Készült az ERFP – DD2002 – HU – B – 01 szerzősésszámú projekt támogatásával

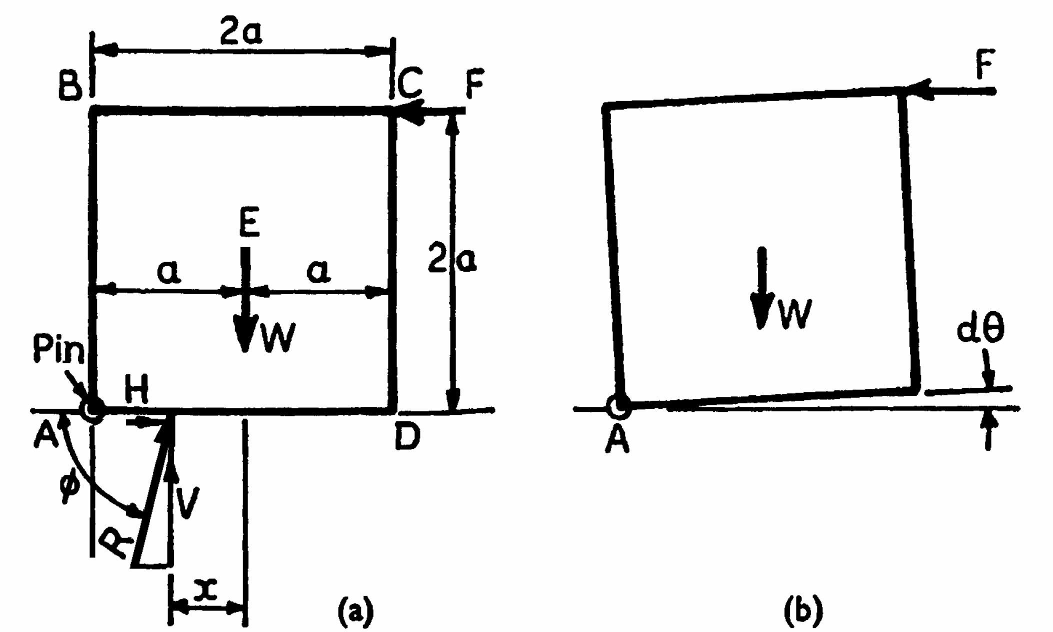
# Chapter 1

# Introduction

# 1.1 Notions of Instability

## The Stability of a Rigid Block Resting on a Table

*Equilibrium Method*




*Equilibrium of Block on Table*

Statical equilibrium of a system:

$$\Sigma H = 0; \quad H - F = 0; \quad H = R\cos\theta$$

$$\Sigma V = 0; \quad V - W = 0; \quad V = R\sin\theta$$

$$\Sigma M_E = 0; \; -Vx + Ha + Fa = 0; \; x = \frac{Ha + Fa}{V} = \frac{2Fa}{W}$$

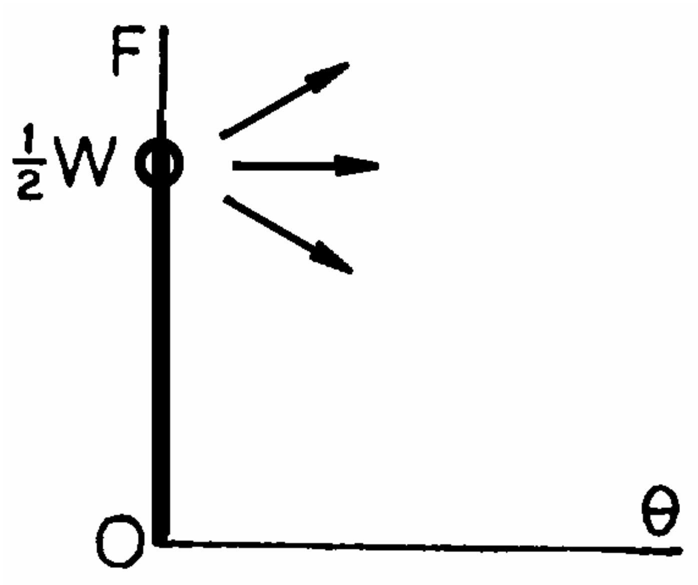
When *x*=*a*, *R* passes through the hinge *A*. The system does not remain in equilibrium, and the block tilts – it becomes <u>unstable</u>:

$$F_{cr} = \frac{1}{2}W$$

Let us now imagine the block to be just tilting – a rotation d$\theta$ has occurred.

*Equilibrium:*

$$\Sigma M_A = 0;\; 2aF - Wa = 0;\; F_{cr} = \frac{1}{2}W$$




*Small displacements*

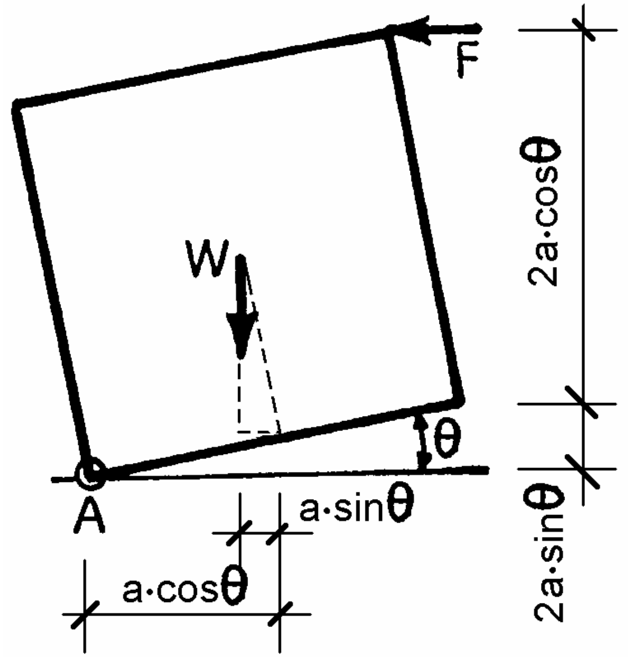
## 1.2 Large Displacements of the Block

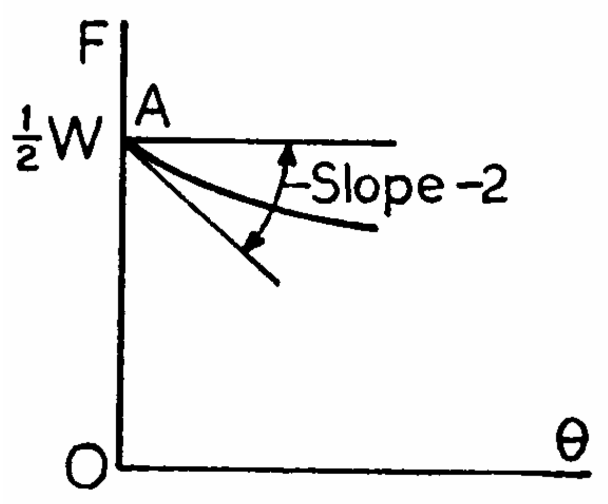
Taking moments about the hinge *A*:

$$\Sigma M_A = 0$$

$$F(2a\cos\theta + 2a\sin\theta) - W(a\cos\theta - a\sin\theta) = 0$$

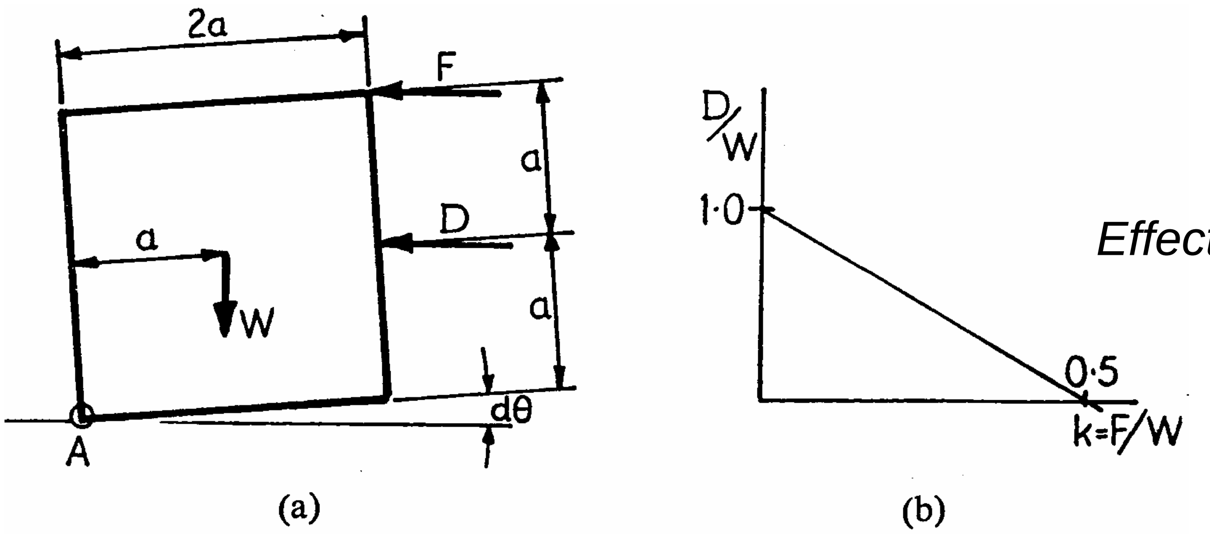
$$F = \frac{1}{2}W\,\frac{\cos\theta - \sin\theta}{\cos\theta + \sin\theta}$$



## 1.3 The Effect of a Disturbance of the System

Suppose the block is acted on by the force *F*, and put *F=kW*.




*Effect of a Disturbing Force*

Taking moments about *A*:

$$\Sigma M_A = 0$$

$$Da + 2Fa - Wa = 0$$

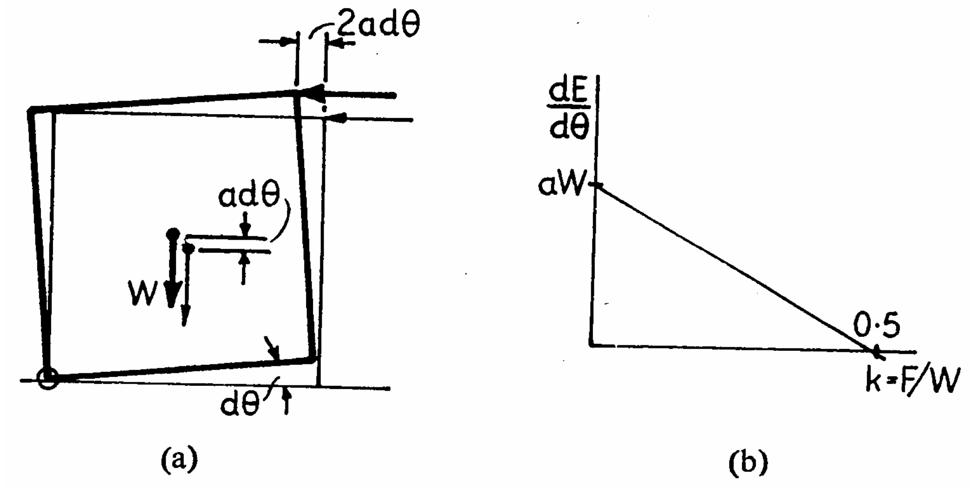
$$D = W - 2F = W(1 - 2k)$$

$$\frac{D}{W} \rightarrow 0 \;;\; k = \frac{1}{2} = \frac{F}{W} \;;\; F_{cr} = \frac{1}{2}W$$

The system has <u>zero resistance</u> to disturbance at $F_{cr}$.

## 1.4 Energy Method

Suppose we now apply an unspecified disturbance to the block, sufficient to rotate it to an angle of inclination dθ from the table, and we examine the energy changes involved.




*Application of Energy Method*

The energy increase involved:

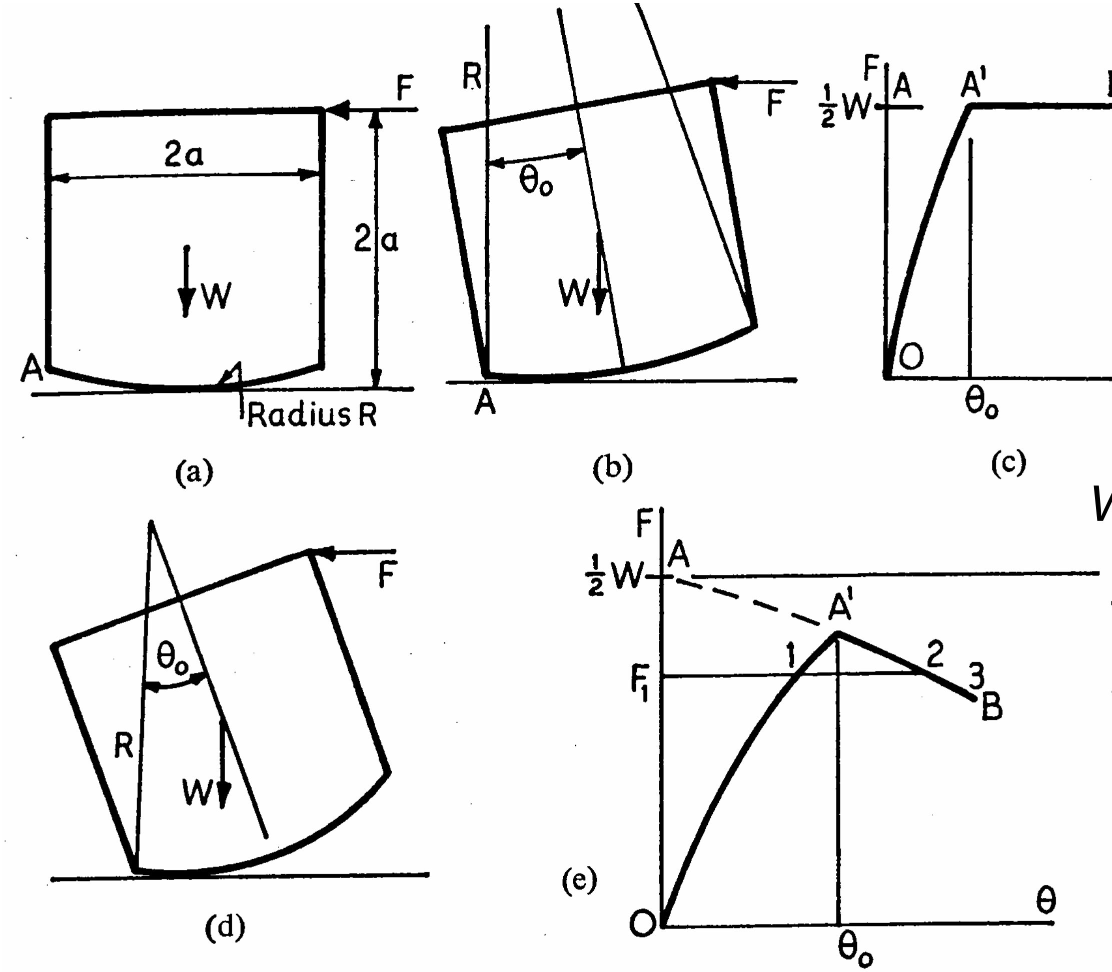
$$dE = -F \cdot 2a \cdot d\theta + W \cdot a \cdot d\theta$$

This quantity of energy must be supplied to cause the movement dθ:

$$\frac{dE}{d\theta} = -F \cdot 2a + W \cdot a = Wa(1-2k) \qquad \frac{dE}{d\theta} \to 0 \qquad k = \frac{1}{2}$$

# 1.5 The Stability of a Slightly Imperfect Block

Suppose now that the block has been accidentally manufactured with a small imperfection:




$\theta_0 \rightarrow$ small

$$2aF - aW = 0$$

$$F = \frac{1}{2}$$

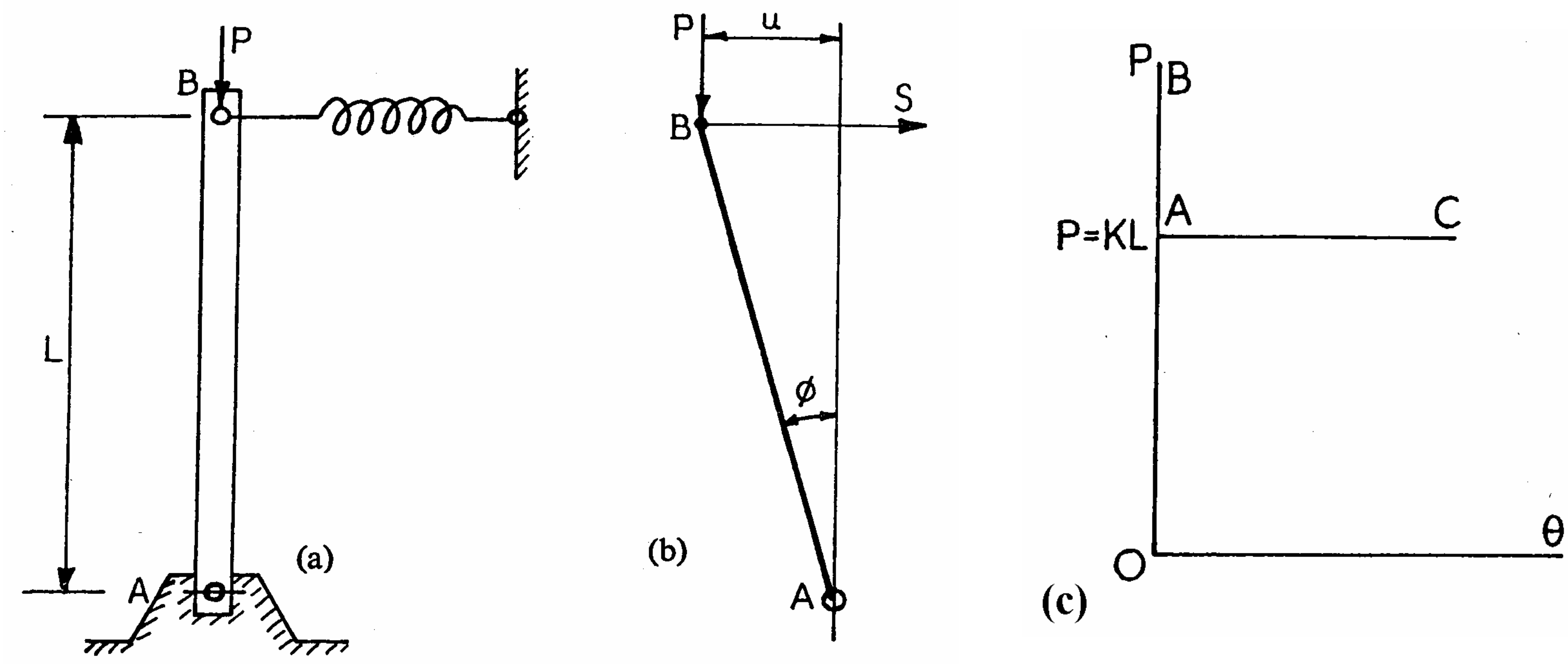
$\theta_0 \rightarrow$ large

$$W(R-a)\sin\theta_0 = 2Fa(\sin\theta_0 + \cos\theta_0)$$

Therefore, at tilting, where $\theta=\theta_0$ and $P$ reaches its maximum value:

$$F_{max} = \frac{W(R-a)\sin\theta_0}{2a(\sin\theta_0 + \cos\theta_0)}$$

# 1.6 The Stability of a Compressed Rigid Bar, Elastically restrained Against Rotation




$$\Sigma M_A = 0$$

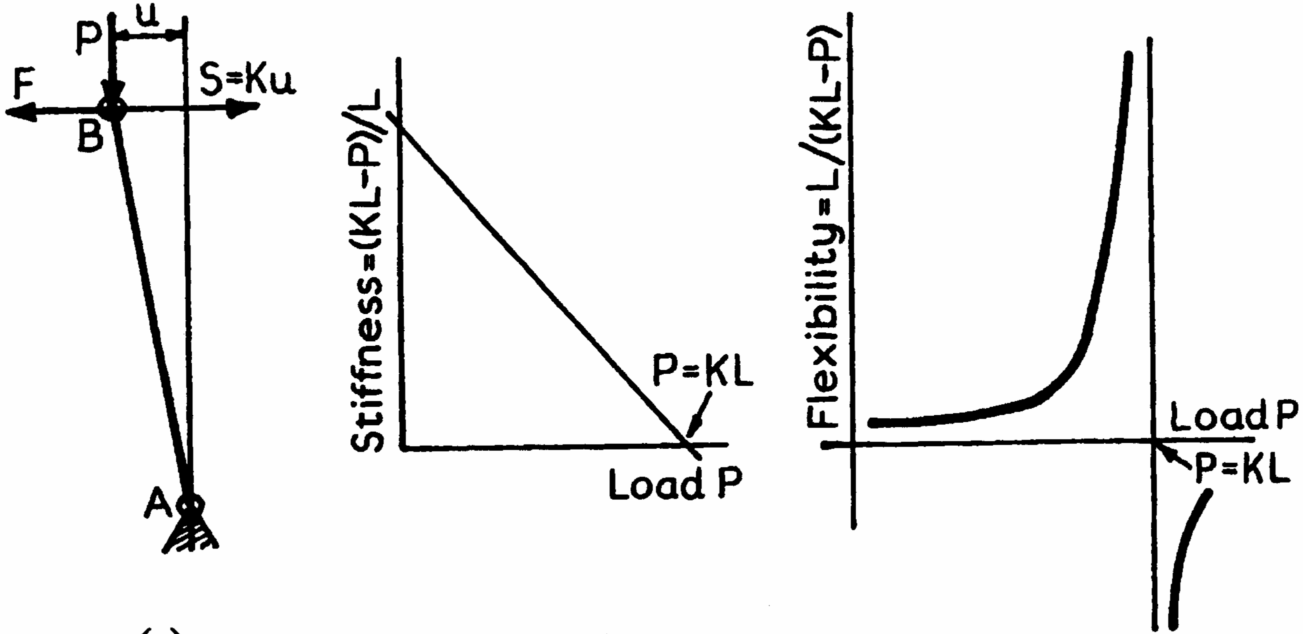
Spring: $S = K \cdot u$

$$Pu - KuL = 0$$

*K:* linear spring of stiffness

$$P_{cr} = K \cdot L$$

## 1.7 Methodology




**(a)** **Disturbing force *F* applied to bar**

**(b)** **Graph of Stiffness against load**

**(c)** **Graph of Flexibility against Load**

(a) Consider the stiffness of the rod and spring system against a disturbance force *F* applied at *B* in the direction normal to *P*.

*Equilibrium:*

$$Pu - KuL + FL = 0$$

(b) The <u>stiffness</u> of the system against the force *F* required to impose a given displacement at *B* equal to *u* is:

$$\frac{F}{u}=\frac{KL-P}{L}=K\left(1-\frac{P}{KL}\right)=K\left(1-\frac{P}{P_{cr}}\right)$$

As *P* approaches the critical value KL, the stiffness *F/u* tends to zero.

(c) Alternatively, we may investigate the response of the system to a given applied disturbance force *F,* in which case the <u>flexibility</u> of the system, as measured by the ratio of the resulting deflection to applied force is:

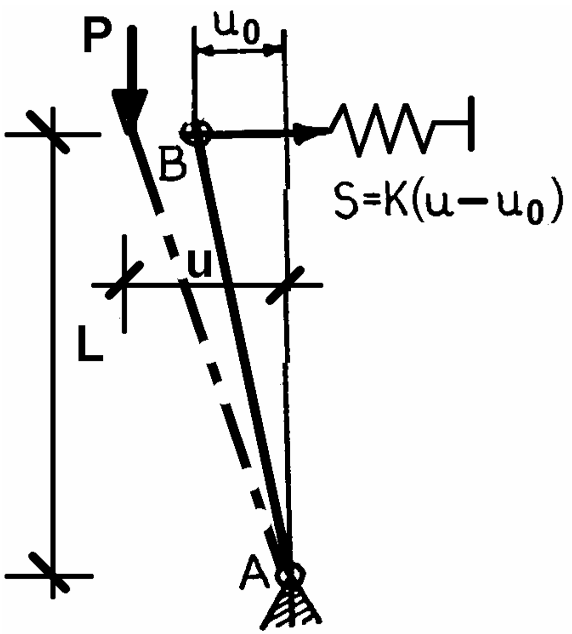
$$\frac{u}{F}=\frac{L}{KL-P}=\frac{1}{K}\left(\frac{1}{1-\frac{P}{KL}}\right)=\frac{1}{K}\left(1-\frac{1}{1-\frac{P}{P_{cr}}}\right)$$

As *P* approaches the critical value KL, the flexibility *u/F* tends to infinity.

## 1.8 Southwell-plot

Consider the effect of the rod being initially out of line with the direction of *P* (imperfection).

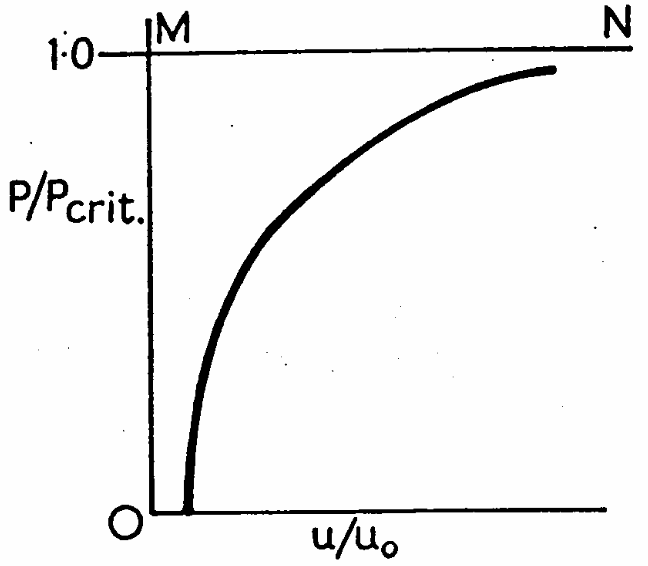
That is, when *P* is zero, *u* has the value $u_0$.



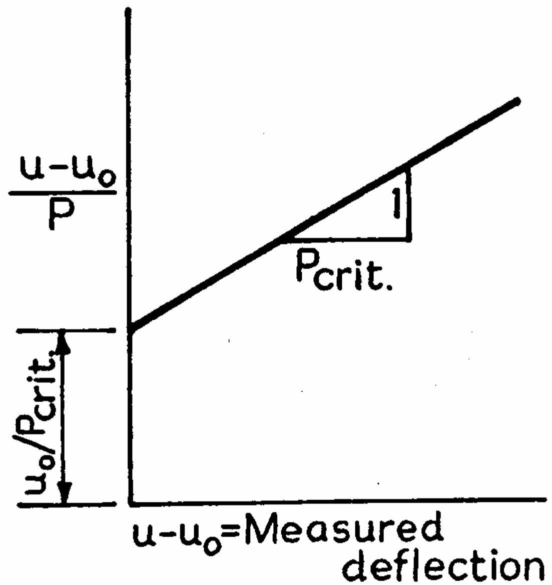

*Equilibrium:*

$$Pu - K(u - u_0)L = 0$$

$$u = \frac{Ku_0L}{KL - P} = \frac{u_0}{1 - \frac{P}{KL}} = \frac{1}{1 - \frac{P}{P_{cr}}}u_0$$




(a)




(b)

$$u\left(1-\frac{P}{P_{cr}}\right)=u_0$$

$$u(P_{cr}-P)=u_0P_{cr}$$

$$(u-u_0)P_{cr}=uP$$

$$\frac{u-u_0}{P}=\frac{u}{P_{cr}}-\frac{u_0}{P_{cr}}+\frac{u_0}{P_{cr}}$$

$$\frac{u-u_0}{P}=\frac{u-u_0}{P_{cr}}+\frac{u_0}{P_{cr}}$$

$$\boxed{\frac{\delta}{P}=\frac{\delta}{P_{cr}}+\frac{u_0}{P_{cr}}}$$