delta S_2 = 2\delta Q$

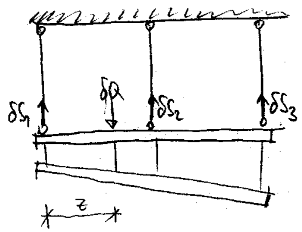
$$\delta \overline{W} = \delta Q \cdot v_K - \delta S_1 \cdot \Delta l_1 - \delta S_2 \cdot \Delta l_2 =$$
$$= \delta Q \cdot v_K + \delta Q \cdot \Delta l_1 - 2\delta Q \cdot \Delta l_2 =$$
$$= \delta Q \left(v_K + \Delta l_1 - 2\Delta l_2 \right) = 0$$
$$\boxed{v_K = 2\Delta l_2 - \Delta l_1}$$

$$\Delta l_1 = \frac{S_1 l}{EA} \; ; \; \Delta l_2 = \frac{S_2 l}{EA} \; ; \; S_1 = -\frac{1}{2}F \; ; \; S_2 = \frac{3}{2}F$$

$$v_K = 2 \cdot \frac{3}{2} F \cdot \frac{l}{E \cdot A} + \frac{1}{2} F \cdot \frac{l}{EA} = \boxed{\frac{7}{2}\frac{F l}{EA}}$$

2. STATIKAILAG HATÁROZATLAN MÓDON KAPCSOLT MEREV TEST ELMOZDULÁSAI





$$\left.\begin{aligned} &\delta Q - \delta S_1 - \delta S_2 - \delta S_3 = 0 \\ &\delta Q \cdot z - \delta S_2 \cdot a - \delta S_3 \cdot 2a = 0 \end{aligned}\right\} \begin{array}{l} 2 \text{ EGYENLET} \\ 3 \text{ ISMERETLEN} \end{array}$$

$\Downarrow$

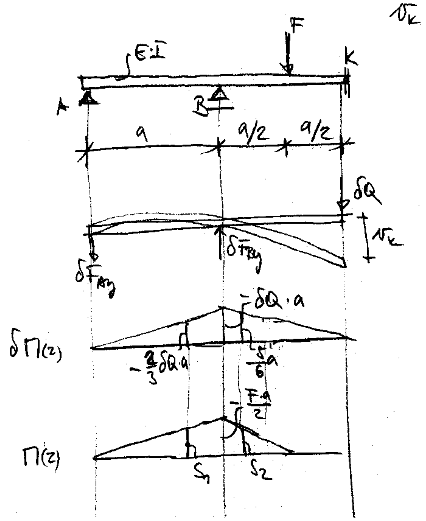
A VIRTUÁLIS ERŐRENDSZER 1 SZABAD PARAMÉTERREL RENDELKEZIK

LEGYEN EZ A PARAMÉTER AZ δS_3-AS RÚDERŐ

LEGYEN $\delta S_3 = \frac{1}{2}\delta Q$

3. STATIKAILAG HATÁROZOTT RUGALMAS GERENDA ELMOZDULÁSAI

$v_K = ?$



$\delta F_{Ay} = \delta Q \; ; \; \delta F_{By} = -2\delta Q$

A VIRT. KÜLSŐ REAKCIÓERŐK HELYÉN NINCS MUNKA!

$$\delta \overline{W}_k = \delta Q \cdot v_K$$

$$\delta \overline{W}_b = -\int_0^{2a} \delta M(z) \cdot \kappa(z)\, dz \qquad \kappa(z): \text{fajlagos elfordulás}$$

$$\kappa(z) = \frac{M(z)}{E \cdot I}$$

$$\delta \overline{W}_b = -\int_0^{2a} \delta M(z) \cdot \frac{M(z)}{E \cdot I} = -\frac{1}{E \cdot I} \int_0^{2a} \delta M(z) \cdot M(z)\, dz$$

$$\delta \overline{W} = \delta Q \cdot v_K - \frac{1}{EI} \cdot \int_0^{2a} \delta M(z) \cdot M(z)\, dz = 0$$

MIVEL A $\delta M(z)$ FÜGGVÉNY IS δQ FÜGGVÉNYE, ÉS δQ NAGYSÁGA TETSZŐLEGES, EZÉRT LEGYEN $\delta Q = 1$

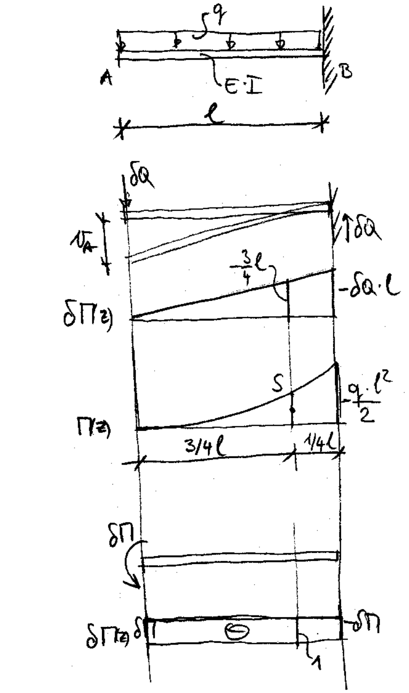
$$v_K = \frac{1}{E \cdot I} \int \delta M(z) \cdot M(z)$$

A SZÁMÍTÁS AZ ÁBRÁK SZORZATA ALAPJÁN:

$$v_K = \frac{1}{E \cdot I}\left[\left(-\frac{F \cdot a}{2} \cdot a \cdot \frac{1}{2}\right)\cdot\left(-\frac{2}{3}a\right) + \left(-\frac{F \cdot a}{2} \cdot \frac{a}{2} \cdot \frac{1}{2}\right)\cdot\left(-\frac{5}{6}a\right)\right]$$

$$v_K = \frac{13}{48} \frac{F \cdot a^3}{E \cdot I}$$

4. F-8.20



$$\delta\overline{W}_k = \delta Q \cdot v_A$$

$$\delta\overline{W}_b = -\int_0^l \delta\Pi(z) \cdot K(z)\, dz$$

$$K(z) = \frac{\Pi(z)}{E \cdot I}$$ (fajlagos elfordulás)

$$\delta\overline{W}_b = -\frac{1}{E\cdot I}\int_0^l \delta\Pi(z) \cdot \Pi(z)\, dz$$

$$\delta Q = 1$$

$$\delta\overline{W} = \delta Q \cdot v_A - \frac{1}{EI}\int \delta\Pi(z) \cdot \Pi(z)\, dz$$

$$v_A = \frac{1}{EI} \cdot \int \delta\Pi(z) \cdot \Pi(z)\, dz$$

AZ ÁBRÁK SZORZATA ALAPJÁN:

$$v_A = \frac{1}{E\cdot I}\cdot\left[\left(-\frac{q\cdot l^2}{2}\cdot l \cdot \frac{1}{3}\right)\left(-\frac{3}{4}l\right)\right] = \frac{1}{EI}\cdot\frac{q\cdot l^4}{8}$$

$$v_A = \frac{q\cdot l^4}{8EI}$$

$$\delta\overline{W}_k = \delta\Pi \cdot \varphi_A$$

$$\delta\overline{W}_b = -\int_0^l \delta\Pi(z) \cdot K(z)\, dz$$

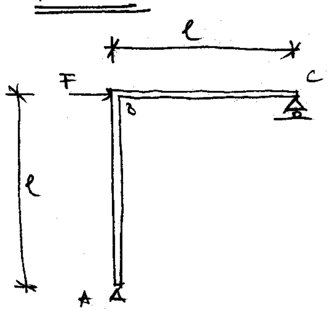
$$\delta\overline{W} = \delta\Pi \cdot \varphi_A - \frac{1}{E\cdot I} \cdot \int \delta\Pi_z \cdot \Pi_z\, dz$$

$$\delta\Pi = 1$$

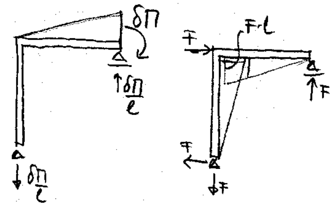
$$\varphi_A = \frac{1}{EI}\cdot\left(-\frac{q\cdot l^2}{2}\cdot l \cdot \frac{1}{3}\,(-1)\right) = -\frac{q\cdot l^3}{6EI}$$

5. HATÁROZZA MEG AZ ÁLLANDÓ EI MEREVSÉGŰ TARTÓ ‚C' TÁMASZKERESZTMETSZETÉNEK ELFORDULÁSÁT ÉS ELTOLÓDÁSÁT

F-8.27.



1. ELFORDULÁS SZÁMÍTÁSA



$$\delta\overline{W}_k = \delta\Pi \cdot \varphi_C$$

$$\delta\overline{W}_b = -\int \delta\Pi(z)\, K(z)$$

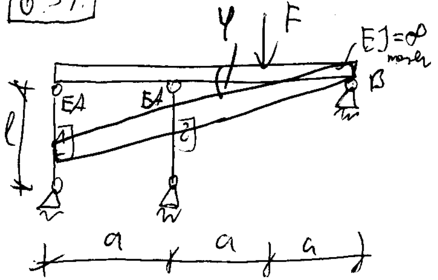
$$\varphi_C = \frac{1}{E\cdot I}\cdot\left(F\cdot l\cdot l\cdot\frac{1}{2}\cdot\frac{1}{3}\right) = \frac{F\cdot l^2}{6EI} \circlearrowright$$

2. ELTOLÓDÁS SZÁMÍTÁSA

$$v_C = \frac{1}{E\cdot I}\cdot\left(F\cdot l\cdot l\cdot\frac{1}{\cancel{2}}\cdot\frac{\cancel{2}}{3}\cdot l\right)\cdot 2 = \frac{2F\cdot l^3}{3EI} \rightarrow$$

Potenciális energia stacionaritása

B.39.



Határozza meg a reakcióerőket a pot. en. stac. tétele segítségével!

φ – 1 szabadságfokú rendszer

$\Delta l_1 = 3a\varphi$

$\Delta l_2 = 2a\varphi$

$\Delta l = \frac{Sl}{EA} \rightarrow S = \frac{EA}{l}\Delta l$

$$\Pi_k = -F \cdot a\varphi$$

$$\Pi_b = \sum \frac{1}{2} S_i \cdot \Delta l_i = \frac{1}{2} S_1 \cdot \Delta l_1 + \frac{1}{2} S_2 \cdot \Delta l_2 = \frac{1}{2}\frac{EA}{l}\cdot \Delta l_1^2 + \frac{1}{2}\frac{EA}{l}\cdot\Delta l_2^2 =$$

$$= \frac{1}{2}\frac{EA}{l}\left(9a^2\varphi^2 + 4a^2\varphi^2\right) = \frac{13}{2}\frac{EAa^2}{l}\varphi^2$$

$$\Pi = \Pi_k + \Pi_b = -F\cdot a \cdot\varphi + \frac{13}{2}\frac{EAa^2}{l}\cdot\varphi^2 = \text{stac!}$$

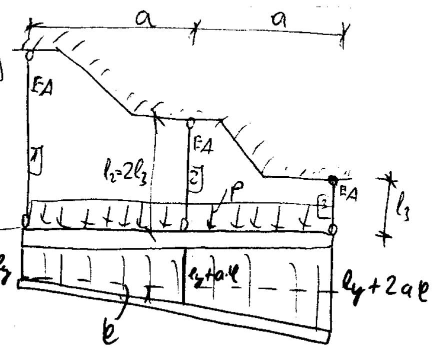
$$\frac{\partial \Pi}{\partial \varphi} = 0 = -aF + \frac{13EA\cdot a^2}{l}\cdot\varphi \rightarrow \boxed{\varphi = \frac{F\cdot a\cdot l}{13EA\cdot a^2} = \frac{F\cdot l}{13EA\cdot a}}$$

$$\boxed{S_1} = \frac{\cancel{EA}}{l}\cdot 3a\,\frac{Fl}{13\cancel{EA}\cdot a} = \frac{3}{13}F\ (-)$$

$$\boxed{S_2} = \frac{\cancel{EA}}{l}\cdot 2a\,\frac{Fl}{13\cancel{EA}\cdot a} = \frac{2}{13}F\ (-)$$

$$\boxed{B_y} = F - S_1 - S_2 = F - \frac{3}{13}F - \frac{2}{13}F = \boxed{\frac{8}{13}F}\ (\uparrow)$$

B.46.



Hat. meg a reakcióerőket a pot. en. stac.!

e_y, φ – 2 szab. fok

$S_i = \frac{EA}{l_i}\cdot\Delta l_i$

$$\Delta l_1 = e_y\ ;\quad \Delta l_2 = e_y + a\cdot\varphi\ ;\quad \Delta l_3 = e_y + 2a\varphi$$

$$\Pi_k = -\int p(z)\,v(z)\,dz = -\left(2a\cdot e_y\cdot p + \frac{\cancel{2}a\cdot 2a\varphi}{\cancel{2}}\cdot p\right) = -2\cdot ap\cdot e_y - 2a^2p\cdot\varphi$$

$$\Pi_b = \sum\frac{1}{2}S_i\cdot\Delta l_i = \frac{EA}{2}\sum\frac{\Delta l_i^2}{l_i} = \frac{EA}{2l_3}\left(\frac{e_y^2}{3} + \frac{e_y^2 + 2a\varphi\cdot e_y + a^2\varphi^2}{2} + e_y^2 + 4a\varphi e_y + 4a^2\varphi^2\right) = \frac{EA}{2l_3}\left(\frac{2e_y^2}{6} + \frac{3e_y^2 + 6a\varphi\cdot e_y + 3a^2\varphi^2}{6} + \frac{6e_y^2 + 24a\varphi\cdot e_y + 24a^2\varphi^2}{6}\right) =$$

$$= \frac{EA}{12l_3}\left(11e_y^2 + 30a\cdot\varphi\cdot e_y + 27a^2\varphi^2\right)$$

$$\Pi = -2ap\cdot e_y - 2a^2p\cdot\varphi + \frac{EA}{12l_3}\left(11e_y^2 + 30a\varphi\cdot e_y + 27a^2\varphi^2\right) = \text{stac.!}$$

$$\frac{\partial\Pi}{\partial e_y} = -2ap + \frac{EA}{12l_3}\left(22e_y + 30a\varphi\right) = 0$$

$$\frac{\partial\Pi}{\partial\varphi} = -2a^2p + \frac{EA}{12l_3}\left(30a\cdot e_y + 54a^2\varphi\right) = 0$$

$$\varphi = \left(2a^2p - \frac{EA}{12l_3}30\cdot a\cdot e_y\right)\frac{12\,l_3}{54\,a^2EA} = \frac{2a^2p\cdot 12l_3}{54a^2EA} - \frac{30a\cdot e_y\,EA\cdot 12l_3}{12l_3\cdot 54a^2EA} =$$

$$= \underline{\underline{\frac{24\,p\,l_3}{54\,EA} - \frac{30}{54}\frac{e_y}{a}}}$$

$$-2ap + \frac{EA}{12l_3}\cdot 22e_y + \frac{30}{12}\frac{EA\cdot a}{l_3}\,\frac{24\,p\,l_3}{54EA} - \frac{30}{12}\frac{EA\cdot a}{l_3}\cdot\frac{30}{54}\frac{e_y}{a} = \emptyset$$

$$-2ap + \frac{22\,EAe_y}{12\,l_3} + \frac{60}{54}ap - \frac{900}{12\cdot 54}\frac{EA\cdot e_y}{l_3} = \emptyset$$

$$\left(\frac{60}{54}-2\right)a\cdot p = \left(\frac{900}{12\cdot 54} - \frac{22}{12}\right)\frac{EA}{l_3}\cdot e_y$$

$$\boxed{e_y} = \frac{48}{54}a\cdot p\cdot\frac{l_3}{EA}\,\frac{12\cdot 54}{288} = \boxed{\frac{2\cdot a\cdot p\cdot l_3}{EA}}$$

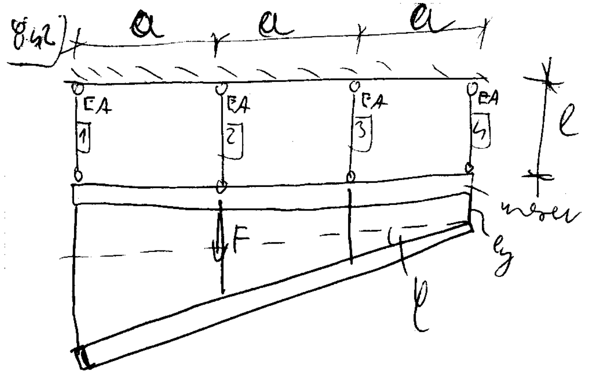
$$\varphi = \frac{24}{54}\frac{pl_3}{EA} - \frac{30}{54}\cdot\frac{2a\,pl_3}{EA\cdot a} = -\frac{36}{54}\frac{pl_3}{EA} = \underline{\underline{-\frac{2\,p\,l_3}{3\,EA}}}$$

$$S_1 = \frac{EA}{3l_3}\cdot\frac{2\cdot a\cdot p\cdot l_3}{EA} = \boxed{\frac{2}{3}\cdot a\cdot p} \quad = \underline{\underline{\frac{6}{9}a\cdot p}}$$

$$S_2 = \frac{EA}{2l_3}\cdot\left(\frac{2\cdot a\cdot p\cdot l_3}{EA} + \frac{-2p\cdot a\cdot l_3}{3EA}\right) = a\cdot p - \frac{1}{3}a\cdot p = \underline{\underline{\frac{2}{3}ap}}$$

$$S_3 = \frac{EA}{l_3}\cdot\left(\frac{2\cdot a\cdot p\cdot l_3}{EA} + \frac{-4}{3}\frac{p\cdot a\cdot l_3}{EA}\right) = 2ap - \frac{4}{3}ap = \underline{\underline{\frac{2}{3}ap}}$$

8.42)



Rúderők a pot. en. áll. tételével!

e_y, φ – 2 paraméter

$$S_i = \frac{EA}{l_i}\Delta l_i$$

$$\Delta l_1 = e_y + 3a\varphi\,;\ \Delta l_2 = e_y + 2a\varphi\,;\ \Delta l_3 = e_y + a\varphi\,;\ \Delta l_4 = e_y$$

$$\Pi_k = -F\cdot(e_y + 2a\varphi)$$

$$\Pi_b = \frac{EA}{2l}\sum\Delta l_i^2 = \frac{EA}{2l}\left(e_y^2 + 6a\varphi\cdot e_y + 9a^2\varphi^2 + e_y^2 + 4a\varphi e_y + 4a^2\varphi^2 + e_y^2 + 2a\varphi e_y + a^2\varphi^2 + e_y^2\right) = \frac{EA}{2l}\left(4e_y^2 + 12a\varphi\cdot e_y + 14a^2\varphi^2\right)$$

$$\Pi = -F e_y - 2a\varphi\cdot F + \frac{EA}{2l}\left(4e_y^2 + 12a\varphi\cdot e_y + 14a^2\varphi^2\right)$$

$$\frac{\partial\Pi}{\partial\varphi} = -2aF + \frac{EA}{2l}\left(12a\cdot e_y + 28a^2\varphi\right) = 0$$

$$\frac{\partial\Pi}{\partial e_y} = -F + \frac{EA}{2l}\left(8e_y + 12a\varphi\right) = 0$$

$$\hookrightarrow \varphi = \left(F - \frac{EA}{2l}8\cdot e_y\right)\frac{2l}{12EA\,a} = \underline{\frac{2F\cdot l}{12EA\cdot a} - \frac{2}{3}\frac{e_y}{a}}$$

$$-2aF + \frac{EA}{2l}\cdot 12\cdot a\cdot e_y + \frac{EA}{2l}\cdot 28a^2\cdot\frac{2F\cdot l}{12EA\cdot a} - \frac{EA}{2l}\cdot 28a^2\cdot\frac{2}{3}\frac{e_y}{a} = 0$$

$$-2aF + \frac{6EA\cdot a\cdot e_y}{l} + \frac{7}{3}\cdot a\cdot F - \frac{28EA\cdot a\cdot e_y}{3\cdot l} = 0$$

$$\left(\frac{17}{3}-2\right)a\cdot F+\frac{EA\cdot a}{\ell}\cdot \ell_y\left(6-\frac{28}{3}\right)=0$$

$$0=\frac{a\cdot F}{\cancel{3}}+\frac{-10}{\cancel{3}}\,\frac{EA\cdot a}{\ell}\,\ell_y \longrightarrow \ell_y=\frac{\cancel{a}\cdot F\cdot\ell}{10EA\cdot\cancel{a}}=\boxed{\frac{F\cdot\ell}{10EA}}$$

$$\varphi=\frac{2F\ell}{12EA\cdot a}-\frac{2}{3}\cdot\frac{F\ell}{10EA\cdot a}=\left(\underset{\frac{5}{30}}{\frac{1}{6}}-\frac{2}{30}\right)\frac{F\cdot\ell}{EA\cdot a}=\boxed{\frac{F\cdot\ell}{10EA\cdot a}}$$

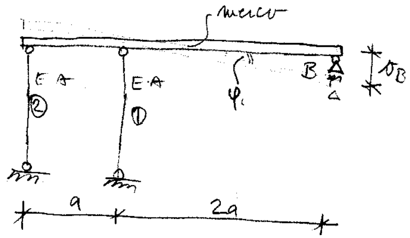
$$S_1=\frac{\cancel{EA}}{\cancel{\ell}}\left(\frac{F\cdot\cancel{\ell}}{10\cancel{EA}}+\frac{3\cdot\cancel{a}\cdot F\cdot\cancel{\ell}}{10\cancel{EA}\cdot\cancel{a}}\right)=\boxed{\frac{4}{10}\cdot F}$$

$$S_2=\frac{\cancel{EA}}{\cancel{\ell}}\left(\frac{F\cdot\cancel{\ell}}{10\cancel{EA}}+\frac{2\cdot\cancel{a}\cdot F\cdot\cancel{\ell}}{10\cancel{EA}\cdot\cancel{a}}\right)=\boxed{\frac{3}{10}F}$$

$$S_3=\frac{\cancel{EA}}{\cancel{\ell}}\left(\frac{F\cancel{\ell}}{10\cancel{EA}}+\frac{\cancel{a}\cdot F\cancel{\ell}}{10\cancel{EA}\cdot\cancel{a}}\right)=\boxed{\frac{2}{10}F}$$

$$S_4=\frac{EA}{\ell}\cdot\frac{F\ell}{10EA}=\boxed{\frac{1}{10}F}$$

F-8.40



ELMOZDULÁSI SZABADSÁGFOK : 1

φ

$\Delta\ell_1=\varphi\cdot 2a-v_B$; $\Delta\ell_2=\varphi\cdot 3a-v_B$

$S_1=\frac{E\cdot A}{\ell}\cdot\Delta\ell_1$; $S_2=\frac{EA}{\ell}\cdot\Delta\ell_2$

MIVEL KÜLSŐ ERŐ NEM MŰKÖDIK, ÍGY A KÜLSŐ POTENCIÁL ZÉRUS

$\Pi_k=0$

A BELSŐ POTENCIÁL:

$$\Pi_b=\frac{1}{2}S_1\cdot\Delta\ell_1+\frac{1}{2}S_2\cdot\Delta\ell_2$$

A POTENCIÁLIS ENERGIA EGYENLŐ A BELSŐ POTENCIÁLLAL.

$$\Pi=\Pi_b=\frac{1}{2}\frac{E\cdot A}{\ell}\left((\varphi\cdot 2a-v_B)^2+(\varphi\cdot 3a-v_B)^2\right)=\frac{EA}{2\ell}\left(\varphi^2 4a^2-4\varphi a v_B+v_B^2+9\varphi^2a^2-6\varphi a\cdot v_B+v_B^2\right)$$

$$=\frac{EA}{2\ell}\left(13\varphi^2a^2-10\varphi\cdot a\cdot v_B+2v_B^2\right)=\text{STACIONÁRIUS!}$$

$$\frac{\partial\Pi}{\partial\varphi}=\frac{EA}{2\ell}\left(26\varphi a^2-10a\cdot v_B\right)=0 \Longrightarrow \varphi=\frac{v_B\cdot 10a}{26a^2}=\boxed{\frac{10v_B}{26a}}$$

VISSZAHELYETTESÍTVE:

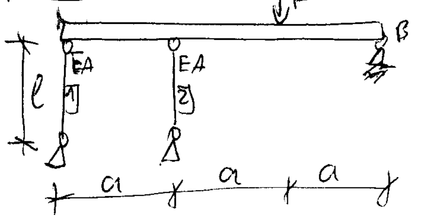
$$\Delta\ell_1=2\cancel{a}\cdot\frac{10v_B}{\underset{13}{\cancel{26}}\cancel{a}}-v_B=-\frac{3}{13}v_B \quad ; \quad \Delta\ell_2=3\cancel{a}\cdot\frac{10v_B}{26\cancel{a}}-v_B=\frac{2}{13}v_B$$

$$S_1=\frac{E\cdot A}{\ell}\cdot -\frac{3}{13}v_B=\frac{3}{13}\frac{E\cdot A}{\ell}\cdot v_B \text{ (nyomó)} \uparrow \qquad S_2=\frac{EA}{\ell}\cdot\frac{2}{13}v_B \text{ (húzó)} \downarrow$$

A FÜGGŐLEGES ERŐK EGYENSÚLYA MIATT: $F_{By}=\frac{1}{13}\frac{E\cdot A}{\ell}v_B \downarrow$

8.50

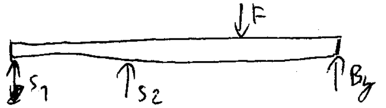
Kiegészítő potenciális energia minimum tétele



$F - S_1 - S_2 - B_y = 0$

$S_1 \cdot 3a + S_2 \cdot 2a - F \cdot a = 0$

$$\boxed{S_2 = \frac{F - 3S_1}{2}}$$



$F - S_1 - \frac{F - 3S_1}{2} - B_y = 0$

$$\boxed{B_y = \frac{2F - 2S_1 - F + 3S_1}{2} = \frac{F + S_1}{2}}$$

$\Delta l_i = \frac{S_i\, l_i}{E_i A_i}$

$\tilde{\Pi}_k = \emptyset$

$$\tilde{\Pi}_b = \Sigma \frac{1}{2} S_i \cdot \Delta l_i = \Sigma \frac{1}{2} \frac{l_i}{E_i A_i} S_i^2 = \frac{\ell}{2EA} \Sigma S_i^2 =$$

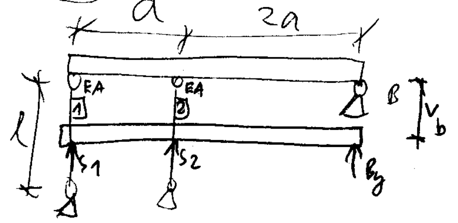
$$= \frac{\ell}{2EA}\left(S_1^2 + \frac{F^2 - 6S_1F + 9S_1^2}{4}\right) = \min! = \tilde{\Pi} = \frac{\ell}{8EA}\left(4S_1^2 + F^2 - 6S_1F + 9S_1^2\right)$$

$$\frac{\partial \tilde{\Pi}}{\partial S_1} = 0 = 26 S_1 - 6F \longrightarrow S_1 = \frac{6}{26} F = \boxed{\frac{3}{13} F} (\leftarrow)$$

$$S_2 = \frac{\frac{13}{13}F - \frac{9}{13}F}{2} = \boxed{\frac{2}{13} F} (\leftarrow)$$

$$B_y = \frac{\frac{13}{13}F + \frac{3}{13}F}{2} = \boxed{\frac{8}{13} F} (\uparrow)$$

8.51



$\Sigma F_y = S_1 + S_2 + B_y = 0$

$\Sigma M_B = S_1 \cdot 3a + S_2 \cdot 2a = 0$

$S_1 = -\frac{2}{3} S_2$

$B_y = +\frac{2}{3} S_2 - S_2 = -\frac{1}{3} S_2$

$\tilde{\Pi}_k = -\left(-S_1 \cdot v_b - S_2 v_b\right)$

$$\tilde{\Pi}_b = \frac{1}{2}\frac{\ell}{EA}\left(S_1^2 + S_2^2\right) = \frac{\ell}{2EA}\left(\frac{4}{9}S_2^2 + S_2^2\right) = \frac{13}{18}\frac{\ell}{EA} \cdot S_2^2$$

$$\tilde{\Pi} = S_1 \cdot v_b + S_2 v_b + B_y \cdot v_b + \frac{13}{18}\frac{\ell}{EA} S_2^2 = \min$$

$$\tilde{\Pi} = -\frac{2}{3} S_2 \cdot v_b + S_2 \cdot v_b + \frac{13}{18}\frac{\ell}{EA} S_2^2$$

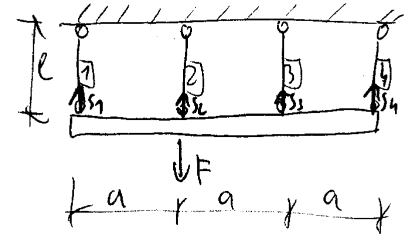
$$\tilde{\Pi} = \frac{1}{3} v_b \cdot S_2 + \frac{13}{18}\frac{\ell}{EA} S_2^2 = \min!$$

$$\frac{\partial \tilde{\Pi}}{\partial S_2} = \frac{1}{3} v_b + \frac{13}{9}\frac{\ell}{EA} S_2 = 0 \longrightarrow S_2 = -\frac{1}{\cancel{3}} v_b \cdot \frac{\cancel{9}\,3}{13}\frac{EA}{\ell} = \boxed{\frac{-3}{13}\frac{EA}{\ell} \cdot v_b} (-)$$

$$S_1 = -\frac{2}{\cancel{3}} \cdot \frac{-\cancel{3}}{13}\frac{EA}{\ell} \cdot v_b = \boxed{\frac{2}{13}\frac{EA}{\ell} \cdot v_b} (+)$$

$$B_y = \frac{+1}{\cancel{3}} \cdot \frac{+\cancel{3}}{13}\frac{EA}{\ell} \cdot v_b = \boxed{\frac{1}{13}\frac{EA}{\ell} \cdot v_b} (\downarrow)$$

$\boxed{8.53}$



$EA \quad \Sigma F_{iy} = F - S_1 - S_2 - S_3 - S_4 = 0$

$\Sigma M_y = F a - S_2 \cdot a - 2 S_3 a - 3 S_4 \cdot a = 0$

$S_4 = F - S_1 - S_2 - S_3 = F - S_1 - S_2 - 2F + 3S_1 + 2S_2 = \boxed{2S_1 + S_2 - F}$

$F - S_2 - 2S_3 - 3F + 3S_1 + 3S_2 + 3S_3 = 0$

$3S_1 + 2S_2 + S_3 - 2F = 0$

$\boxed{S_3 = 2F - 3S_1 - 2S_2}$

$\Delta l = \dfrac{S \cdot l}{EA}$

$\tilde{\Pi}_k = 0$

$\tilde{\Pi}_b = \Sigma \frac{1}{2} S \Delta l = \Sigma \frac{1}{2} \frac{l}{EA} S^2 = \frac{l}{2EA}\left[S_1^2 + S_2^2 + (2F - 3S_1 - 2S_2)^2 + (2S_1 + S_2 - F)^2\right] =$

$= \frac{l}{2EA}\Big(S_1^2 + S_2^2 + 4F^2 - 6FS_1 - 4FS_2 - 6FS_1 + 9S_1^2 + 6S_1S_2 - 4FS_2 + 6S_1S_2 + 4S_2^2 +$

$+ 4S_1^2 + 2S_1S_2 - 2FS_1 + 2S_1S_2 + S_2^2 - FS_2 - 2FS_1 - FS_2 + F^2\Big) =$

$= \frac{l}{2EA}\left(14S_1^2 + 6S_2^2 + 5F^2 - 16FS_1 - 10FS_2 + 16S_1S_2\right) = \min$

$\left.\begin{aligned} \frac{\partial \tilde{\Pi}}{\partial S_1} &= 28S_1 - 16F + 16S_2 = 0 \\ \frac{\partial \tilde{\Pi}}{\partial S_2} &= 12S_2 - 10F + 16S_1 = 0 \end{aligned}\right\}$

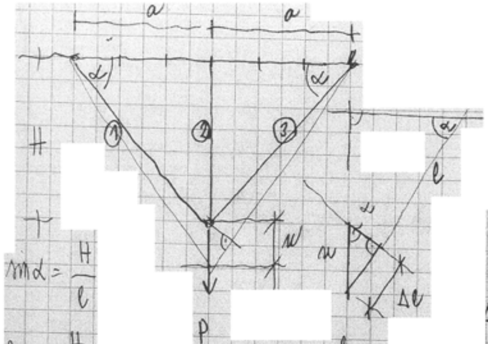
$S_3 = \frac{20}{10}F - \frac{12}{10}F - \frac{6}{10}F = \boxed{\frac{2}{10}F}$

$S_4 = \frac{8}{10}F + \frac{3}{10}F - \frac{10}{10}F = \boxed{\frac{1}{10}F}$

$7S_1 - 4F - 4S_2 = 0 \rightarrow \boxed{S_2 = \frac{4F - 7S_1}{4}} = F - \frac{7}{4}\frac{4}{10}F = \boxed{\frac{3}{10}F}$

$\frac{6}{4}(4F - 7S_1) - 5F + 8S_1 = 0$

$24F - 42S_1 - 20F + 32S_1 = 0 \rightarrow 4F - 10S_1 = 0 \rightarrow \boxed{S_1 = \frac{4}{10}F}$



Hooke törvény

$\Delta l = \frac{Sl}{EA} \Rightarrow S = \frac{EA}{l} \cdot \Delta l$

$\sin\alpha = \frac{H}{l}$

$l = \frac{H}{\sin\alpha}$

vagy: $\operatorname{tg}\alpha = \frac{l_2}{u}$

$\sin\alpha = \frac{\Delta l}{u}$

$\Delta l = u \sin\alpha$

$a \operatorname{tg}\alpha = l_2$

$\frac{a}{\cos\alpha} = l_1$

$l_1 = \frac{a}{\cos\alpha} \qquad \Delta l_1 = u \sin\alpha$

$\Delta l_1^2 = u^2 \sin^2\alpha$

$\frac{\Delta l_1^2}{l_1} = \frac{u^2 \sin^2\alpha}{a} \cdot \cos\alpha$

$l_2 = a \operatorname{tg}\alpha$

$\Delta l_2 = u$

$\Delta l_2^2 = u^2$

$\cos\alpha = \frac{a}{l_1}$

$\Pi = -uP + \frac{1}{2} EA \sum_{i=1}^{3} \frac{\Delta l_i^2}{l_i}$

$\Pi = -uP + \frac{1}{2} EA \left\{ \underbrace{2 \cdot \frac{u^2 \cdot \sin^2\alpha \cos\alpha}{a} + \frac{u^2 \cos\alpha}{a \sin\alpha}}_{\frac{u^2 \cdot \cos\alpha}{a}\left(2\sin^2\alpha + \frac{1}{\sin\alpha}\right)} \right\}$

$\frac{\partial \Pi}{\partial u} = -P + \frac{1}{2} EA \cdot 2u \frac{\cos\alpha}{a}\left(2\sin^2\alpha + \frac{1}{\sin\alpha}\right) = 0$

$\frac{\cos\alpha}{\sin\alpha}\left(2\sin^3\alpha + 1\right)$

$\boxed{u = \frac{P \cdot a}{EA} \overset{l_2}{\operatorname{tg}\alpha} \frac{1}{2\sin^3\alpha + 1}}$

$\boxed{S_2 = P \frac{1}{2\sin^3\alpha + 1}}$; $\left\{\begin{aligned} \Delta l_1 &= u \cdot \sin\alpha \\ l_1 &= \frac{a}{\cos\alpha} \end{aligned}\right.$

$\Delta l_1 = \frac{P a \cdot \sin^2}{EA \cos\alpha} \cdot \frac{1}{2\sin^3\alpha + 1}$

$S_1 = S_3 = P \frac{\sin^2\alpha}{\ldots}$

Ha $\alpha = 45°$

$\operatorname{ctg}\alpha = 1$

$\sin\alpha = \frac{\sqrt{2}}{2} \qquad \sin^2\alpha = \frac{2}{4} \quad \frac{1}{2}$

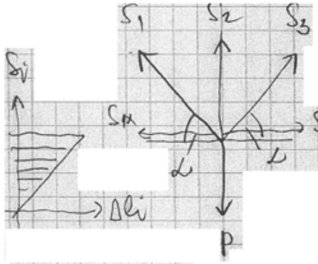
$\sin^3\alpha = \frac{2\sqrt{2}}{8} = \frac{\sqrt{2}}{4}$

$1+2\sin^3\alpha = \frac{\sqrt2}{2}+1 = \frac{2+\sqrt2}{2}$

$\boxed{S_2 = P\frac{2}{2+\sqrt2}}$

$\boxed{S_1} = P\frac{1}{\cancel{2}}\cdot\frac{\cancel{2}}{2+\sqrt2} = \boxed{P\frac{1}{2+\sqrt2}}$

$\cos\alpha = \frac{S_{1x}}{S_1}$



$\Sigma F_{ix}=0 \quad (-S_1+S_3)\cos\alpha = 0$

$\Sigma F_{iy}=0 \quad (S_1+S_3)\sin\alpha + S_2 = P$

$S_1 = S_3$

$2S_1\sin\alpha + S_2 = P$

$\overline{\Pi} = \overline{\Pi}_k + \overline{\Pi}_b$

$S_2 = P - 2S_1\sin\alpha$

$\overline{\Pi}_b = \frac{1}{2}\frac{1}{EA}\sum_{i=1}^{3} l_i S_i^2$

$S_2^2 = P^2 - 4PS_1\sin\alpha + 4S_1^2\sin^2\alpha$

$l_1 = \frac{a}{\cos\alpha}$ $\qquad l_2 = a\frac{\sin\alpha}{\cos\alpha}$

$\overline{\Pi} = \overline{\Pi}_b = \frac{1}{2}\frac{1}{EA}\left(2\cdot\frac{a}{\cos\alpha}\cdot S_1^2 + a\frac{\sin\alpha}{\cos\alpha}\left(P^2 - 4PS_1\sin\alpha + 4S_1^2\sin^2\alpha\right)\right)$

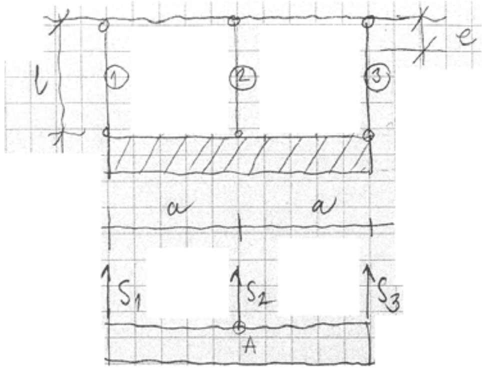
$\frac{\partial\overline{\Pi}}{\partial S_1} = \frac{1}{2}\frac{a}{EA}\cdot\frac{1}{\cos\alpha}\left(4S_1 + \underbrace{\sin\alpha\left(-4P\sin\alpha + 8S_1\sin^2\alpha\right)}_{0}\right) = 0$

$S_1 = \frac{\cancel{4}P\cdot\sin^2\alpha}{\cancel{4}+\overset{2}{\cancel{8}}\sin^3\alpha}$

$\boxed{S_1 = P\frac{\sin^2\alpha}{1+2\sin^3\alpha}}$

za $\alpha = 45^\circ$ $\quad S_1 = P\frac{2\cancel{2}\cdot\frac{1}{2}}{1+\overset{4}{\cancel{8}}\cdot\frac{\sqrt2}{\cancel{8}}}$

$\boxed{S_1 = P\frac{2}{1+2\sqrt2}}$



$\Sigma F_{iy}=0$

$S_1 + S_2 + S_3 = 0$

$\Sigma M_i^{(A)} = 0$

$S_1 a = S_3 a$

$\boxed{S_3 = S_1}$

$\overline{\Pi} = -(S_3\cdot e) + \frac{1}{2}\sum S_i\Delta l_i$

$2S_1 + S_2 = 0$

$\boxed{S_2 = -2S_1}$

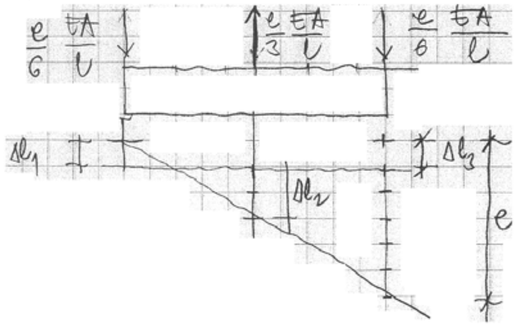
$\Delta l_i = \frac{S_i l}{EA}$

$\overline{\Pi} = +S_1\cdot e + \frac{l}{2EA}\{\underbrace{S_1^2 + 4S_1^2 + S_1^2}_{6S_1^2}\}$

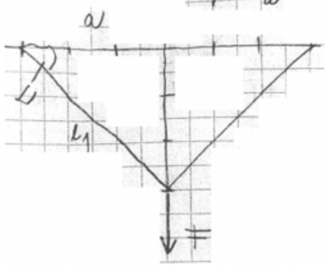
$\frac{\partial\overline{\Pi}}{\partial S_1} = +e + \frac{l}{2EA}\cdot\overset{6}{\cancel{12}}S_1 = 0$

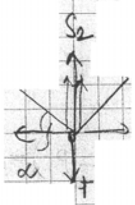
$S_1 = -e\frac{EA}{6l} = S_3$

$S_2 = +e\frac{EA}{3l}$



$\mathrm{tg}\alpha = \frac{\dots}{a}$ $\quad \sin\alpha =$ $\quad \cos\alpha = \frac{a}{l_1}$





$S_1 = S_3$

$$S_1\sin\alpha + S_3\sin\alpha + S_2 = F$$

$$2S_1\sin\alpha + S_2 = F$$

$$S_2 = F - 2S_1\sin\alpha$$

$$S_2^2 = F^2 - 4FS_1\sin\alpha + 4S_1^2\sin^2\alpha$$

$$l_1 = \frac{a}{\cos\alpha}$$

$$l_2 = a\,\mathrm{tg}\alpha = a\frac{\sin\alpha}{\cos\alpha}$$

$$\Delta l_i = \frac{S_i l_i}{EA} \qquad S_i = \frac{\Delta l_i}{l_i}\cdot EA$$

$$\Pi = \frac{1}{2EA}\sum_{i=1}^{3} S_i^2 l_i = \frac{1}{2EA}\left\{2\cdot S_1^2\cdot\frac{a}{\cos\alpha} + a\frac{\sin\alpha}{\cos\alpha}\left(F^2 - 4FS_1\sin\alpha + 4S_1^2\sin^2\alpha\right)\right\}$$

$$\Pi = \frac{a}{\cos\alpha}\cdot\frac{1}{2EA}\left\{2S_1^2 + \sin\alpha\left(F^2 - 4FS_1\sin\alpha + 4S_1^2\sin^2\alpha\right)\right\}$$

$$\frac{\partial\Pi}{\partial S_1} = \frac{a}{\cos\alpha}\,\frac{1}{2EA}\left\{\underbrace{\cancel{4}S_1 + \sin\alpha\left(-\cancel{4}F\sin\alpha + \cancel{4}\cdot 2\cdot S_1\sin^2\alpha\right)}_{\phi}\right\} = 0$$

$$S_1 + 2S_1\sin^3\alpha - F\sin^2\alpha = 0$$

na $\alpha = 45^\circ$

$$\sin^2\alpha = \left(\frac{\sqrt2}{2}\right)^2 = \frac{2}{4} = \frac{1}{2}$$

$$\sin^3\alpha = \frac{1}{2}\frac{\sqrt2}{2} = \frac{\sqrt2}{4}$$

$$S_1\left(1 + 2\sin^3\alpha\right) = F\sin^2\alpha$$

$$\boxed{\begin{aligned} S_1 &= F\,\frac{\sin^2\alpha}{1+2\sin^3\alpha} \\ S_2 &= F\left(1 - \frac{2\sin^3\alpha}{1+2\sin^3\alpha}\right) = F\,\frac{1}{1+2\sin^3\alpha}\end{aligned}}$$

$$\cancel{\frac{2}{1+4\sqrt2}}$$

$$\frac{1}{2+\sqrt2}\cdot\frac{2}{2} = \frac{2}{4+2\sqrt2}$$

$$S_1 + 4\sin\alpha\left(2S_1\sin^2\alpha - P\right) = 0$$

$$S_1\left(1 + 8\sin^3\alpha\right) = 4P\sin^2\alpha$$

$$S_1\left(1 + 2\sqrt2\right) = 2P$$

$$\frac{2}{1+2\sqrt2} = \frac{1}{\frac{1+2\sqrt2}{2}}$$

$$\sin^2\alpha = \frac{1}{2}$$

$$\sin^3\alpha = \frac{1}{2}\cdot\frac{\sqrt2}{2} = \frac{\sqrt2}{4}$$

$$8\cdot\sin^3\alpha = \frac{8}{4}\sqrt2 = 2\sqrt2$$