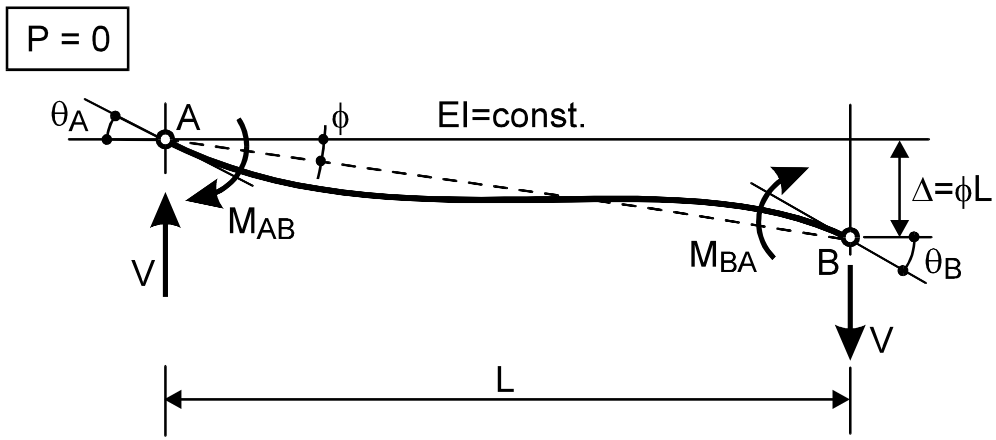
Készült az ERFP – DD2002 – HU – B – 01 szerzősésszámú projekt támogatásával

C h a p t e r 3

Stability Functions

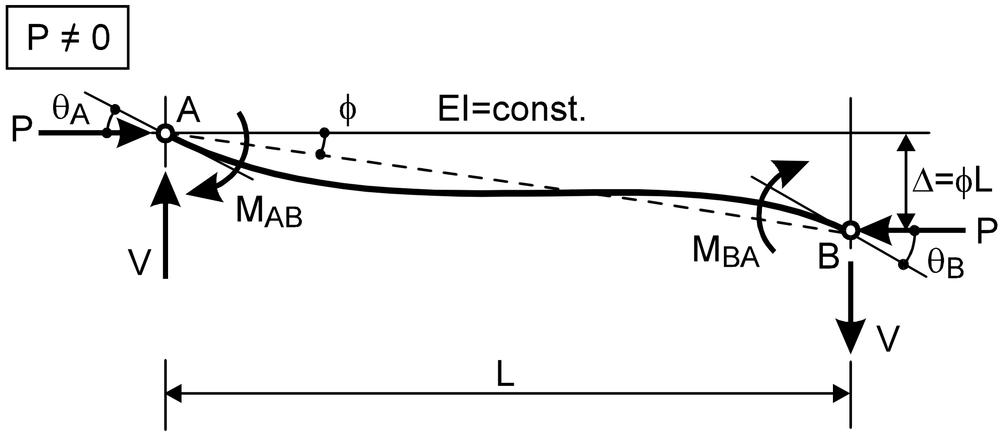
3.1. Members Subjected to Bending Moments

[Horne, Merchant, 1965]



In case of $P = 0$:

$$M_{AB} = \frac{EI}{L} \cdot \left(4\theta_A + 2\theta_B - \frac{6}{L} \cdot \Delta \right)$$



In case of $P \neq 0$:

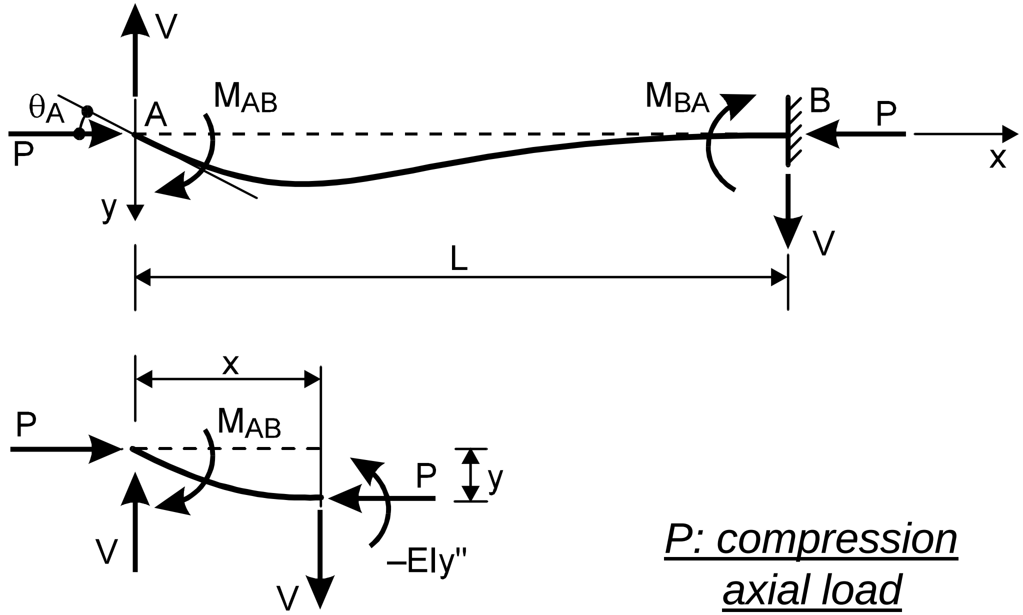
$$M_{AB} = \frac{EI}{L} \cdot \left(s \cdot \theta_A + s \cdot c \cdot \theta_B - \frac{s \cdot (1+c)}{L} \cdot \Delta \right)$$

s – stiffness function

c – carry-over function

3.2. Effect of Axial Load on Member Stiffness

3.2.1 Functions s and c: End Rotation, Far End Fixed



P: compression axial load

Equilibrium equation:

$$P \cdot y + M_{AB} + V \cdot x = -EI \cdot y''$$

Differential equation:

$$y'' + \kappa^2 \cdot y = \frac{M_{AB}}{EI} \cdot \left(\frac{x}{L} - 1 \right) + \frac{M_{BA}}{EI} \cdot \frac{x}{L}$$

$$\kappa^2 = \frac{P}{EI} \qquad \kappa \cdot L = 2\alpha$$

General solution:

$$y = A \cdot \sin \kappa x + B \cdot \cos \kappa x + \frac{M_{AB}}{P} \cdot \left(\frac{x}{L} - 1 \right) + \frac{M_{BA}}{P} \cdot \frac{x}{L}$$

Boundary conditions:

$$x = 0 \quad \rightarrow \quad y = 0; \qquad x = L \quad \rightarrow \quad y = 0.$$

$$B = \frac{M_{AB}}{P} \qquad A = -\frac{M_{AB}}{P} \cdot \frac{\cos \kappa L}{\sin \kappa L} - \frac{M_{BA}}{P} \cdot \frac{1}{\sin \kappa L}$$

$$y'(x=L)=0 \quad \rightarrow$$ Stability function *c*

$$c=\frac{M_{BA}}{M_{AB}}=\frac{\kappa L-\sin\kappa L}{\sin\kappa L-\kappa\cdot L\cdot\cos\kappa L}=\frac{2\alpha-\sin 2\alpha}{\sin 2\alpha-2\alpha\cdot\cos 2\alpha}$$

$$y'(x=L)=\theta_a \quad \rightarrow$$ Stability function *s*

$$s=\frac{(1-\kappa\cdot L\cdot\operatorname{ctg}\kappa L)\cdot\frac{1}{2}\cdot\kappa\cdot L}{\operatorname{tg}\frac{1}{2}\kappa L-\frac{1}{2}\cdot\kappa\cdot L}=\frac{(1-2\alpha\cdot\operatorname{ctg}2\alpha)\cdot\alpha}{\operatorname{tg}\alpha-\alpha}$$

$$k=EI/L \quad \rightarrow \quad M_{AB}=s\cdot k\cdot\theta_A$$

$$M_{BA}=s\cdot c\cdot k\cdot\theta_A$$

$$V=-\frac{M_{AB}+M_{BA}}{L}=-s\cdot(1+c)\cdot\frac{k}{L}\cdot\theta_A$$

$$\rho=P/P_E \qquad \alpha=\frac{\kappa L}{2}=\frac{\pi}{2}\cdot\sqrt{\rho}$$

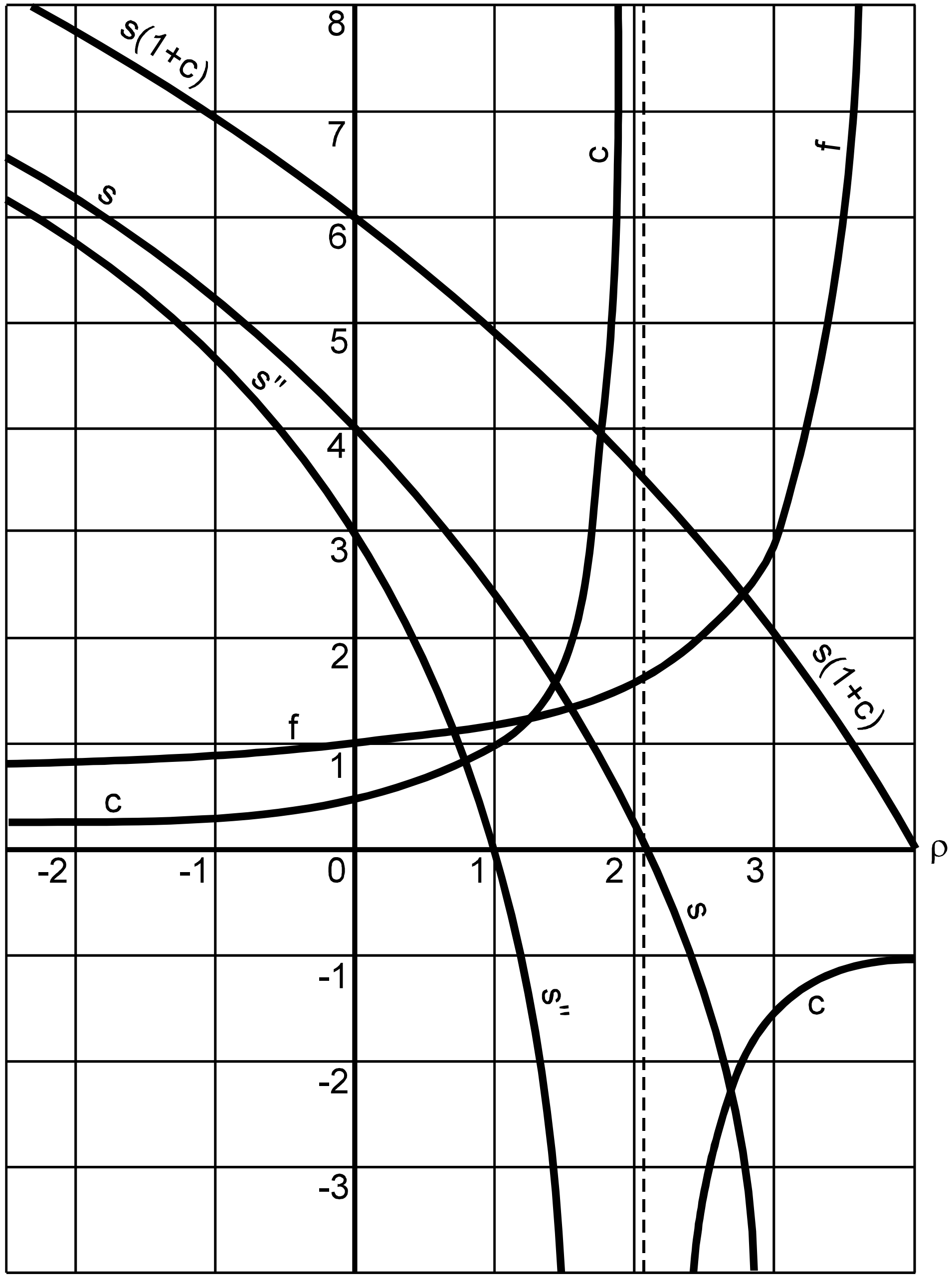
<u>If P: tension axial load:</u>

$$\gamma=\frac{\pi}{2}\cdot\sqrt{-\rho}=\mathrm{i}\cdot\alpha$$

$$c=\frac{2\gamma-\operatorname{sh}2\gamma}{\operatorname{sh}2\gamma-2\gamma\cdot\operatorname{ch}2\gamma}$$

$$s=\frac{(1-2\gamma\cdot\operatorname{cth}2\gamma)\cdot\gamma}{\operatorname{th}\gamma-\gamma}$$

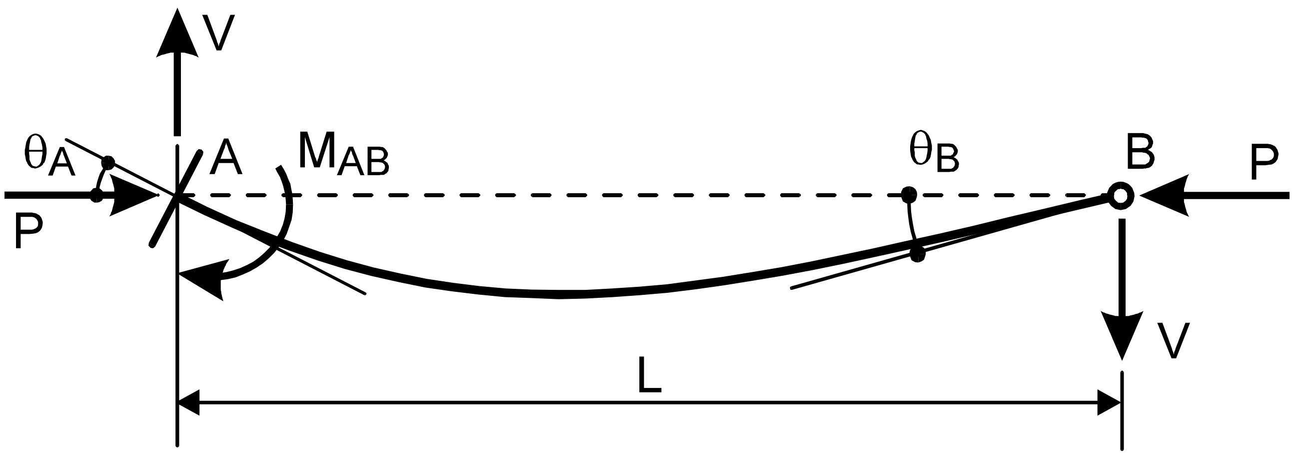
Stability Functions



$\rho = P / P_E$ (tension)

$\rho = P / P_E$ (compression)

3.2.2 Function s”: End Rotation, Far End Pinned



Equilibrium equations:

$$M_{AB} = s \cdot k \cdot \theta_A + s \cdot c \cdot k \cdot \theta_B$$

$$M_{BA} = s \cdot c \cdot k \cdot \theta_A + s \cdot k \cdot \theta_B = 0$$

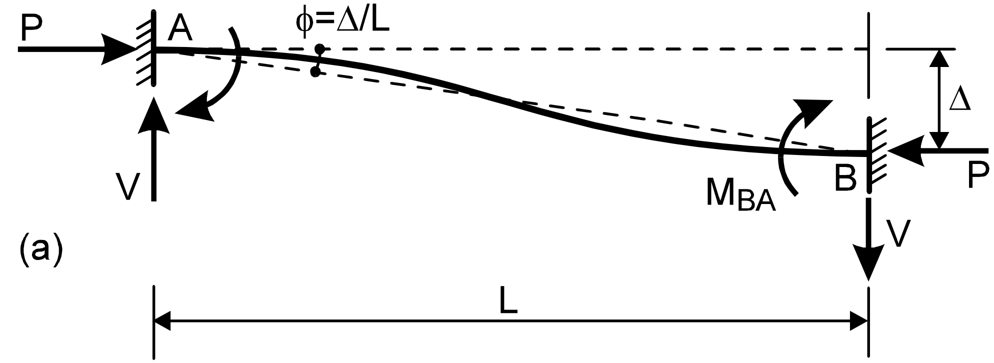
$$M_{AB} = s \cdot (1 - c^2) \cdot k \cdot \theta_A = s'' \cdot k \cdot \theta_A$$

Pinned far end:

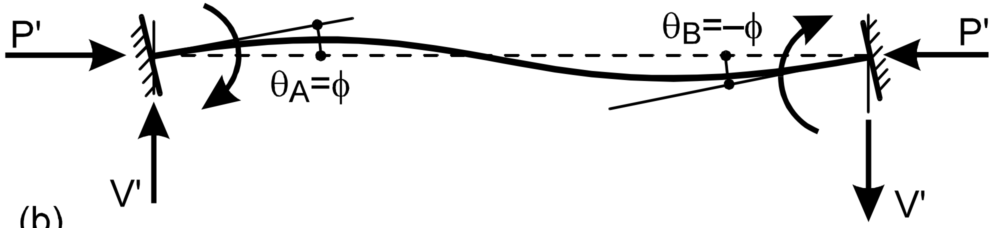
$$\boxed{s'' = s \cdot (1 - c^2)}$$

$$M_{BA} = 0 \quad \rightarrow \quad \theta_B = -c \cdot \theta_A$$

3.2.3 Sway Function s(1+c) and m: Joint Translation. Both Ends Fixed



P≈P'   V≈V'



Equilibrium equations:

$$M_{AB} = M_{BA} = -s\cdot(1+c)\cdot k\cdot\phi = $$

$$= -s\cdot(1+c)\cdot k\cdot\frac{\Delta}{L}$$

$$M_{AB} + M_{BA} + V\cdot L + P\cdot\Delta = 0$$

$$V = \left[2\cdot s\cdot(1+c)\cdot k\cdot\frac{\Delta}{L} - \frac{\pi^2\cdot\rho\cdot k}{L}\cdot\Delta\right]\cdot\frac{1}{L} =$$

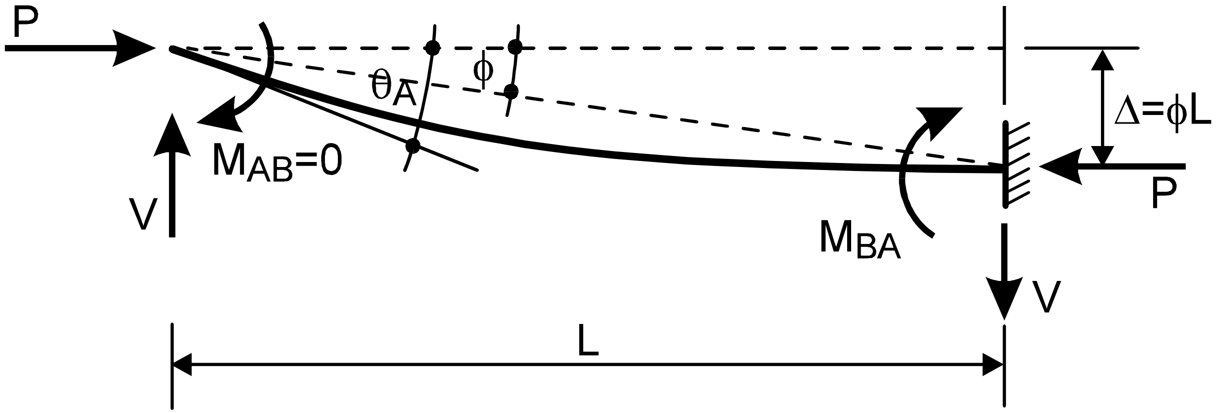
$$= \left[2s\cdot(1+c) - \pi^2\cdot\rho\right]\cdot\frac{k\cdot\Delta}{L^2}$$

$$\boxed{m = \frac{2s\cdot(1+c)}{2s\cdot(1+c)-\pi^2\cdot\rho}}$$

$$M_{AB} = M_{BA} = -m\cdot\frac{V\cdot L}{2}$$

$$V = \frac{2s\cdot(1+c)}{m}\cdot\frac{k}{L^2}\cdot\Delta = \frac{2s\cdot(1+c)}{m}\cdot\frac{k}{L}\cdot\phi$$

3.2.4 Functions for Joint Translation: One End Sway



$$M_{BA} = -s'' \cdot k \cdot \phi = -\frac{s''}{s'' - \pi^2 \cdot \rho} \cdot V \cdot L$$

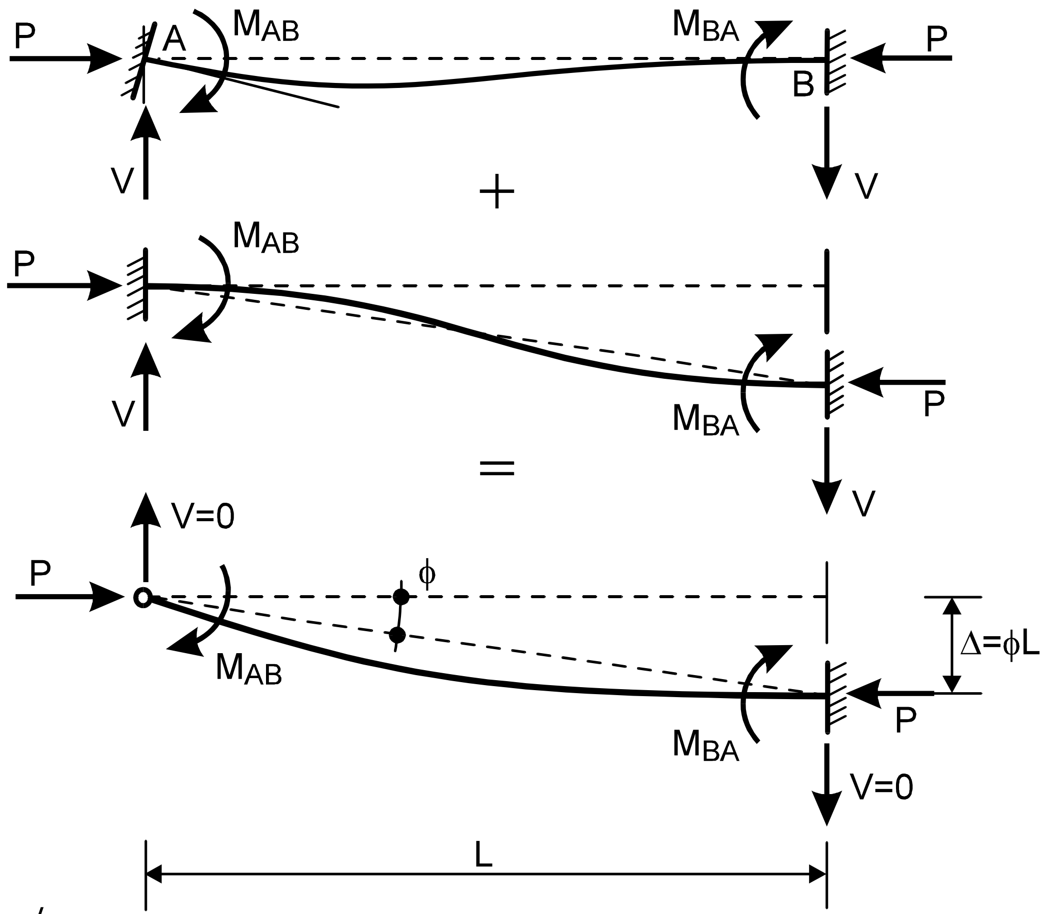
$$V = (s'' - \pi^2 \rho) \cdot \frac{k}{L} \cdot \phi$$

$$\theta_A = (1 + c) \cdot \phi$$

3.2.5 No Shear Function n and o: No-shear Translation

$$M_{AB} = s \cdot k \cdot \theta_A - s \cdot (1+c) \cdot k \cdot \phi = $$
$$= n \cdot k \cdot \theta_A$$

$$M_{BA} = c \cdot s \cdot k \cdot \theta_A - s \cdot (1+c) \cdot k \cdot \phi = $$
$$= -o \cdot k \cdot \theta_A$$



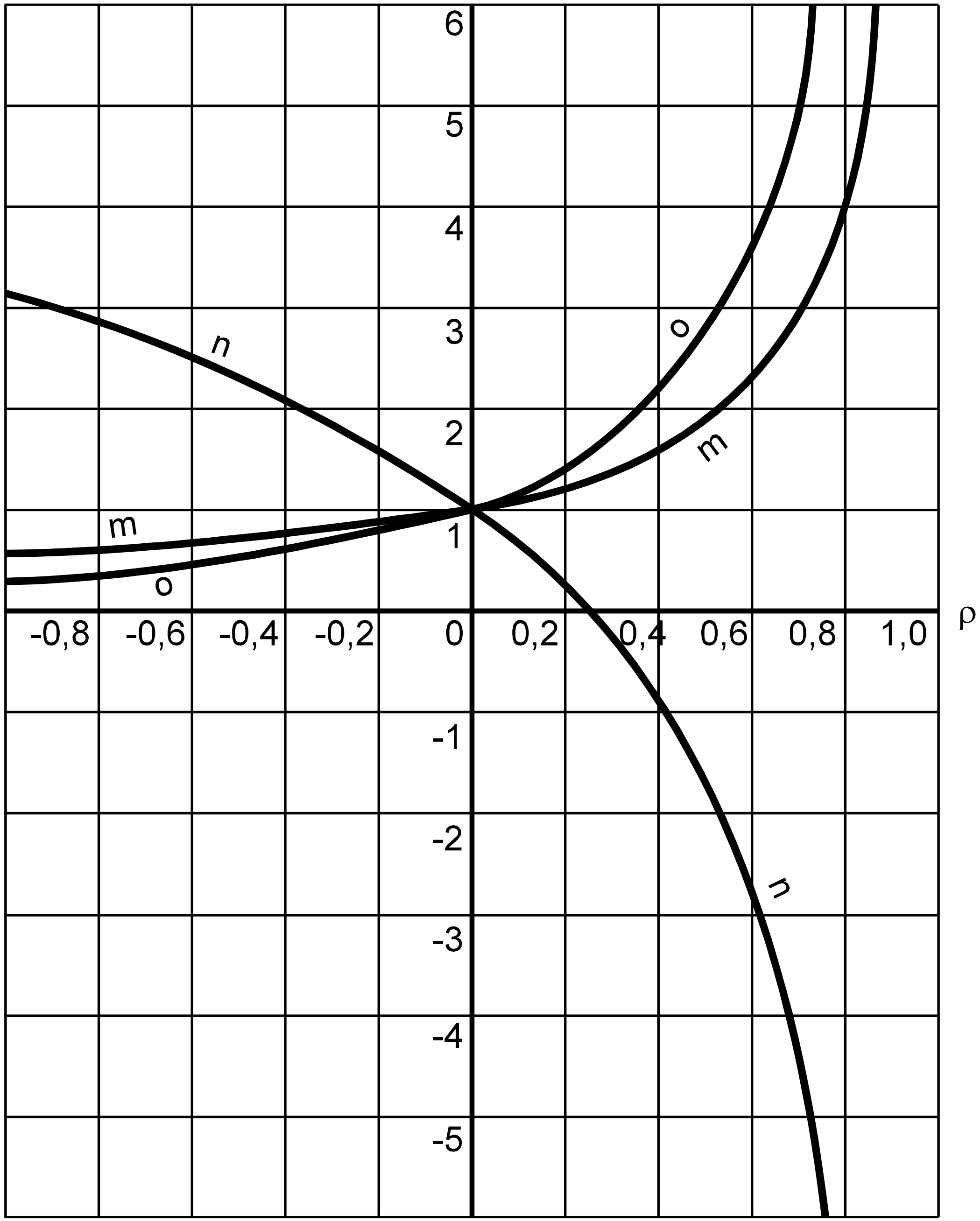
$$\boxed{n = s \cdot \left[1 - \frac{m \cdot (1+c)}{2}\right]}$$

$$\boxed{o = s \cdot \left[\frac{m \cdot (1+c)}{2} - c\right]}$$

$$V = -\frac{s \cdot (1+c) \cdot k \cdot \theta_A}{L} + \frac{2 \cdot s \cdot (1+c) \cdot k \cdot \phi}{m \cdot L} = 0$$

$$\phi = \frac{m}{2} \cdot \theta_A$$

Stability Functions



$\rho = P / P_E$ (tension)

$\rho = P / P_E$ (compression)

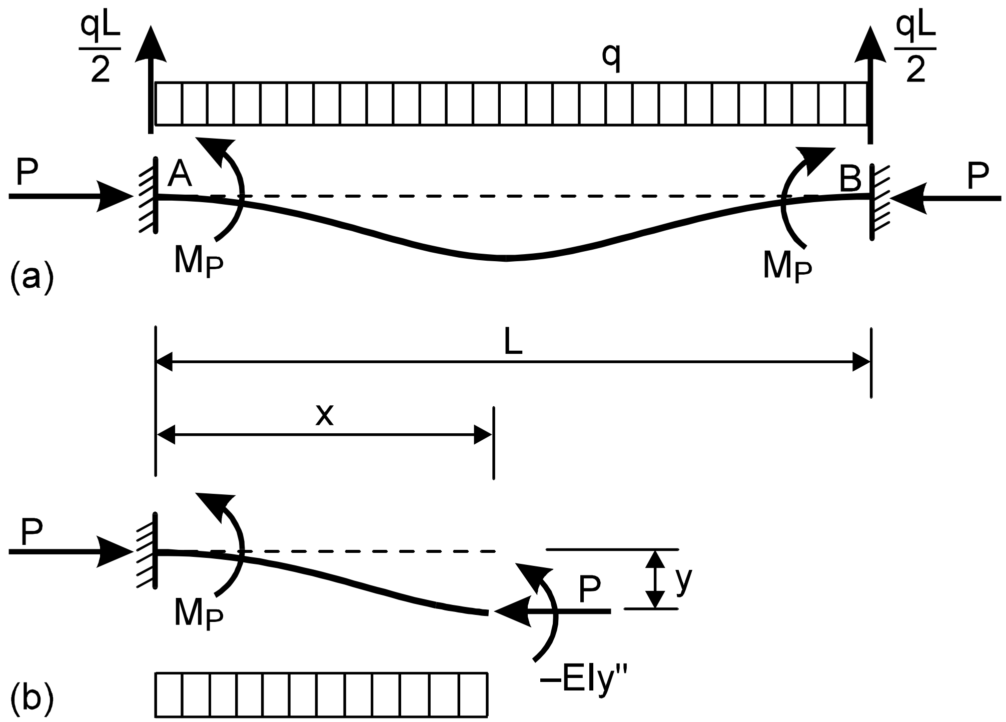
3.2.6 Uniformly Distributed Load

(a) Both Ends Fixed

$$P \cdot x - M_P + \frac{q \cdot x \cdot (L-x)}{2} = EI \cdot y''$$

$$M_P = f \cdot \frac{q \cdot L^2}{12}$$

$$\boxed{f = \frac{3}{\alpha^2} \cdot (1 - \alpha \cdot \operatorname{ctg} \alpha)}$$



In case of tension:

$\gamma = i \cdot \alpha$ $\quad\quad \operatorname{ctg} \alpha = i \cdot \operatorname{cth}(i \cdot \alpha)$

$$\boxed{f = \frac{3}{\gamma^2} \cdot (\gamma \cdot \operatorname{cth} \gamma - 1)}$$

(b) Far End Pinned

$$M_{PA} = 0$$

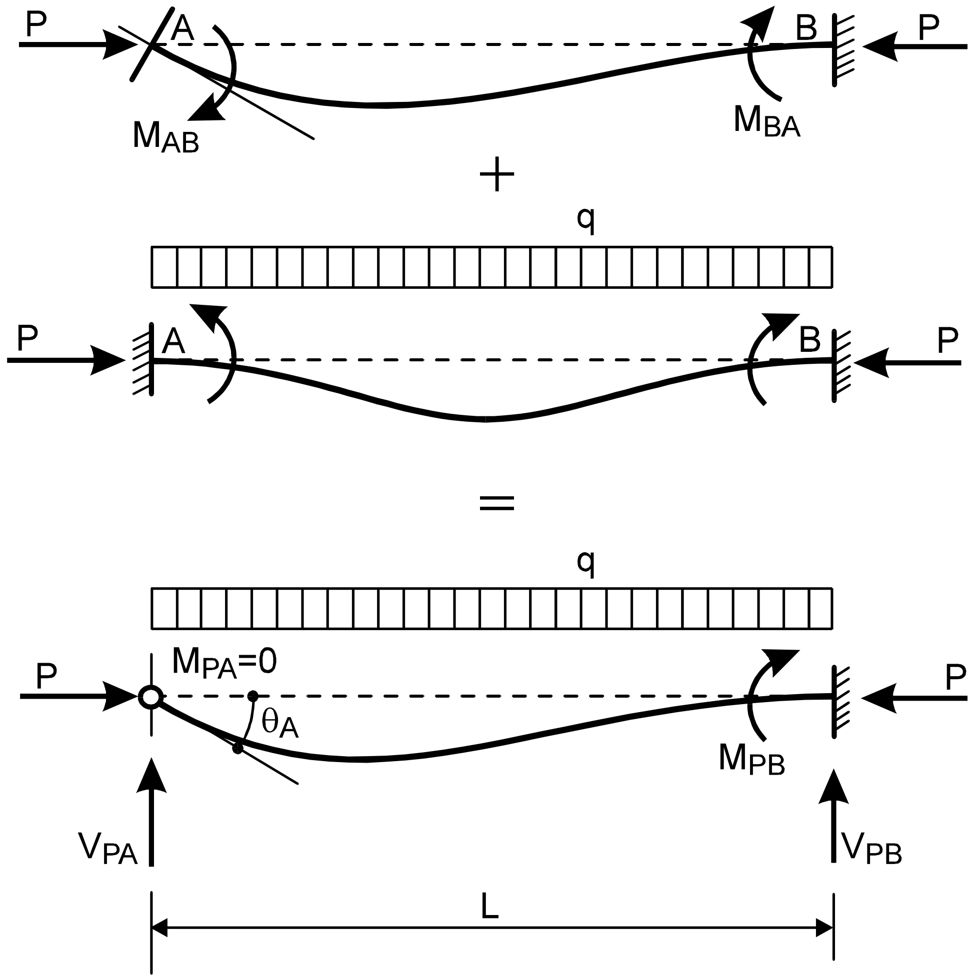
$$M_{PB} = f \cdot (1+c) \cdot \frac{q \cdot L^2}{12}$$

$$\theta_A = \frac{f}{s \cdot k} \cdot \frac{q \cdot L^2}{12}$$

$$V_{PA} = \left[1 - \frac{f \cdot (1+c)}{6}\right] \cdot \frac{q \cdot L}{2}$$

$$V_{PB} = \left[1 + \frac{f \cdot (1+c)}{6}\right] \cdot \frac{q \cdot L}{2}$$

$$\boxed{f'' = f \cdot (1+c)}$$



3.2.7 Concentrated Load

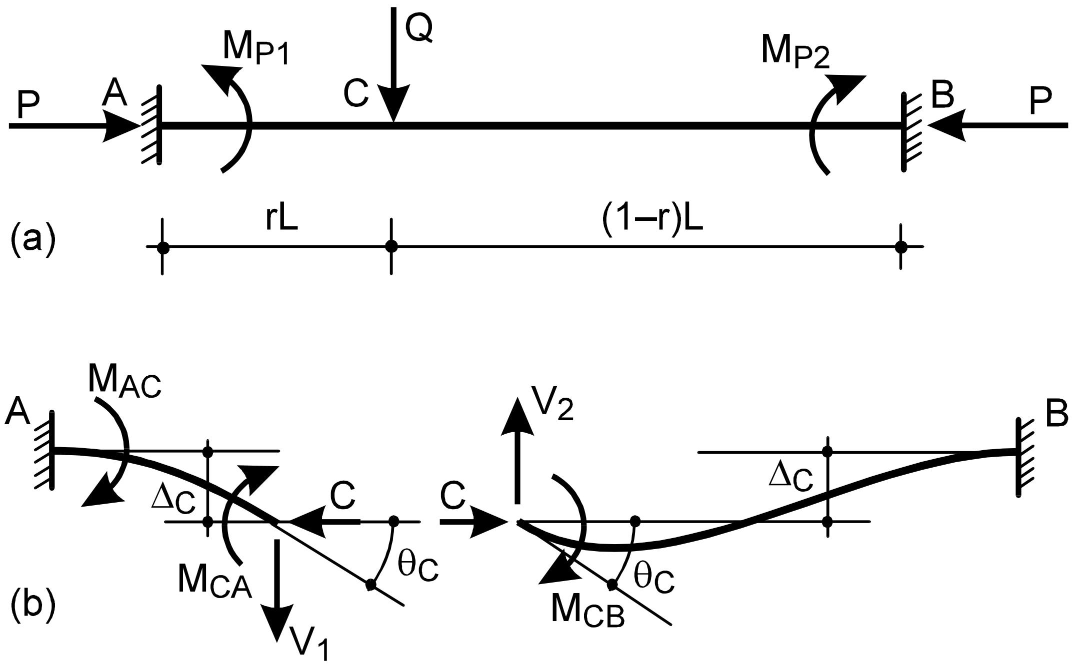
$$\rho = P / P_E \qquad \rho_1 = r^2 \cdot \rho \qquad \rho_2 = (1-r)^2 \cdot \rho$$

$$P_{E1} = \frac{P_E}{r^2} \qquad P_{E2} = \frac{P_E}{(1-r)^2}$$

$$k = \frac{EI}{L} \qquad k_1 = \frac{EI}{r \cdot L} = \frac{k}{r}$$

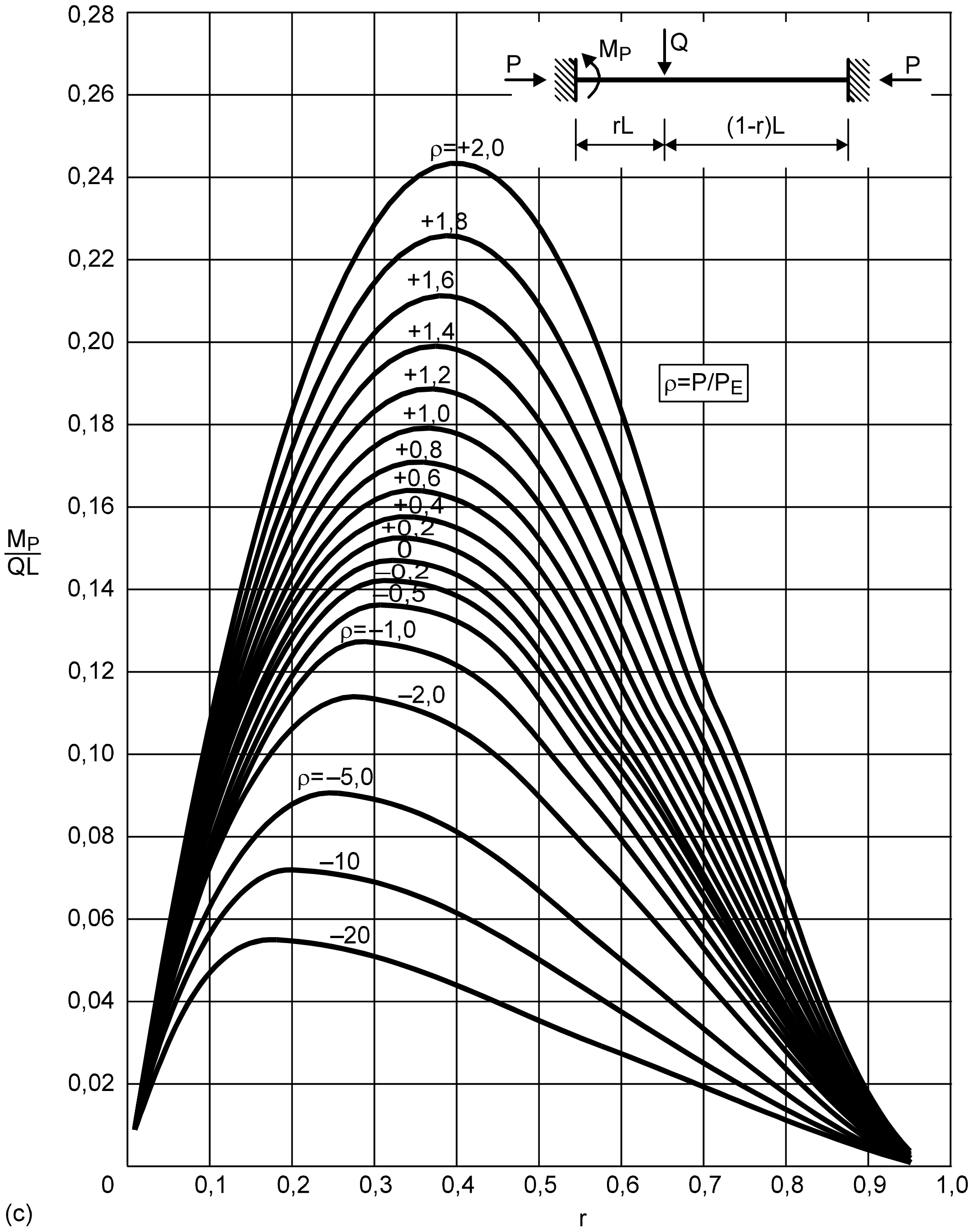
$$k_2 = \frac{EI}{(1-r) \cdot L} = \frac{k}{1-r}$$

$$\frac{M_{P1}}{Q \cdot L} = \frac{(1+c_1) \cdot A - c_1 \cdot r \cdot B}{2A \cdot C - B^2} \cdot \frac{s_1}{r^2}$$



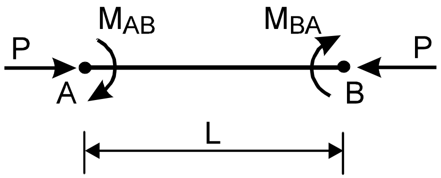
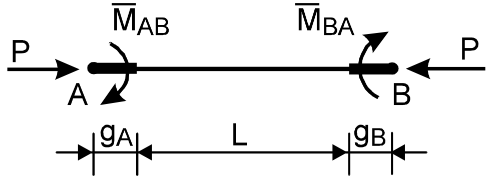
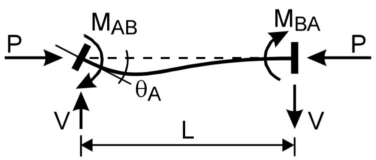
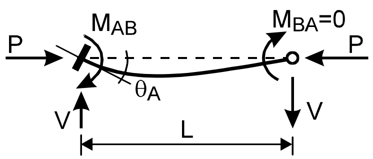
$$A = \frac{s_1}{r} + \frac{s_2}{1-r} \qquad B = \frac{s_1 \cdot (1+c_1)}{r^2} - \frac{s_2 \cdot (1+c_2)}{(1-r)^2} \qquad C = \frac{s_1 \cdot (1+c_1)}{r^2 \cdot m_1} + \frac{s_2 \cdot (1+c_2)}{(1-r)^2 \cdot m_2}$$

Stability Functions



(c)

3.2.8 Summary of Operations

Cases	Uniform member	Uniform member with gussets
	 $k=\frac{EI}{L};\quad \rho=\frac{1}{\pi^2}\cdot\frac{P\cdot L}{k}=\frac{P}{P_E};$ $s''=s\cdot(1-c^2)$	 $k=\frac{EI}{L};\quad \rho=\frac{1}{\pi^2}\cdot\frac{P\cdot L}{k}=\frac{P}{P_E}$ $D=s\cdot(1+c)-\frac{\pi^2}{2}\cdot\rho=\frac{s\cdot(1+c)}{m}$
	$M_{AB}=s\cdot k\cdot\theta_A$ $M_{BA}=c\cdot s\cdot k\cdot\theta_A=c\cdot M_{AB}$ $V=-s\cdot(1+c)\cdot\frac{k}{L}\cdot\theta_A$	$\overline{M}_{AB}=\bar{s}\cdot k\cdot\theta_A;\quad M_{BA}=\bar{c}\cdot\bar{s}\cdot k\cdot\theta_A;$ $V=-(\bar{s}+\bar{s}\cdot\bar{c})\cdot\frac{k}{\overline{L}}\cdot\theta_A;$ $\overline{L}=L+g_A+g_B;$ $\bar{s}=s+\frac{2g_A\cdot g_B}{L^2}\cdot D+s\cdot(1+c)\cdot\frac{g_A+g_B}{L};$ $\bar{s}+\bar{s}\cdot\bar{c}=\left[s\cdot(1-c)+\frac{2g_A}{L}\cdot D\right]\cdot\frac{\overline{L}}{L}$
	$M_{AB}=s''\cdot k\cdot\theta_A$ $V=-s''\cdot\frac{k}{L}\cdot\theta_A$ $\theta_B=-c\cdot\theta_A$	

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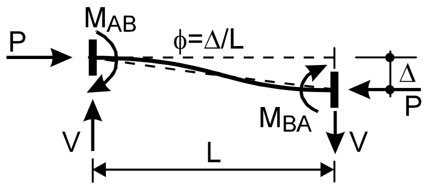
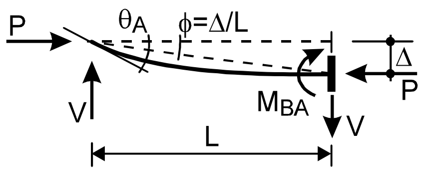
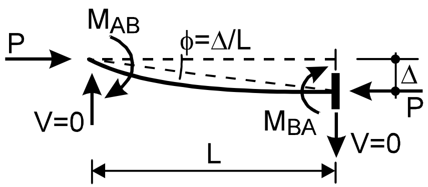
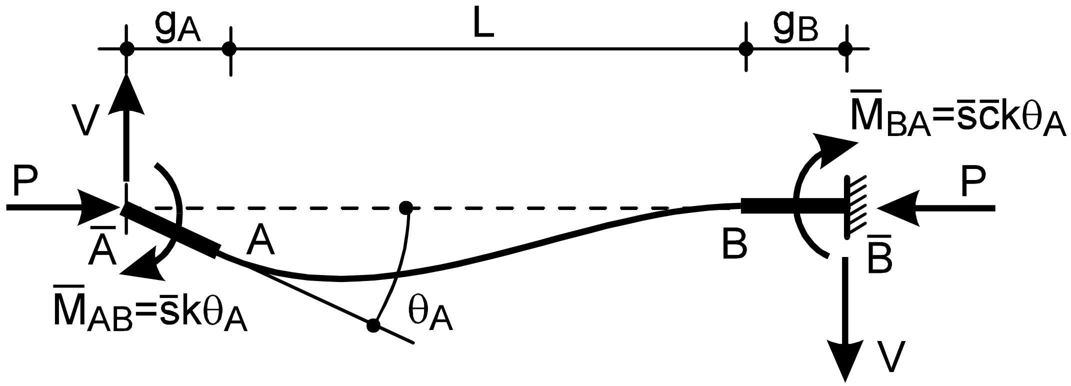
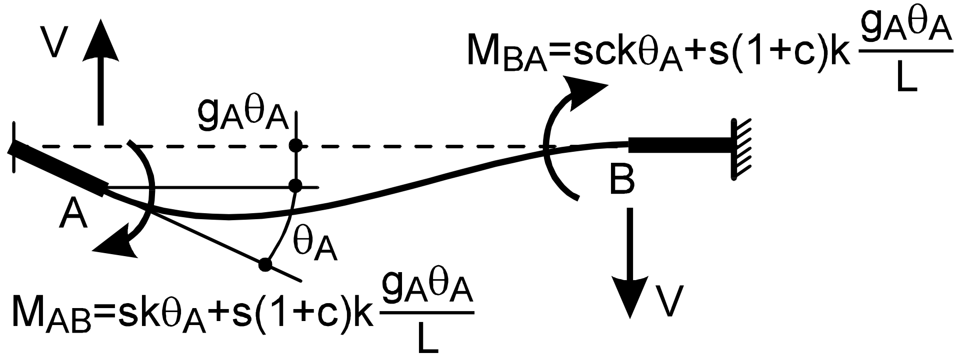
Cases	Uniform member	Uniform member with gussets
	$M_{AB} = M_{BA} = s \cdot (1+c) \cdot k \cdot \phi = -m \cdot \frac{V \cdot L}{2}$; $V = \frac{2s \cdot (1+c)}{m} \cdot \frac{k}{L} \cdot \phi$;	$\overline{M}_{AB} = -\overline{m}_A \cdot \frac{V \cdot L}{2}$; $\overline{M}_{BA} = -\overline{m}_B \cdot \frac{V \cdot L}{2}$; $V = D \cdot \frac{k}{L^2} \cdot \Delta$; $\overline{m}_A = m + \frac{2g_A}{L}$; $\overline{m}_B = m + \frac{2g_B}{L}$
	$M_{BA} = -s'' \cdot k \cdot \phi = -\frac{s''}{s'' - \pi^2 \cdot \rho} \cdot V \cdot L$; $V = (s'' - \pi^2 \cdot \rho) \cdot \frac{k}{L} \cdot \phi$; $\theta_A = (1+c) \cdot \phi$	
	$M_{AB} = n \cdot k \cdot \theta_A$; $M_{BA} = o \cdot k \cdot \theta_A$; $\phi = \frac{m}{2} \cdot \theta_A$	$\overline{M}_{AB} = \overline{n} \cdot k \cdot \theta_A$; $\overline{M}_{BA} = \overline{o} \cdot k \cdot \theta_A$; $\phi = \frac{\overline{m}_A}{2} \cdot \theta_A$; $\overline{n} = n - \pi^2 \cdot \rho \cdot \frac{g_A}{L}$; $\overline{o} = o$

3.2.9 Effect of Gusset Plates



(a)



$$\bar{s} = s + \frac{2g_A}{L} \cdot \left(1 + \frac{g_A}{L}\right) \cdot D$$

(b)

$$\bar{s} \cdot \bar{c} = s \cdot c + s \cdot (1+c) \cdot \left(\frac{g_A}{L} + \frac{g_B}{L}\right) + 2 \cdot \frac{g_A}{L} \cdot \frac{g_B}{L} \cdot D$$

$$\bar{s} \cdot (1+\bar{c}) = \left[s \cdot (1+c) + \frac{2g_A}{L} \cdot D\right] \cdot \left(1 + \frac{g_A}{L} + \frac{g_B}{L}\right)$$

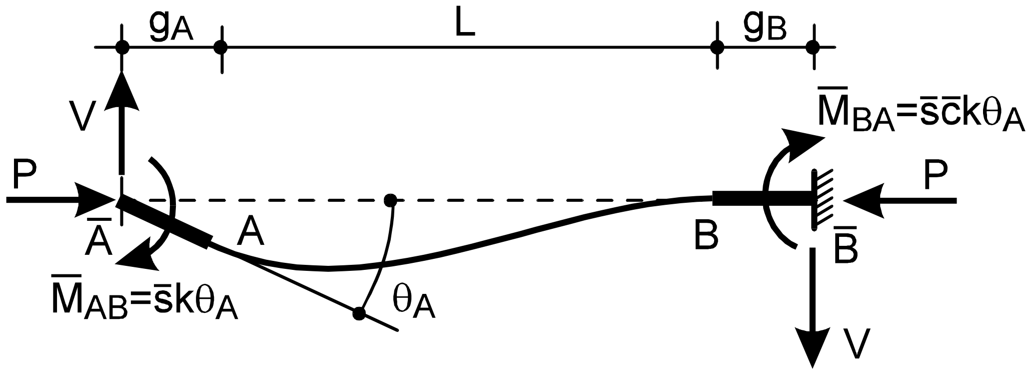
$$D = s \cdot (1+c) - \frac{\pi^2}{2} \cdot \rho$$

3.2.10 Effect of Flexible Connections

$$M_{AB} = k \cdot (s \cdot \theta_A + s \cdot c \cdot \theta_B)$$

$$M_{BA} = k \cdot (s \cdot c \cdot \theta_A + s \cdot \theta_B)$$

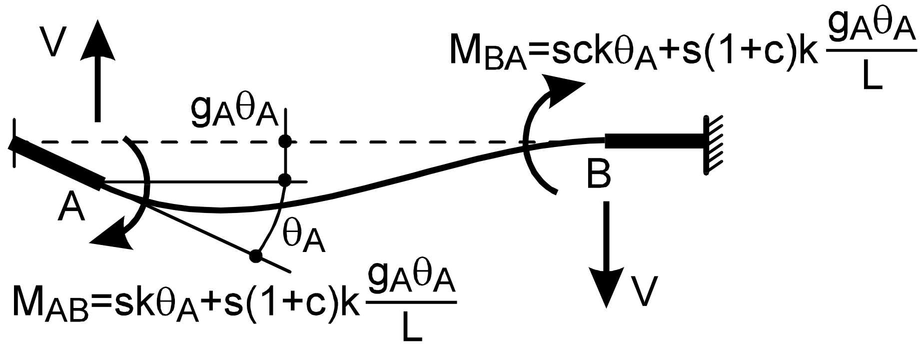
$$k = EI / L$$



(a)

$$M_{AB} = G_A \cdot (\theta'_A - \theta_A)$$

$$G_A = C_A \cdot k \qquad G_B = C_B \cdot k$$



(b)

$$M_{AB} = C_A \cdot k \cdot (\theta'_A - \theta_A)$$

$$M_{BA} = C_B \cdot k \cdot (\theta'_B - \theta_B)$$

$$M_{AB} = k \cdot [s'_A \cdot \theta'_A + (s \cdot c)' \cdot \theta'_B]$$

$$M_{BA} = k \cdot [(s \cdot c)' \cdot \theta'_A + s'_B \cdot \theta'_B]$$

$$p = 1 + s \cdot \left(\frac{1}{C_A} + \frac{1}{C_B} \right) + \frac{s^2 \cdot (1 - c^2)}{C_A \cdot C_B}$$

$$s'_A = \left(s + \frac{s^2 \cdot (1 - c^2)}{C_B} \right) \cdot \frac{1}{p}$$

$$s'_B = \left(s + \frac{s^2 \cdot (1 - c^2)}{C_A} \right) \cdot \frac{1}{p}$$

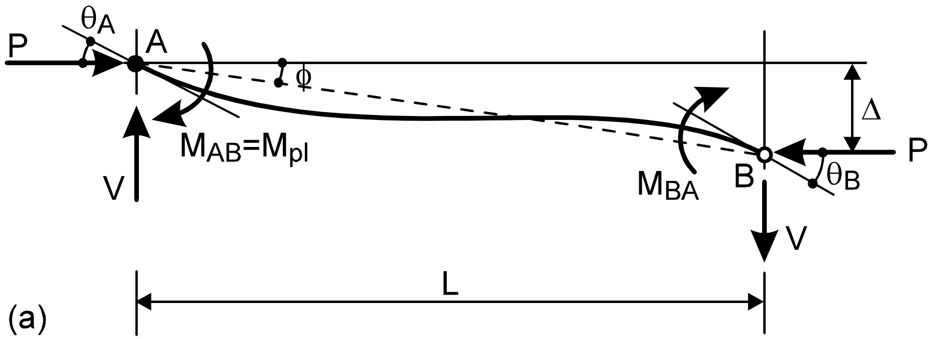
$$(s \cdot c)' = \frac{s \cdot c}{p}$$

3.2.11 Effect of Plastic Hinges

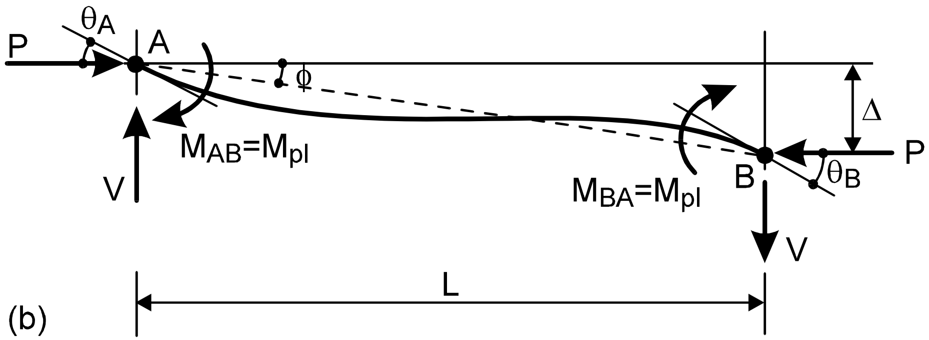
$$M_{AB} = s \cdot k \cdot \theta_A + c \cdot s \cdot k \cdot \theta_B - s \cdot (1+c) \cdot k \cdot \phi$$

$$M_{BA} = c \cdot s \cdot k \cdot \theta_A + s \cdot k \cdot \theta_B - s \cdot (1+c) \cdot k \cdot \phi$$

$$V = s \cdot (1+c) \cdot \frac{k}{L} \cdot \theta_A + s \cdot (1+c) \cdot \frac{k}{L} \cdot \theta_B - \frac{2s \cdot (1+c) \cdot k}{m \cdot L} \cdot \phi$$



(a)



(b)

(a) In case of $M_{AB} = M_{pl}$

$$s \cdot k \cdot \theta_A + c \cdot s \cdot k \cdot \theta_B - s \cdot (1+c) \cdot k \cdot \phi = M_{pl}$$

$$c \cdot s \cdot k \cdot \theta_A = c \cdot M_{pl} - c^2 \cdot s \cdot k \cdot \theta_B + s \cdot c \cdot (1+c) \cdot k \cdot \phi$$

$$M_{BA} = c \cdot M_{pl} + s \cdot (1-c^2) \cdot k \cdot (\theta_B - \phi)$$

$$V = \frac{s \cdot (1+c)}{L} \cdot \left[\frac{M_{pl}}{s} + k \cdot \theta_B \cdot (1-c) - k \cdot \phi \cdot \left(\frac{2}{m} - 1 - c \right) \right]$$

(b) In case of $M_{BA} = M_{AB} = M_{pl}$

$$V = \frac{1}{L} \cdot (2M_{pl} + P \cdot L \cdot \phi)$$

3.2.12 Effect of Variable Cross-section

Moment of inertia: $I(x) = I_A \cdot \left[1 + (a-1)\cdot \frac{x}{L}\right]^{m_1}$ $\quad a = (I_B / I_A)^{1/m_1}$ $\quad m_1 = \frac{\log(I_B / I_A)}{\log(d_B / d_A)}$

(Values of m_1 are in the next slide)

$$M_{AB} = k_A \cdot \left[s_A \cdot \theta_A + \tilde{s} \cdot \tilde{c} \cdot \theta_B - (s_A + \tilde{s} \cdot \tilde{c}) \cdot \frac{\Delta}{L} \right]$$

$$k_A = \frac{EI_A}{L}$$

$$M_{BA} = k_A \cdot \left[\tilde{s} \cdot \tilde{c} \cdot \theta_A + s_B \cdot \theta_B - (s_B + \tilde{s} \cdot \tilde{c}) \cdot \frac{\Delta}{L} \right]$$

$$\rho_A = \frac{P}{P_{EA}}$$

$$V = \frac{k_A}{L} \cdot \left[-(s_A + \tilde{s} \cdot \tilde{c}) \cdot \theta_A - (s_B + \tilde{s} \cdot \tilde{c}) \cdot \theta_B + (s_A + s_B + 2\tilde{s} \cdot \tilde{c} - \pi^2 \cdot \rho_A) \cdot \frac{\Delta}{L} \right]$$

$$\rho = \rho_A \cdot a^{-m_1/2}$$

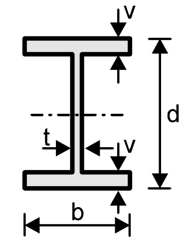
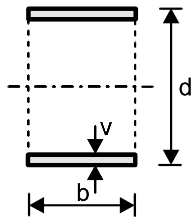
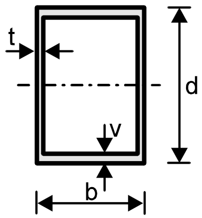
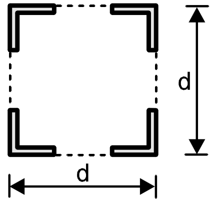
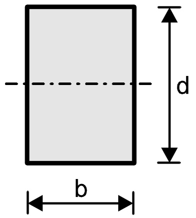
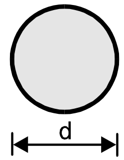
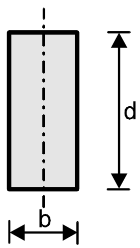
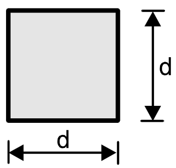
$$P_{EA} = \frac{\pi^2 \cdot EI_A}{L^2}$$

$$s_A = s \cdot a^{\alpha \cdot m_1/4} \qquad \tilde{s} \cdot \tilde{c} = s \cdot c \cdot a^{(1+\alpha)\cdot m_1/4} \qquad s_B = s \cdot a^{(1+\alpha/2)\cdot m_1/2}$$

$$m_1 = 4 \quad \rightarrow \quad \alpha = 1 \qquad\qquad m_1 \le 3 \quad \rightarrow \quad \alpha = 1{,}04 + 0{,}08 \cdot (3 - m_1)$$

$$\boxed{s''_A = s'' \cdot a^{\alpha \cdot m_1/4}} \qquad V_A = \left(s'' \cdot a^{\alpha \cdot m_1/4} - \pi^2 \cdot \rho \cdot a^{\alpha \cdot m_1/2}\right) \cdot k_A \cdot \Delta$$

$$\boxed{s''_B = s'' \cdot a^{(1+\alpha/2)\cdot m_1/2}} \qquad V_B = \left(s'' \cdot a^{(1+\alpha/2)\cdot m_1/2} - \pi^2 \cdot \rho \cdot a^{\alpha \cdot m_1/2}\right) \cdot k_A \cdot \Delta$$

Cross-section		m_1	Cross-section		m_1
	I cross-section *b*, *t*, *v* const., *d* variable	2.1...2.6		Sandwich cross-section *b*, *v* const., *d* variable	2
	Closed cross-section *b*, *t*, *v* const., *d* variable	2.1...2.6		Lattice cross-section területe const., *d* variable	2
	Solid cross-section *b* const., *d* variable	3		Solid cross-section *d* variable	4
	Solid cross-section *b* const., *d* variable	1		Solid cross-section *d* variable	4

Values of m_1 for variable cross-sections

3.2.13 Relationship Between the Stability Functions

$$c = \frac{2\alpha - \sin 2\alpha}{\sin 2\alpha - 2\alpha \cdot \cos 2\alpha}$$

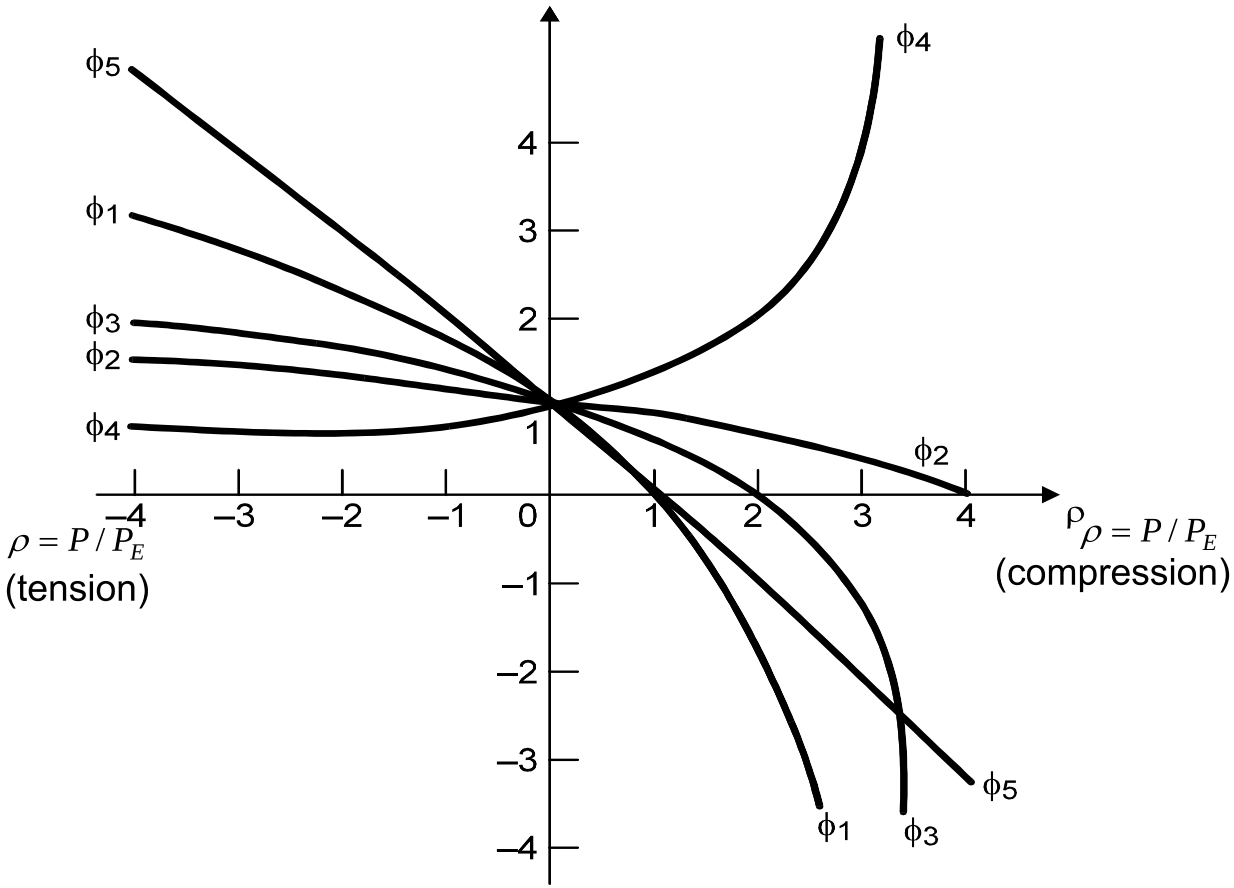
$$s = \frac{(1 - 2\alpha \cdot \mathrm{ctg}\, 2\alpha) \cdot \alpha}{\mathrm{tg}\, \alpha - \alpha}$$

$$\alpha = \frac{1}{2} \cdot \pi \cdot \sqrt{\rho}$$

$$s = \frac{0{,}25 \cdot \pi^2 \cdot \rho + \Phi_1 - \Phi_1^2}{1 - \Phi_1}$$

$$c = \frac{1}{4s} \cdot \frac{\pi^2 \cdot \rho - 4\Phi_1 + 4\Phi_1^2}{1 - \Phi_1}$$

$$\Phi_1 = \alpha \cdot \mathrm{ctg}\, \alpha$$



[Livesley, Chandler, 1956]

$$M_{AB} = M_{BA} = -m \cdot \frac{V \cdot L}{2} = -\frac{V \cdot L}{2\Phi_1}$$

$$m = \frac{1}{\Phi_1}$$

$$\Phi_2 = \frac{\alpha^2}{3 \cdot (1 - \Phi_1)} \qquad \Phi_3 = \frac{3\Phi_2 + \Phi_1}{4}$$

$$\Phi_4 = \frac{3\Phi_2 - \Phi_1}{2} \qquad \Phi_5 = \Phi_1 \cdot \Phi_2$$

$$M_{AB} = 4\Phi_3 \cdot k \cdot \theta_A + 2\Phi_4 \cdot k \cdot \theta_B$$

$$M_{BA} = 2\Phi_4 \cdot k \cdot \theta_A + 4\Phi_3 \cdot k \cdot \theta_B$$

$$V = -6\Phi_2 \cdot k \cdot (\theta_A + \theta_B) / L$$

[Majid, 1972]

$$\rho = P / P_E = \frac{P \cdot L^2}{\pi^2 \cdot EI}$$

$$P = \rho \cdot P_E = \frac{\rho \cdot \pi^2 \cdot k}{L}$$

$$\Phi_1 = \frac{\pi}{2} \cdot \sqrt{\rho} \cdot \mathrm{ctg}\frac{\pi \cdot \sqrt{\rho}}{2} = \alpha \cdot \mathrm{ctg}\alpha = \frac{1}{m}$$

$$\Phi_2 = \frac{\alpha^2}{3 \cdot (1 - \Phi_1)} = \frac{s \cdot (1+c)}{6}$$

$$\Phi_3 = \frac{3\Phi_2 + \Phi_1}{4} = \frac{s}{4}$$

$$\Phi_4 = \frac{3\Phi_2 - \Phi_1}{2} = \frac{1}{2} \cdot s \cdot c$$

$$\Phi_5 = \Phi_1 \cdot \Phi_2 = \frac{s \cdot (1+c)}{6m}$$

$$\Phi_6 = \frac{\Phi_3}{\Phi_2 \cdot (2\Phi_3 - \Phi_4)} = \frac{3}{s \cdot (1 - c^2)}$$

$$\Phi_7 = \Phi_4 \cdot \Phi_6 \cdot \Phi_3 = \frac{6c}{s \cdot (1 - c^2)}$$

$$s = \frac{(1 - 2\alpha \cdot \mathrm{ctg}2\alpha) \cdot \alpha}{\mathrm{tg}\alpha - \alpha} = \frac{0{,}25\pi^2 \cdot \rho + \Phi_1 - \Phi_1^2}{1 - \Phi_1} = 4\Phi_3 = \frac{12\Phi_6}{4\Phi_6^2 - \Phi_7^2}$$

$$c = \frac{2\alpha - \sin 2\alpha}{\sin 2\alpha - 2\alpha \cdot \cos 2\alpha} = \frac{1}{4s} \cdot \frac{\pi^2 \cdot \rho - 4\Phi_1 + 4\Phi_1^2}{1 - \Phi_1} = \frac{1}{2} \cdot \frac{\Phi_4}{\Phi_3} = \frac{\Phi_7}{2\Phi_6}$$

$$m = \frac{2s \cdot (1+c)}{2s \cdot (1+c) - \pi^2 \cdot \rho} = \frac{1}{\Phi_1}$$

$$n = s \cdot \left[1 - \frac{m \cdot (1+c)}{2}\right] = 4\Phi_3 - 3 \cdot \frac{\Phi_2}{\Phi_1}$$

$$o = s \cdot \left[\frac{m \cdot (1+c)}{2} - c\right] = \frac{3\Phi_2}{\Phi_1} - 2\Phi_4$$

$$s \cdot (1 - c^2) = \frac{\pi^2 \cdot \rho}{1 - n}$$

$$s \cdot (1 - c) = 2 / m = 2\Phi_1$$

$$s \cdot (1 + c) = \frac{o - n}{m - 1} = \frac{m \cdot \pi^2 \cdot \rho}{2 \cdot (m - 1)} = 6\Phi_2$$

$$s = \frac{1-n}{m-1}$$

$$1 - m \cdot n = m \cdot \pi^2 \cdot \rho / 4$$

$$o + n = 2/m$$

$$o - n = m \cdot \pi^2 \cdot \rho / 2$$

$$o^2 - n^2 = \pi^2 \cdot \rho$$

$$\Phi_1 = \frac{64 - 60\rho + 5\rho^2}{(16-\rho)\cdot(4-\rho)} - \sum_{n=1}^{7} \frac{a_n \cdot \rho^n}{2^{3n}}$$

$$a_1 = 1.57973627$$

$$a_2 = 0.15858587$$

$$a_3 = 0.02748899$$

$$a_4 = 0.00547540$$

$$a_5 = 0.00115281$$

$$a_6 = 0.00024908$$

$$a_7 = 0.00005452$$

Livesley devised a method whereby this function is calculated as the sum of a power series in ρ and a rational function. This arrangement absorbs the two singularities nearest to the working range $-4<\rho<4$.

Készült az ERFP – DD2002 – HU – B – 01 szerzősésszámú projekt támogatásával

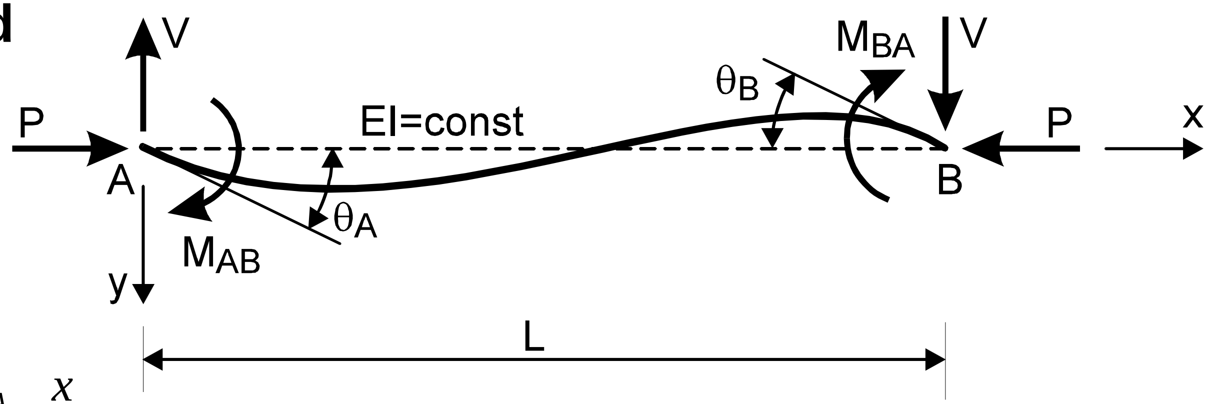
Stability functions for compressive forces

ρ	s	c	sc	$s(1+c)$	m	$\frac{2s(1+c)}{m}$	s''	$s''-\pi^2\rho$	f	f''
0.00	4.000	0.500	2.000	6.000	1.000	12.000	3.000	3.000	1.000	1.500
0.05	3.934	0.513	2.017	5.950	1.043	11.407	2.900	2.406	1.008	1.525
0.10	3.867	0.526	2.034	5.901	1.091	10.814	2.797	1.810	1.017	1.552
0.15	3.799	0.540	2.052	5.850	1.145	10.220	2.691	1.210	1.026	1.580
0.20	3.730	0.555	2.070	5.800	1.205	9.626	2.581	0.607	1.035	1.609
0.25	3.660	0.571	2.089	5.749	1.273	9.030	2.467	0.000	1.044	1.639
0.30	3.589	0.588	2.109	5.697	1.351	8.434	2.350	–0.611	1.053	1.672
0.35	3.517	0.605	2.129	5.646	1.441	7.837	2.228	–1.226	1.063	1.706
0.40	3.444	0.624	2.150	5.594	1.545	7.239	2.102	–1.846	1.073	1.742
0.45	3.370	0.644	2.171	5.541	1.669	6.641	1.971	–2.471	1.083	1.781
0.50	3.294	0.666	2.194	5.488	1.817	6.041	1.834	–3.101	1.093	1.821
0.55	3.218	0.689	2.217	5.435	1.998	5.441	1.691	–3.737	1.104	1.865
0.60	3.140	0.714	2.241	5.381	2.223	4.840	1.541	–4.380	1.115	1.911
0.65	3.061	0.740	2.266	5.327	2.514	4.238	1.385	–5.031	1.126	1.960
0.70	2.981	0.769	2.291	5.272	2.900	3.636	1.220	–5.689	1.138	2.013
0.75	2.899	0.800	2.318	5.217	3.441	3.032	1.046	–6.356	1.150	2.070
0.80	2.816	0.833	2.346	5.162	4.253	2.428	0.862	–7.034	1.162	2.131
0.85	2.731	0.869	2.374	5.106	5.604	1.822	0.667	–7.722	1.175	2.197
0.90	2.645	0.909	2.404	5.049	8.307	1.216	0.460	–8.423	1.188	2.268
0.95	2.557	0.952	2.435	4.992	16.413	0.608	0.238	–9.138	1.202	2.346
1.00	2.467	1.000	2.467	4.935	±∞	–0.002	0.000	–9.870	1.216	2.432
1.10	2.283	1.111	2.536	4.818	–7.902	–1.220	–0.534	–11.391	1.245	2.628
1.20	2.090	1.249	2.610	4.700	–3.847	–2.443	–1.169	–13.013	1.277	2.871
1.30	1.889	1.424	2.691	4.580	–2.495	–3.671	–1.944	–14.774	1.310	3.176
1.40	1.678	1.656	2.779	4.457	–1.818	–4.904	–2.922	–16.740	1.346	3.575
1.50	1.457	1.973	2.875	4.332	–1.411	–6.141	–4.215	–19.019	1.385	4.118
1.60	1.224	2.435	2.980	4.204	–1.139	–7.383	–6.032	–21.823	1.427	4.902
1.70	0.978	3.166	3.096	4.074	–0.944	–8.630	–8.825	–25.604	1.473	6.135
1.80	0.717	4.497	3.224	3.941	–0.798	–9.882	–13.783	–31.548	1.522	8.368
1.90	0.439	7.661	3.367	3.806	–0.683	–11.140	–25.351	–44.104	1.576	13.653
2.00	0.143	24.682	3.525	3.668	–0.591	–12.404	–86.858	–106.59	1.636	42.013
2.10	–0.176	–21.074	3.702	3.526	–0.516	–13.674	77.838	57.112	1.701	–34.154
2.20	–0.519	–7.511	3.901	3.382	–0.452	–14.950	28.782	7.069	1.774	–11.551
2.30	–0.893	–4.623	4.127	3.234	–0.398	–16.232	18.185	–4.515	1.855	–6.721
2.40	–1.301	–3.370	4.383	3.083	–0.352	–17.522	13.472	–10.215	1.946	–4.613
2.50	–1.750	–2.673	4.678	2.928	–0.311	–18.818	10.754	–13.920	2.049	–3.429
2.60	–2.249	–2.231	5.018	2.769	–0.275	–20.123	8.948	–16.713	2.167	–2.668
2.70	–2.809	–1.928	5.415	2.606	–0.243	–21.435	7.631	–19.017	2.302	–2.136
2.80	–3.445	–1.708	5.884	2.439	–0.214	–22.756	6.606	–21.029	2.459	–1.742
2.90	–4.176	–1.543	6.444	2.268	–0.188	–24.086	5.767	–22.855	2.645	–1.436

Stability functions for tension forces

ρ	s	c	sc	$s(1+c)$	m	$\frac{2s(1+c)}{m}$	s''	$s''-\pi^2\rho$	f	f''
0.00	4.000	0.500	2.000	6.000	1.000	12.000	3.000	3.000	1.000	1.500
–0.05	4.065	0.488	1.984	6.049	0.961	12.592	3.097	3.591	0.992	1.476
–0.10	4.130	0.477	1.968	6.098	0.925	13.183	3.192	4.179	0.984	1.453
–0.15	4.194	0.466	1.953	6.147	0.893	13.773	3.284	4.765	0.976	1.431
–0.20	4.257	0.455	1.938	6.195	0.863	14.363	3.374	5.348	0.969	1.410
–0.25	4.319	0.445	1.924	6.243	0.835	14.952	3.462	5.929	0.961	1.389
–0.30	4.380	0.436	1.910	6.290	0.809	15.541	3.548	6.509	0.954	1.370
–0.35	4.441	0.427	1.896	6.337	0.786	16.129	3.632	7.086	0.947	1.351
–0.40	4.501	0.418	1.883	6.384	0.764	16.716	3.714	7.661	0.940	1.333
–0.45	4.561	0.410	1.870	6.431	0.743	17.303	3.794	8.235	0.933	1.316
–0.50	4.619	0.402	1.858	6.477	0.724	17.889	3.872	8.807	0.926	1.299
–0.55	4.678	0.395	1.845	6.523	0.706	18.474	3.950	9.378	0.920	1.283
–0.60	4.735	0.387	1.834	6.569	0.689	19.059	4.025	9.947	0.913	1.267
–0.65	4.792	0.380	1.822	6.614	0.673	19.643	4.099	10.514	0.907	1.252
–0.70	4.848	0.374	1.811	6.659	0.658	20.227	4.172	11.081	0.901	1.238
–0.75	4.904	0.367	1.800	6.704	0.644	20.810	4.243	11.646	0.895	1.223
–0.80	4.959	0.361	1.789	6.749	0.631	21.393	4.314	12.209	0.889	1.210
–0.85	5.014	0.355	1.779	6.793	0.618	21.975	4.383	12.772	0.883	1.197
–0.90	5.068	0.349	1.769	6.837	0.606	22.556	4.451	13.333	0.878	1.184
–0.95	5.122	0.343	1.759	6.881	0.595	23.138	4.518	13.894	0.872	1.171
–1.00	5.175	0.338	1.749	6.924	0.584	23.718	4.583	14.453	0.867	1.159
–1.10	5.280	0.328	1.731	7.010	0.564	24.877	4.712	15.569	0.856	1.136
–1.20	5.382	0.318	1.713	7.096	0.545	26.035	4.837	16.681	0.846	1.115
–1.30	5.483	0.309	1.697	7.180	0.528	27.190	4.959	17.789	0.836	1.094
–1.40	5.583	0.301	1.680	7.263	0.513	28.344	5.077	18.894	0.826	1.075
–1.50	5.681	0.293	1.665	7.346	0.498	29.496	5.192	19.997	0.817	1.056
–1.60	5.777	0.286	1.651	7.427	0.485	30.646	5.305	21.096	0.808	1.039
–1.70	5.871	0.279	1.637	7.508	0.472	31.794	5.415	22.193	0.799	1.022
–1.80	5.965	0.272	1.623	7.588	0.461	32.941	5.523	23.288	0.791	1.006
–1.90	6.056	0.266	1.610	7.667	0.450	34.086	5.628	24.380	0.783	0.991
–2.00	6.147	0.260	1.598	7.745	0.440	35.229	5.731	25.470	0.775	0.976
–2.10	6.236	0.254	1.586	7.822	0.430	36.371	5.832	26.559	0.767	0.962
–2.20	6.324	0.249	1.575	7.899	0.421	37.511	5.932	27.645	0.760	0.949
–2.30	6.411	0.244	1.564	7.975	0.413	38.650	6.029	28.729	0.752	0.936
–2.40	6.496	0.239	1.554	8.050	0.405	39.787	6.125	29.812	0.745	0.924
–2.50	6.581	0.235	1.544	8.125	0.397	40.923	6.219	30.893	0.739	0.912
–2.60	6.664	0.230	1.534	8.198	0.390	42.058	6.311	31.972	0.732	0.900
–2.70	6.747	0.226	1.525	8.271	0.383	43.191	6.402	33.050	0.725	0.889
–2.80	6.828	0.222	1.516	8.344	0.377	44.323	6.491	34.126	0.719	0.879
–2.90	6.908	0.218	1.507	8.416	0.370	45.453	6.579	35.201	0.713	0.869

3.2.14 Flexibility Method



$$y'' + \kappa^2 \cdot y = \frac{M_{AB}}{EI} \cdot \left(\frac{x}{L} - 1 \right) + \frac{M_{BA}}{EI} \cdot \frac{x}{L}$$

$$y = A \cdot \sin \kappa x + B \cdot \cos \kappa x + \frac{M_{AB}}{P} \cdot \left(\frac{x}{L} - 1 \right) + \frac{M_{BA}}{P} \cdot \frac{x}{L}$$

$$k^2 = P / EI$$

$$y(x=0) = 0 \qquad B = \frac{M_{AB}}{P}$$

$$y(x=L) = 0 \qquad A = -\frac{M_{AB}}{P} \cdot \mathrm{ctg}\, kL - \frac{M_{BA}}{P} \cdot \frac{1}{\sin kL}$$

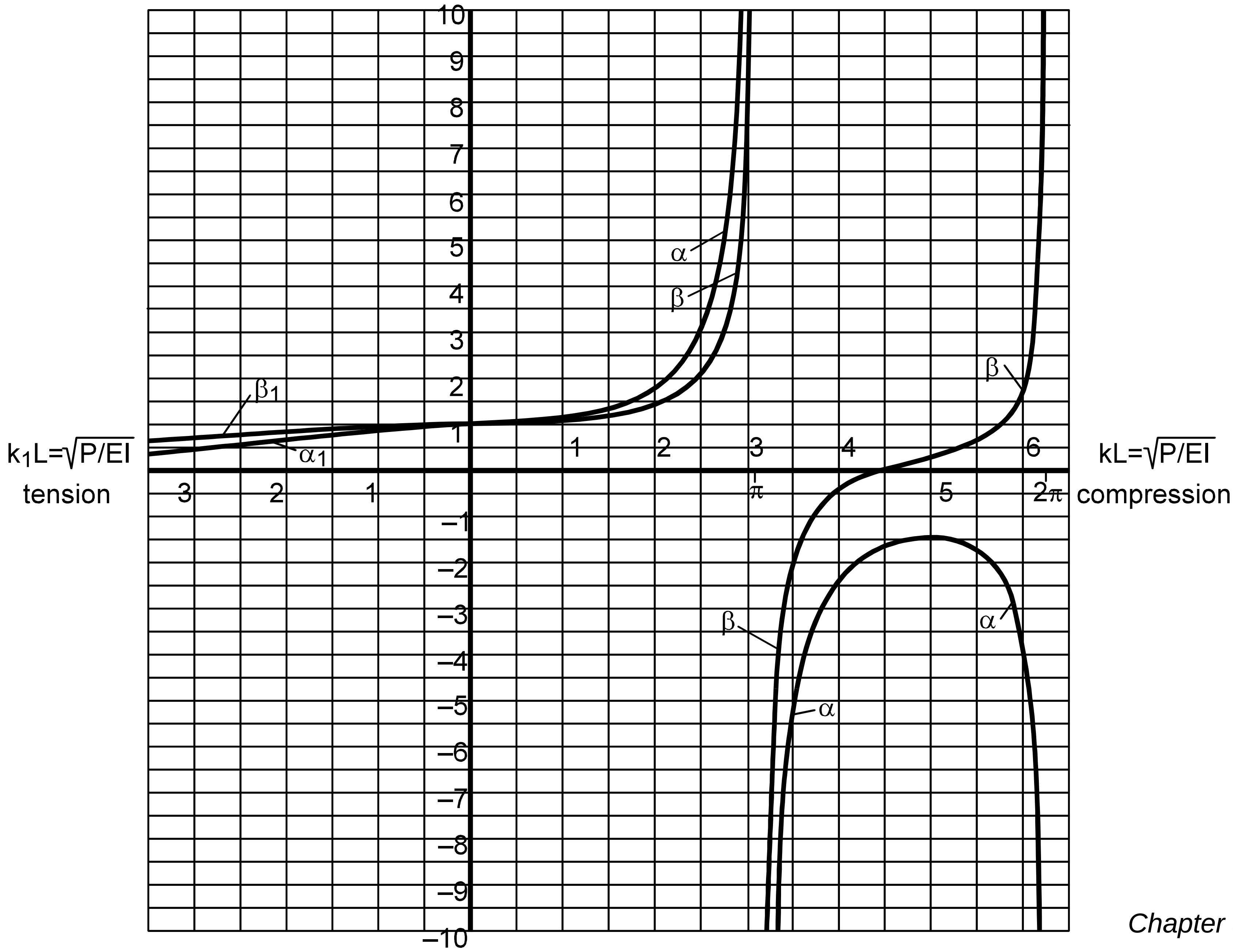
$$\alpha = \frac{6}{k^2 \cdot L^2} \cdot \left(kL \cdot \frac{1}{\sin kL} - 1 \right)$$

$$y'(x=0) = \theta_A \qquad EI \cdot \theta_A = \frac{1}{3} \cdot M_{AB} \cdot L \cdot \beta - \frac{1}{6} \cdot M_{BA} \cdot L \cdot \alpha$$

$$\beta = \frac{3}{k^2 \cdot L^2} \cdot (1 - kL \cdot \mathrm{ctg}\, kL)$$

$$y'(x=L) = \theta_B \qquad EI \cdot \theta_B = -\frac{1}{6} \cdot M_{AB} \cdot L \cdot \alpha + \frac{1}{3} \cdot M_{BA} \cdot L \cdot \beta$$

Flexibility functions



Flexibility and stiffness method:

$$\begin{bmatrix} M_{AB} \\ M_{BA} \end{bmatrix} = k \cdot \begin{bmatrix} 4\phi_3 & 2\phi_4 \\ 2\phi_4 & 4\phi_3 \end{bmatrix} \cdot \begin{bmatrix} \theta_A \\ \theta_B \end{bmatrix}$$

$$k^{-1} \cdot \begin{bmatrix} 4\phi_3 & 2\phi_4 \\ 2\phi_4 & 4\phi_3 \end{bmatrix}^{-1} = \frac{1}{k} \cdot \begin{bmatrix} \phi_6 / 3 & -\phi_7 / 6 \\ -\phi_7 / 6 & \phi_6 / 3 \end{bmatrix}$$

$$\phi_6 = \frac{\phi_3}{\phi_2 \cdot (2\phi_3 - \phi_4)}$$

$$\phi_7 = \phi_4 \cdot \phi_6 \cdot \phi_3$$

$$\begin{bmatrix} \theta_A \\ \theta_B \end{bmatrix} = \frac{L}{6EI} \cdot \begin{bmatrix} 2\phi_6 & -\phi_7 \\ -\phi_7 & 2\phi_6 \end{bmatrix} \cdot \begin{bmatrix} M_{AB} \\ M_{BA} \end{bmatrix}$$

(a)

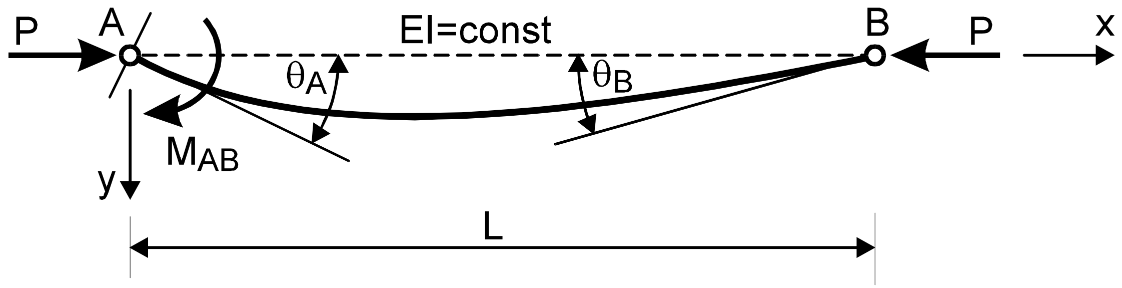
$$EI \cdot \theta_A = \frac{1}{3} \cdot M_{AB} \cdot L \cdot \beta$$

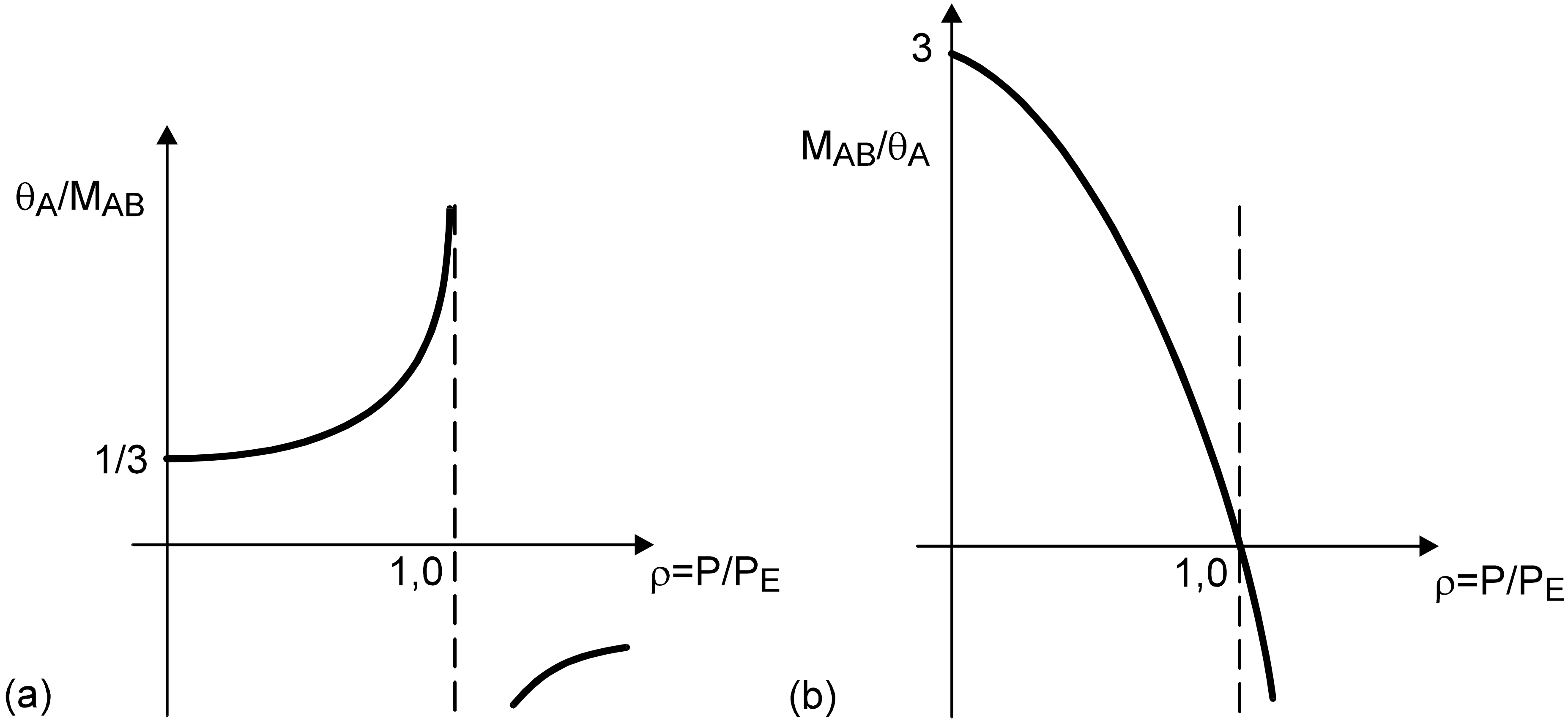
$$\frac{\theta_A}{M_{AB}} = \frac{\beta \cdot L}{3EI} \qquad \frac{\theta_A}{M_{AB}} = \infty$$

(b)

$$\frac{M_{AB}}{\theta_A} = \frac{EI}{L} \cdot (1 - c^2) \cdot s \qquad \frac{M_{AB}}{\theta_A} = 0$$

$$M_{BA} = 0$$

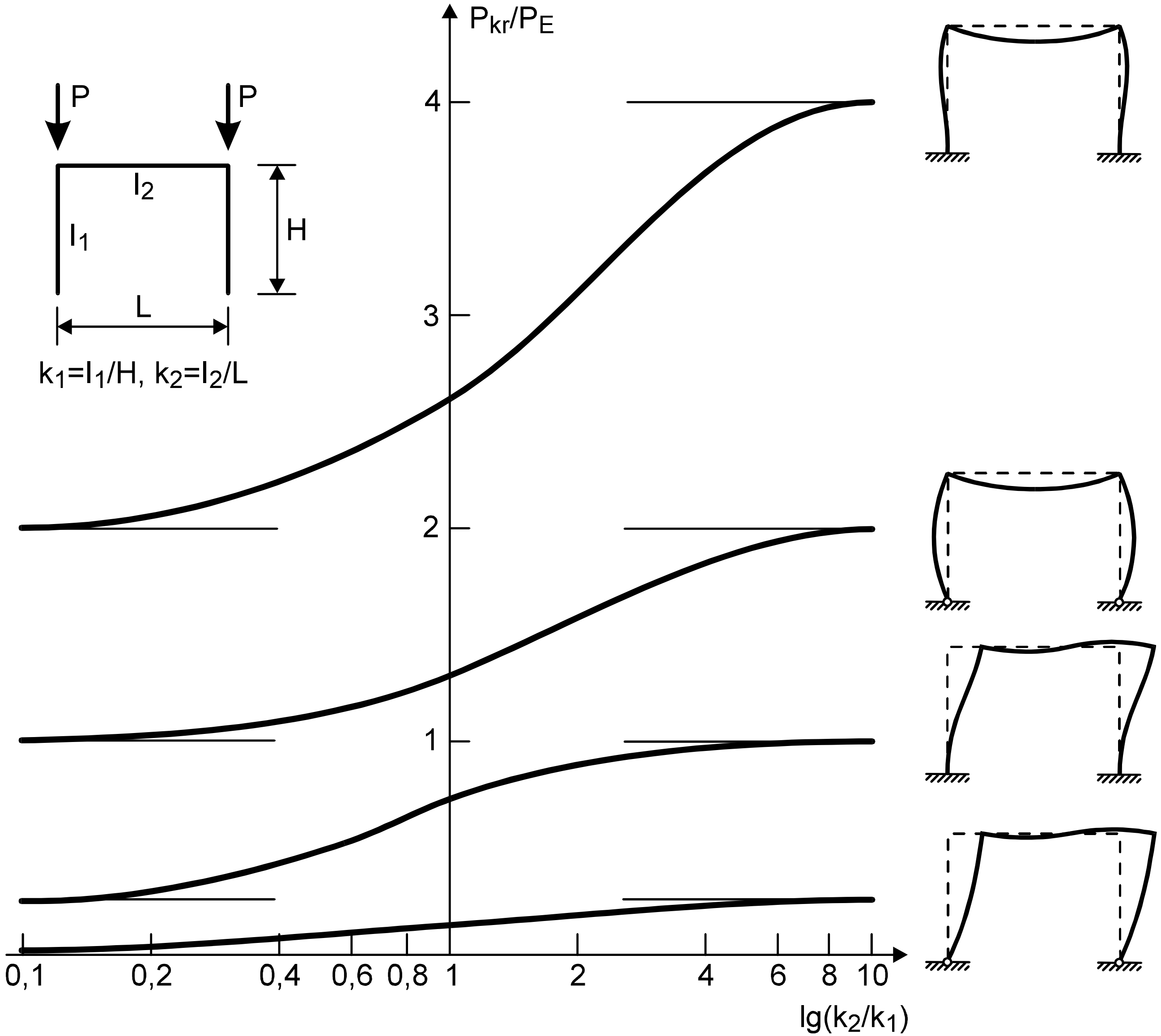




Comparison of Force (Flexibility) and Displacement (Stiffness) Method

3.3. Assessment of Sway-Preventing Action in Frames

Standard cases for single-storey portal frames:



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3.3.1 Sway-Preventing Actions

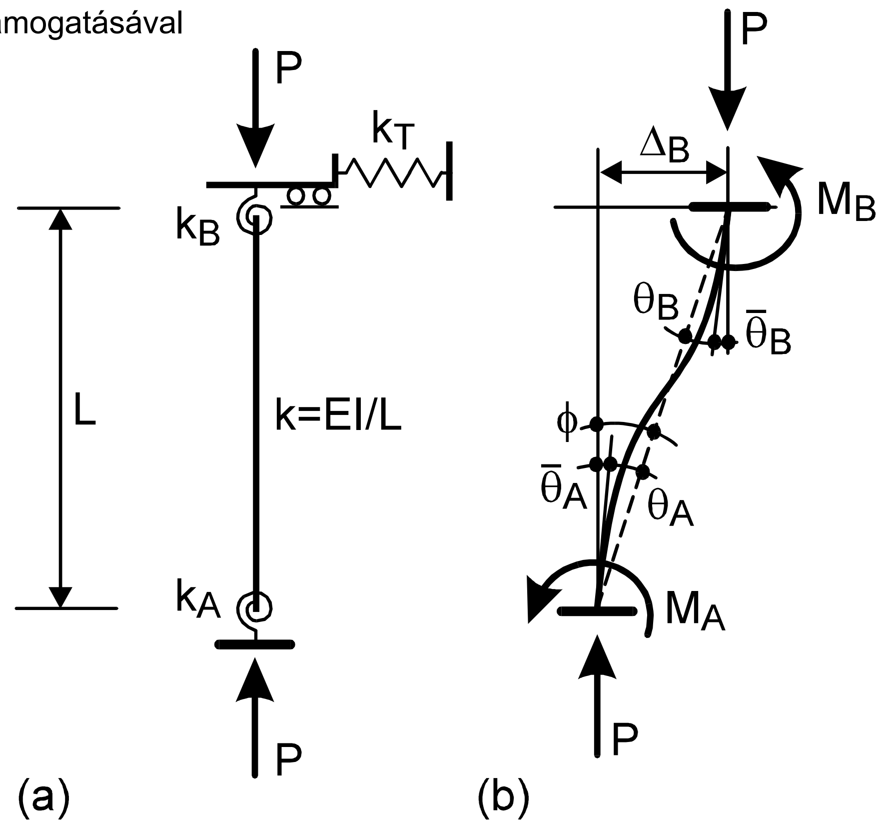
[Lay, 1970]

$$M_A = \frac{EI}{L}\cdot\left[s\cdot\bar{\theta}_A + s\cdot c\cdot\bar{\theta}_B - s\cdot(1+c)\cdot\phi\right];$$

$$M_B = \frac{EI}{L}\cdot\left[s\cdot c\cdot\bar{\theta}_A + s\cdot\bar{\theta}_B - s\cdot(1+c)\cdot\phi\right];$$

$$V\cdot L = P\cdot\Delta_B + M_A + M_B =$$

$$= \frac{EI}{L}\cdot\left\{s\cdot(1+c)\cdot\left(\bar{\theta}_A+\bar{\theta}_B\right) - \left[2s\cdot(1+c)-\pi^2\cdot\rho\right]\cdot\frac{\phi}{L}\right\},$$



$$\bar{\theta}_A = \phi - \theta_A; \quad \theta_A = \frac{M_A}{k_A};$$

$$\bar{\theta}_B = \phi - \theta_B; \quad \theta_B = \frac{M_B}{k_B};$$

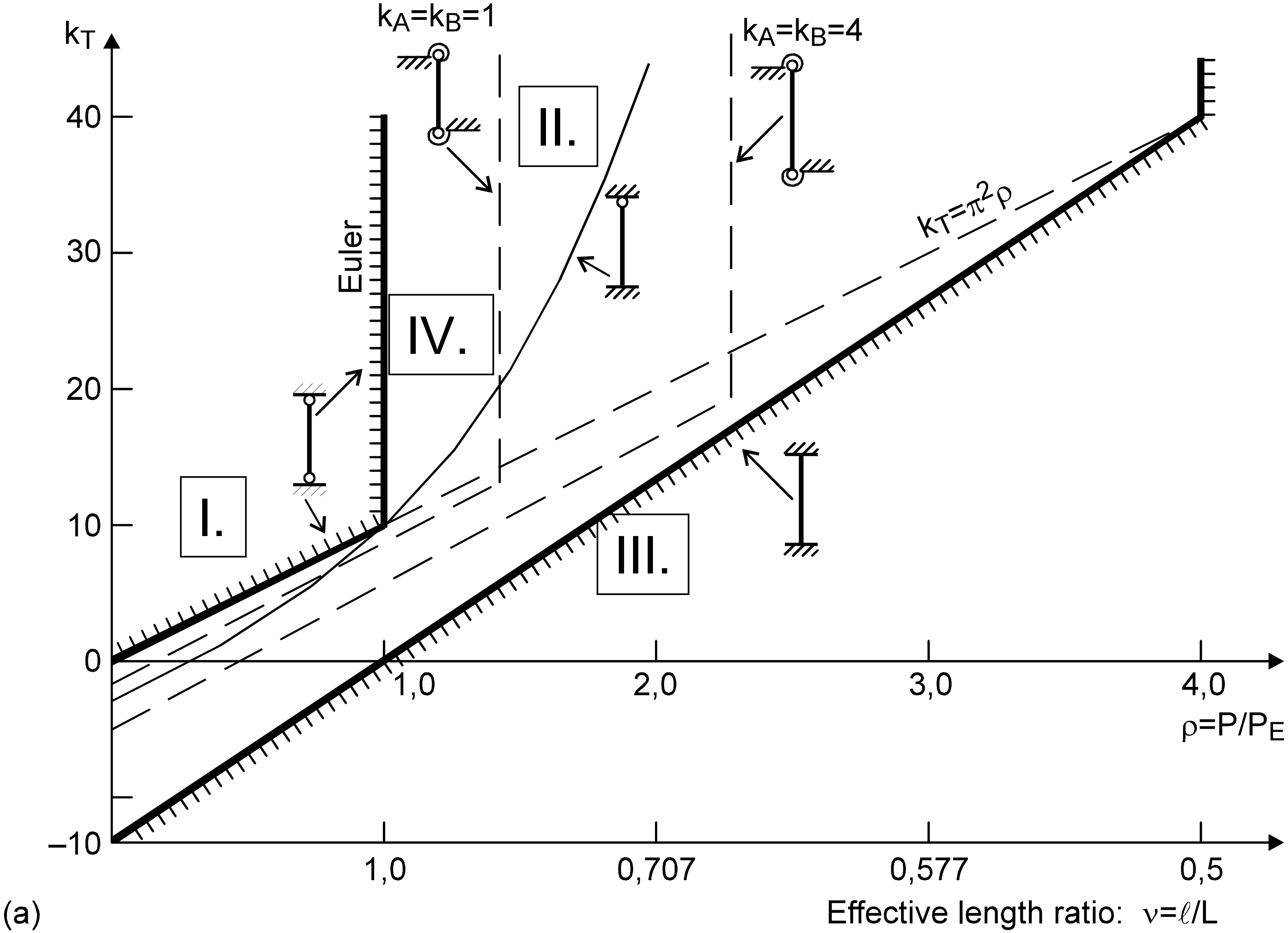
$$\phi = \frac{\Delta_B}{L}; \qquad \Delta_B = \frac{V}{k_T},$$

k_A k_B Rotational spring coefficient

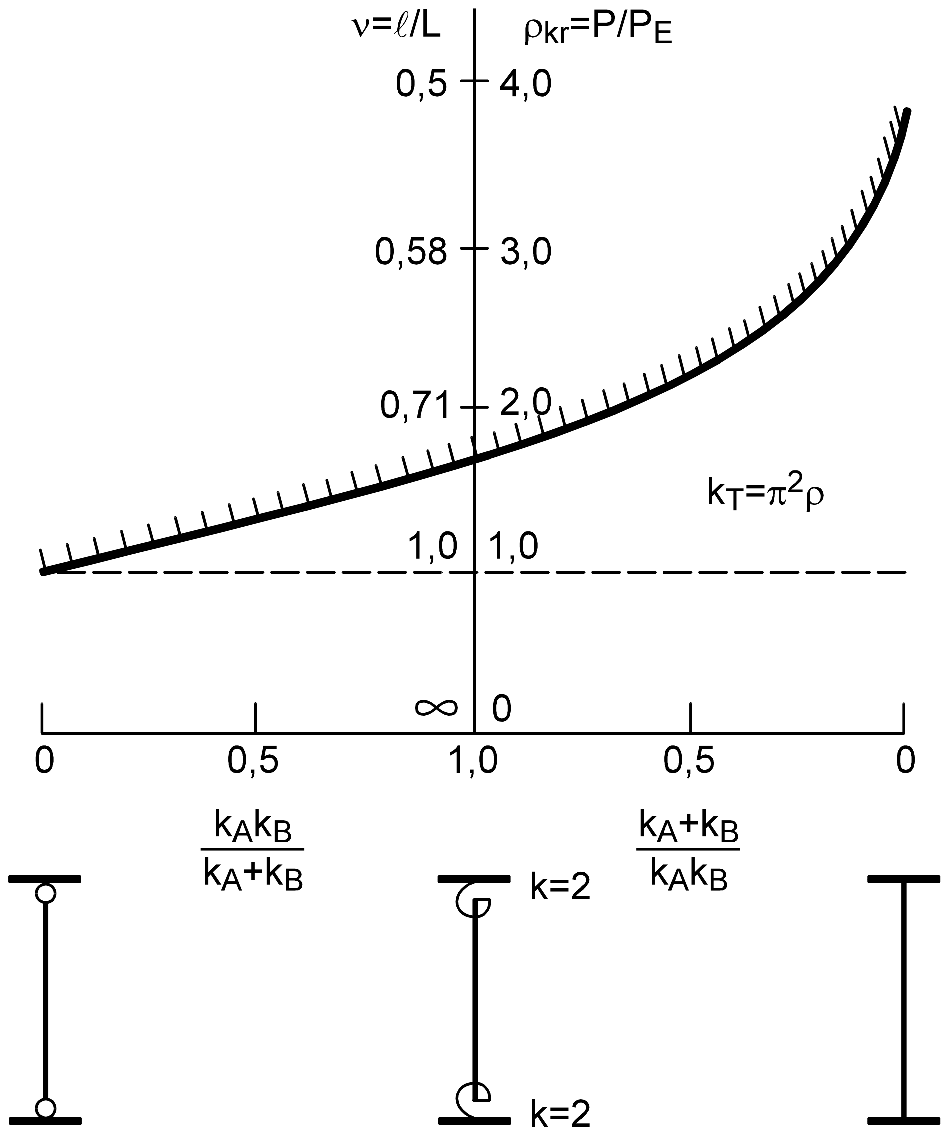
k_T Translational spring coefficient

Final general solution:

$$\pi^2\cdot\rho = k_T + s\cdot(1+c)\cdot\frac{2+s\cdot(1-c)\cdot\left(\frac{1}{k_A}+\frac{1}{k_B}\right)}{1+s\cdot\left(\frac{1}{k_A}+\frac{1}{k_B}\right)+\frac{s^2\cdot(1-c^2)}{k_A\cdot k_B}} \qquad \pi^2\cdot\rho = k_T + K(\rho)$$

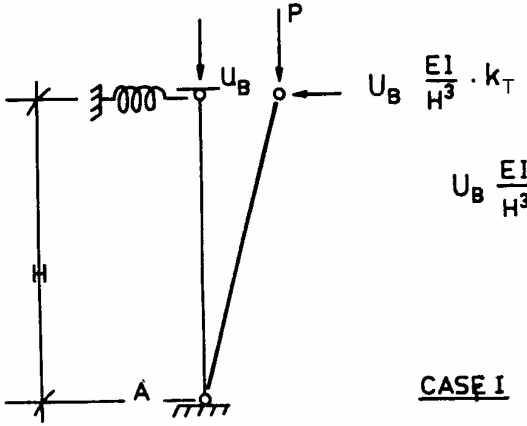


Sway stiffness needed to prevent sway



(b)

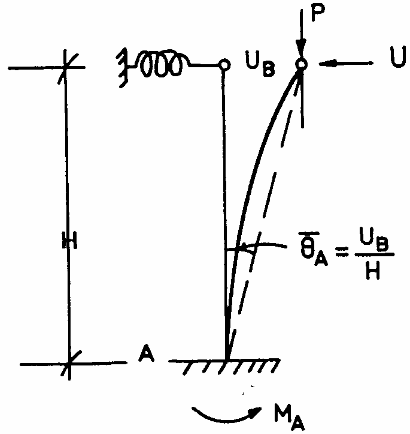
Critical ρ for $k_T=\pi^2\rho$ assumption



$$U_B \frac{EI}{H^3} k_T > Pu_B$$

$$\boxed{k_T > \pi^2 \rho} \quad \left(\text{WHERE } \rho = \frac{PH^2}{\pi^2 EI}\right)$$

CASE I PIN ENDED COLUMN



$$M_A = S(\bar{\theta}_A + C\theta_B)\frac{EI}{H}$$

$$M_B = S(\bar{\theta}_B + C\theta_A)\frac{EI}{H} = 0$$

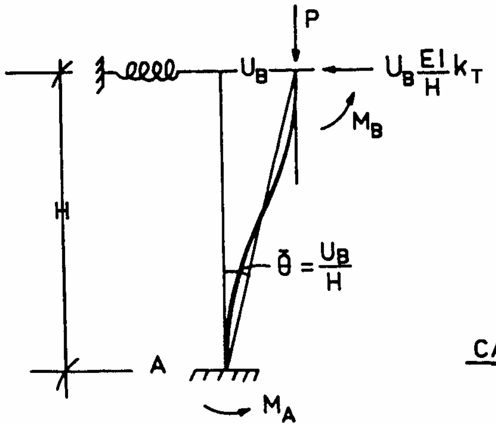
$$\therefore \; C\bar{\theta}_A + \bar{\theta}_B = 0$$

$$\therefore \; M_A = S(1-C^2)\frac{U_B}{H} \cdot \frac{EI}{H}$$

$$U_B \frac{EI}{H^3} k_T + M_A > Pu_B$$

$$\boxed{k_T > \pi^2 \rho - S(1-C^2)}$$

CASE II PIN- FIXED COLUMN



$$M_A = M_B = S\left(\frac{U_B}{H} + C\frac{U_B}{H}\right)\frac{EI}{H}$$

$$M_A = S(1+C)\frac{U_B}{H} \cdot \frac{EI}{H}$$

$$U_B \frac{EI}{H^3} \cdot k_T + 2M_A > Pu_B$$

$$\boxed{k_T > \pi^2 \rho - 2S(1+C)}$$

CASE III FIXED ENDED COLUMN

$$k_A = k_B = k_{AB}$$

$$\boxed{k_T = \pi^2 \rho - \frac{2}{\frac{1}{S(1+C)} + \frac{1}{k_{AB}}}}$$

CASE IV: EQUAL END RESTRAINS

Specific Sway Prevented Derivations

3.3.2 Application of Sway-Stiffness Approximation

(a) Braced Panels

The tension braces considered active

$$N = P_K \cdot \frac{L}{\sqrt{L^2 + (l + \Delta_B)^2}} \approx P_K \cdot \frac{L}{\sqrt{L^2 + l^2}};$$

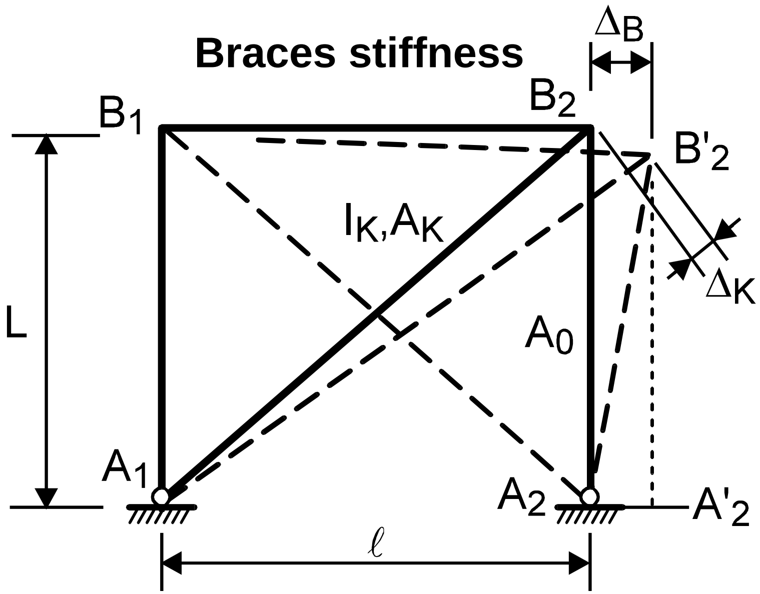
$$P_B = P_K \cdot \frac{l + \Delta_K}{\sqrt{L^2 + (l + \Delta_K)^2}} \approx P_K \cdot \frac{l}{\sqrt{L^2 + l^2}}.$$

$$\frac{\Delta_K}{\Delta_B} = \frac{l + \Delta_B}{\sqrt{L^2 + (l + \Delta_B)^2}} \approx \frac{l}{\sqrt{L^2 + l^2}}$$

$$\varepsilon_K = \frac{\Delta_K}{\sqrt{L^2 + l^2}} = \frac{l}{L^2 + l^2} \Delta_B$$

$$P_K = EA_K \cdot \frac{l}{L^2 + l^2} \cdot \Delta_B$$

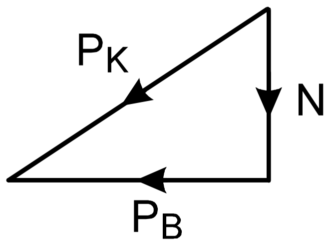
$$K_T = EA_K \cdot \frac{l^2}{(L^2 + l^2)^{3/2}} = \frac{P_B}{\Delta_B} > \frac{A_{oszlop} \cdot \sigma_H}{L}$$

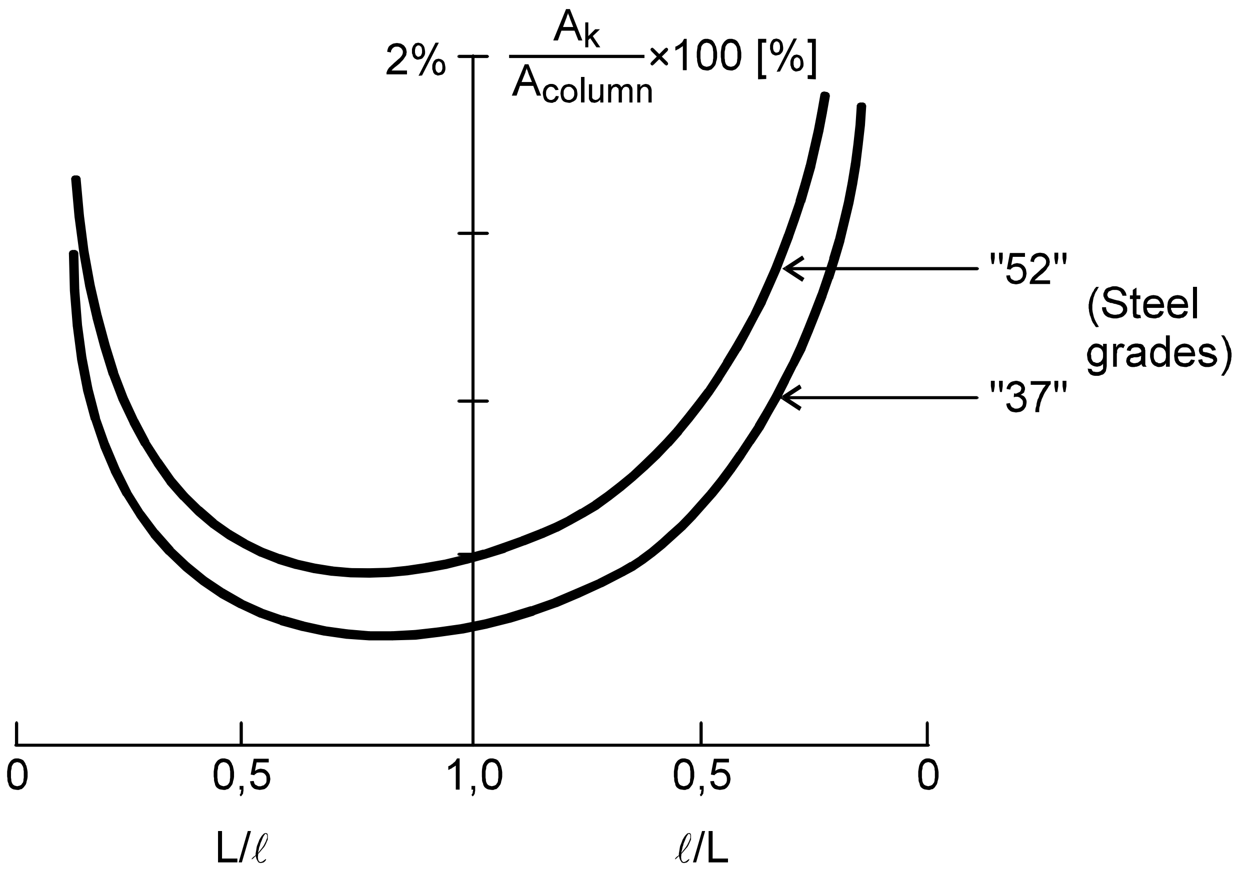


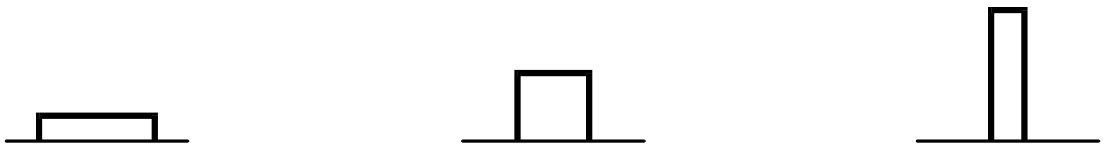
$A_1B_2 = \ell_K = \sqrt{L^2 + \ell^2}$

$A_1B'_2 = \sqrt{L^2 + (\ell + \Delta_B)^2} = A_1B_2 + \Delta_K$

Components of force P_K acting on point B'_2 :







$K_T > P / L$

$P = A_{column} \cdot \sigma_H$

$$\frac{A_K}{A_{column}} > \frac{\sigma_H}{E} \cdot \frac{\left[1 + (L/l)^2\right]^{3/2}}{L/l}$$

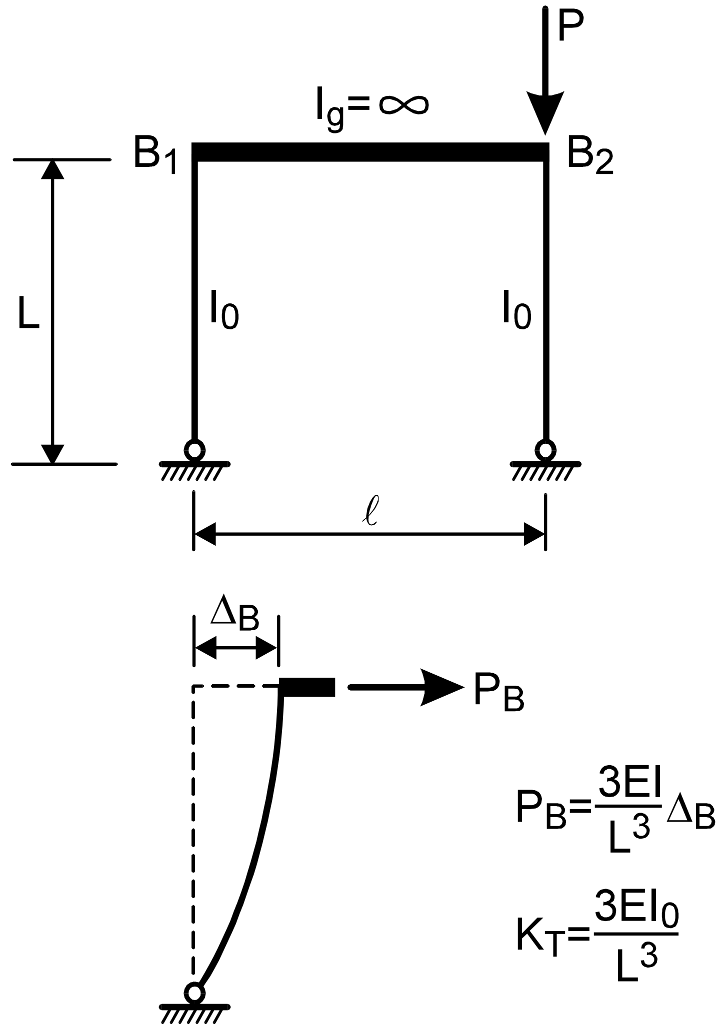
(b) Portals with Single loads

$$K_T = \frac{3EI_0}{L^3} > \frac{P}{L}$$

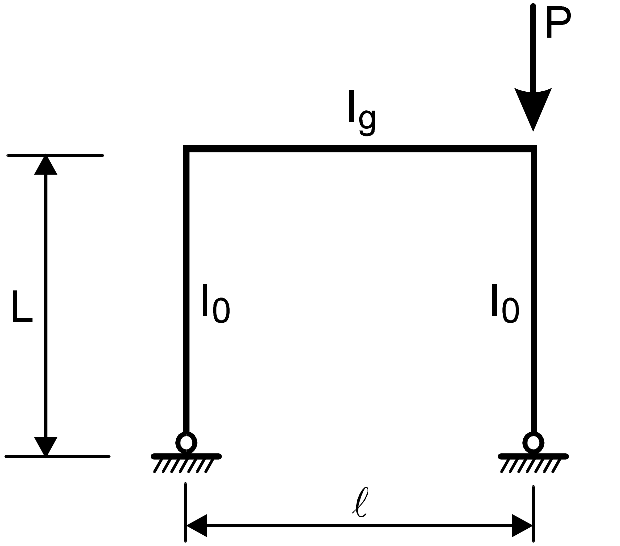
$$\rho_1 = \frac{P \cdot L^2}{\pi^2 \cdot EI_0} < \frac{3}{\pi^2} \approx 0.3$$

$$\rho_1 < 0.3$$

The sway-free design load would be ρ=0.25 and so a min. 20% increase is possible.



Single Portal



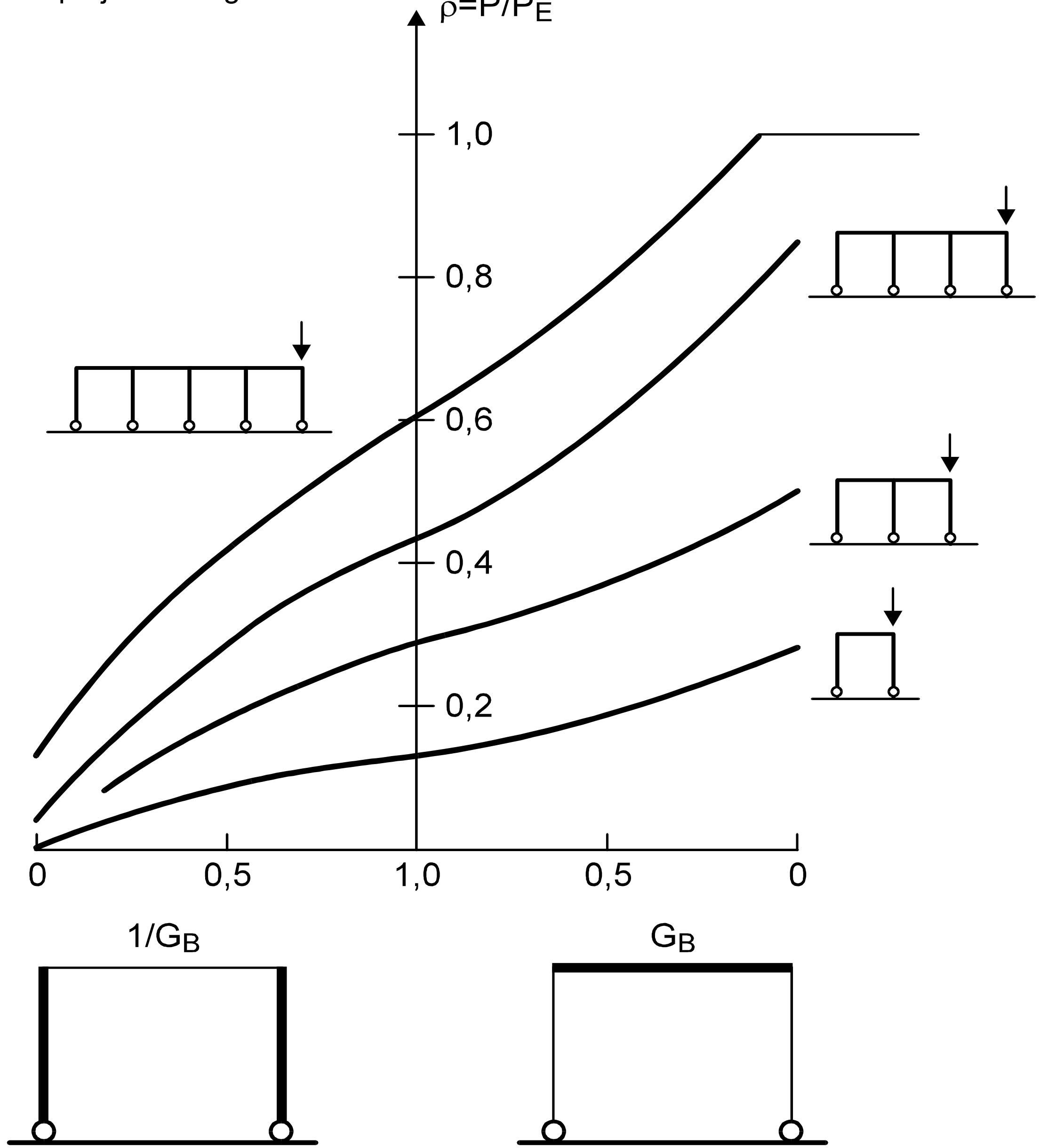
Single Portal with Flexible Beam

$$K_T = \frac{3EI_0}{L^3} \cdot \frac{1}{1 + \dfrac{I_0 \cdot l}{I_g \cdot L}}$$

In case of more (n) unloaded columns:

$$\overline{K}_T = n \cdot K_T \qquad G_B = \frac{I_0 \cdot l}{I_g \cdot L}$$

$$\rho_1 < n \cdot \frac{3}{\pi^2} \cdot \frac{1}{1 + G_B}$$



Unloaded frame effect

3.4. Effect of Semi-Rigid Connections

3.4.1 Member of a Braced Frame

Column c1:

$$M_{A,c1} = \frac{EI_{c1}}{L_{c1}} \cdot (s_i \cdot \theta_A + s_i \cdot c_i \cdot \theta_B)$$

Column c2:

$$M_{A,c2} = \frac{EI_{c2}}{L_{c2}} \cdot (s_i \cdot \theta_A + s_i \cdot c_i \cdot \theta_B)$$

$$M_{B,c2} = \frac{EI_{c2}}{L_{c2}} \cdot (s_i \cdot c_i \cdot \theta_A + s_i \cdot \theta_B)$$

Column c3:

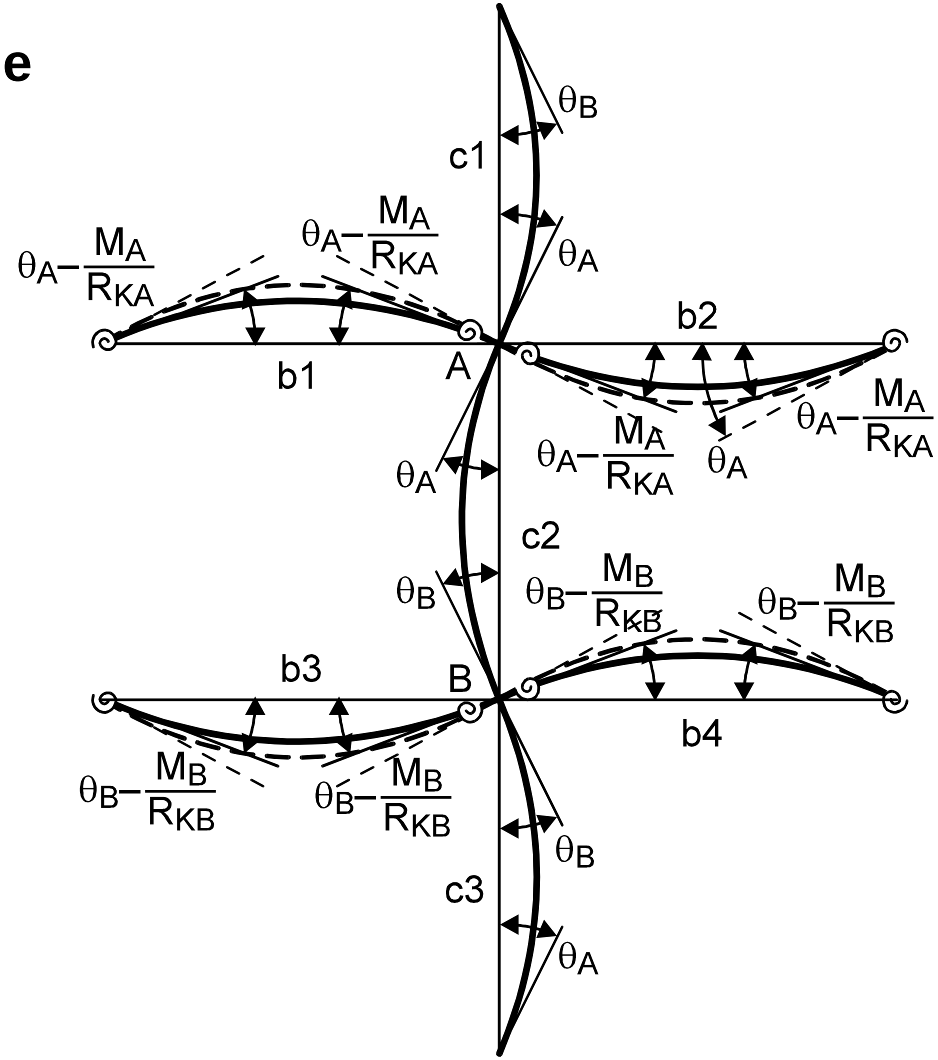
$$M_{B,c3} = \frac{EI_{c3}}{L_{c3}} \cdot (s_i \cdot c_i \cdot \theta_A + s_i \cdot \theta_B)$$

Beam b1:

$$\overline{\theta}_A = \theta_A - \frac{M_{A,b1}}{S_{j,b1}}$$

$$M_{A,b1} = \frac{EI_{b1}}{L_{b1}} \cdot (4\overline{\theta}_A - 2\overline{\theta}_A) = \frac{EI_{b1}}{L_{b1}} \cdot 2\overline{\theta}_A =$$

$$= \frac{EI_{b1}}{L_{b1}} \cdot \frac{1}{1 + \frac{2EI_{b1}}{L_{b1} \cdot S_{b1}}} \cdot 2\theta_A = \frac{EI_{b1}}{L_{b1}} \cdot \alpha_{f,b1} \cdot 2\theta_A$$



Subassembly model for braced frame

$$\alpha_{f,b1} = \frac{1}{1+\dfrac{2EI_{b1}}{L_{b1}\cdot S_{b1}}}$$

Beam b2:

$$M_{A,b2} = \frac{EI_{b2}}{L_{b2}}\cdot\alpha_{f,b2}\cdot 2\theta_A$$

Beam b3:

$$M_{A,b3} = \frac{EI_{b3}}{L_{b3}}\cdot\alpha_{f,b3}\cdot 2\theta_B$$

Beam b4:

$$M_{A,b4} = \frac{EI_{b4}}{L_{b4}}\cdot\alpha_{f,b4}\cdot 2\theta_B$$

For joint equilibrium at A:

$$M_{A,c1} + M_{A,c2} + M_{A,b1} + M_{A,b2} = 0$$

For joint equilibrium at B:

$$M_{A,c2} + M_{A,c3} + M_{A,b3} + M_{A,b4} = 0$$

$$\begin{bmatrix} s_i + \dfrac{2}{G'_A} & s_i\cdot c_i \\ s_i\cdot c_i & s_i + \dfrac{2}{G'_B} \end{bmatrix}\cdot\begin{bmatrix}\theta_A \\ \theta_B\end{bmatrix} = \mathbf{0}$$

$$G'_A = \frac{\sum\limits_{i(A)}\dfrac{EI_{ci}}{L_{ci}}}{\sum\limits_{i(A)}\alpha_{f,bi}\cdot\dfrac{EI_{bi}}{L_{bi}}} \qquad G'_B = \frac{\sum\limits_{i(B)}\dfrac{EI_{ci}}{L_{ci}}}{\sum\limits_{i(B)}\alpha_{f,bi}\cdot\dfrac{EI_{bi}}{L_{bi}}}$$

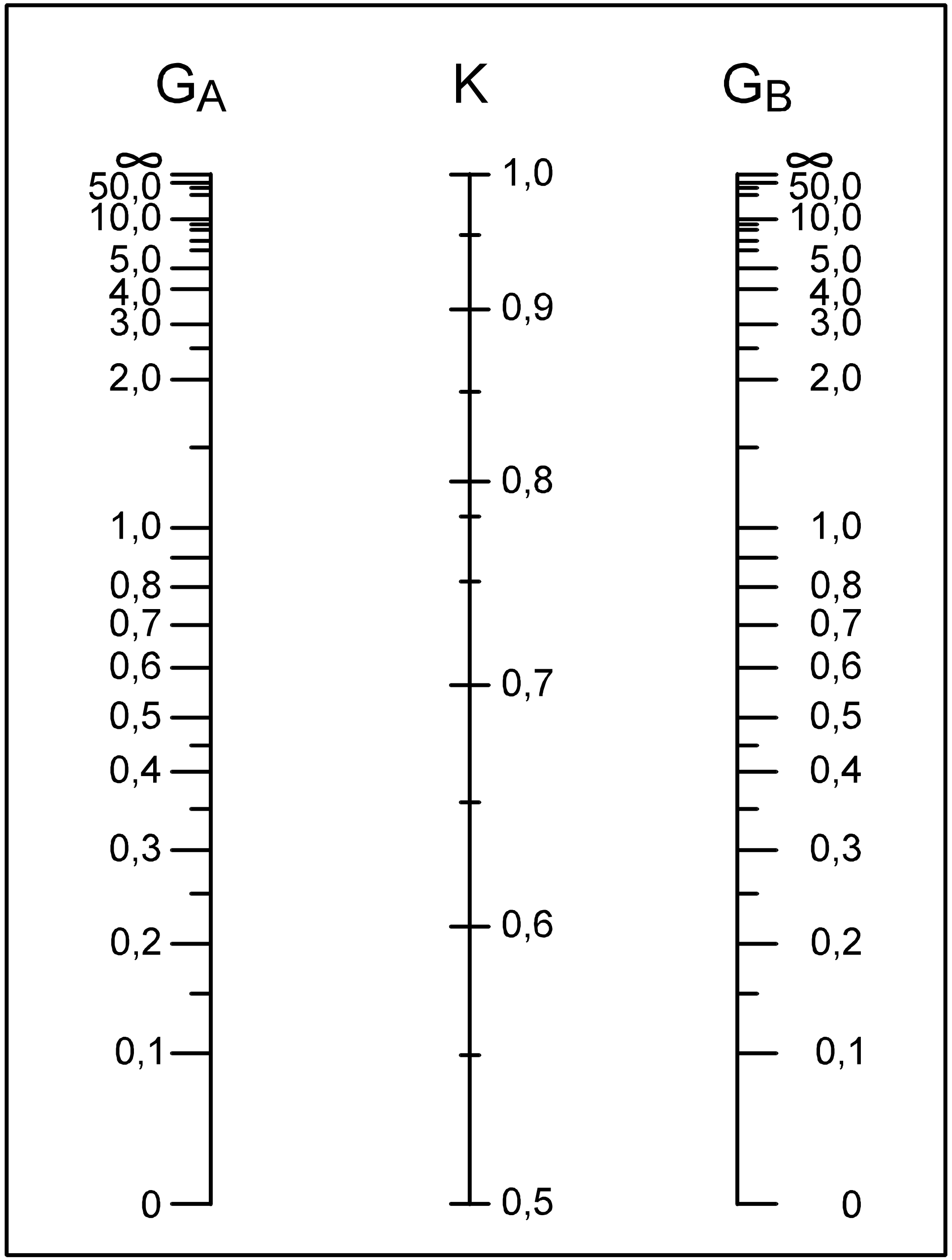
For non-trivial solution:

$$\det\begin{bmatrix} s_i + \dfrac{2}{G'_A} & s_i\cdot c_i \\ s_i\cdot c_i & s_i + \dfrac{2}{G'_B} \end{bmatrix} = 0$$

$$k\cdot L = L\cdot\sqrt{\frac{P}{EI}} = \pi\cdot\sqrt{\frac{P}{P_E}} = \frac{\pi}{\nu}$$

$$\frac{G'_A\cdot G'_B}{4}\cdot\left(\frac{\pi}{\nu}\right)^2 + \frac{G'_A+G'_B}{2}\cdot\left(1-\frac{\dfrac{\pi}{\nu}}{\mathrm{tg}\dfrac{\pi}{\nu}}\right) + 2\cdot\frac{\mathrm{tg}\dfrac{\pi}{\nu}}{\dfrac{\pi}{\nu}} - 1 = 0$$

Nomogram to Determine the Effective Length Factor for Braced Frames



3.4.2 Member of an Unbraced Frame

Column c1:

$$M_{A,c1} = \frac{EI_{c1}}{L_{c1}} \cdot (s_i \cdot \theta_A + s_i \cdot c_i \cdot \theta_B - s_i \cdot (1 + c_i) \cdot \Delta / L_{c1})$$

Column c2:

$$M_{A,c2} = \frac{EI_{c2}}{L_{c2}} \cdot (s_i \cdot \theta_A + s_i \cdot c_i \cdot \theta_B - s_i \cdot (1 + c_i) \cdot \Delta / L_{c2})$$

$$M_{B,c2} = \frac{EI_{c2}}{L_{c2}} \cdot (s_i \cdot c_i \cdot \theta_A + s_i \cdot \theta_B - s_i \cdot (1 + c_i) \cdot \Delta / L_{c2})$$

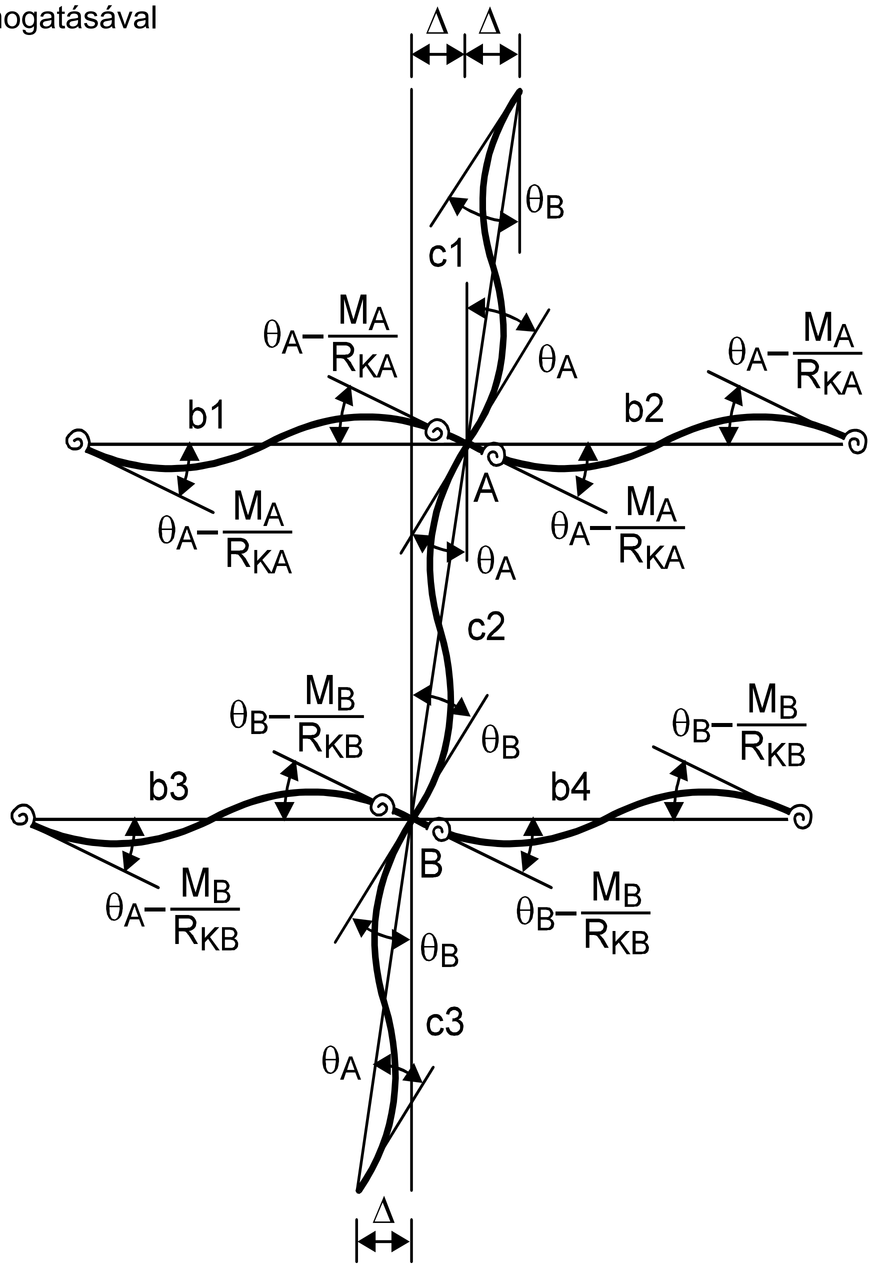
Column c3:

$$M_{B,c3} = \frac{EI_{c3}}{L_{c3}} \cdot (s_i \cdot c_i \cdot \theta_A + s_i \cdot \theta_B - s_i \cdot (1 + c_i) \cdot \Delta / L_{c3})$$

Beam b1:

$$\overline{\theta}_A = \theta_A - \frac{M_{A,b1}}{S_{j,b1}} \qquad \alpha_{k,b1} = \frac{1}{1 + \frac{6EI_{b1}}{L_{b1} \cdot S_{b1}}}$$

$$M_{A,b1} = \frac{EI_{b1}}{L_{b1}} \cdot (4\overline{\theta}_A + 2\overline{\theta}_A) = \frac{EI_{b1}}{L_{b1}} \cdot 6\overline{\theta}_A = \frac{EI_{b1}}{L_{b1}} \cdot \alpha_{k,b1} \cdot 2\theta_A$$



Subassembly model for unbraced frame

Beam b2:

$$M_{A,b2} = \frac{EI_{b2}}{L_{b2}} \cdot \alpha_{k,b2} \cdot 2\theta_A$$

Beam b3:

$$M_{A,b3} = \frac{EI_{b3}}{L_{b3}} \cdot \alpha_{k,b3} \cdot 2\theta_B$$

Beam b4:

$$M_{A,b4} = \frac{EI_{b4}}{L_{b4}} \cdot \alpha_{k,b4} \cdot 2\theta_B$$

For joint equilibrium at A:

$$M_{A,c1} + M_{A,c2} + M_{A,b1} + M_{A,b2} = 0$$

For joint equilibrium at B:

$$M_{A,c2} + M_{A,c3} + M_{A,b3} + M_{A,b4} = 0$$

For storey sway equilibrium:

$$M_{A,c2} + M_{B,c2} + P \cdot \Delta = 0$$

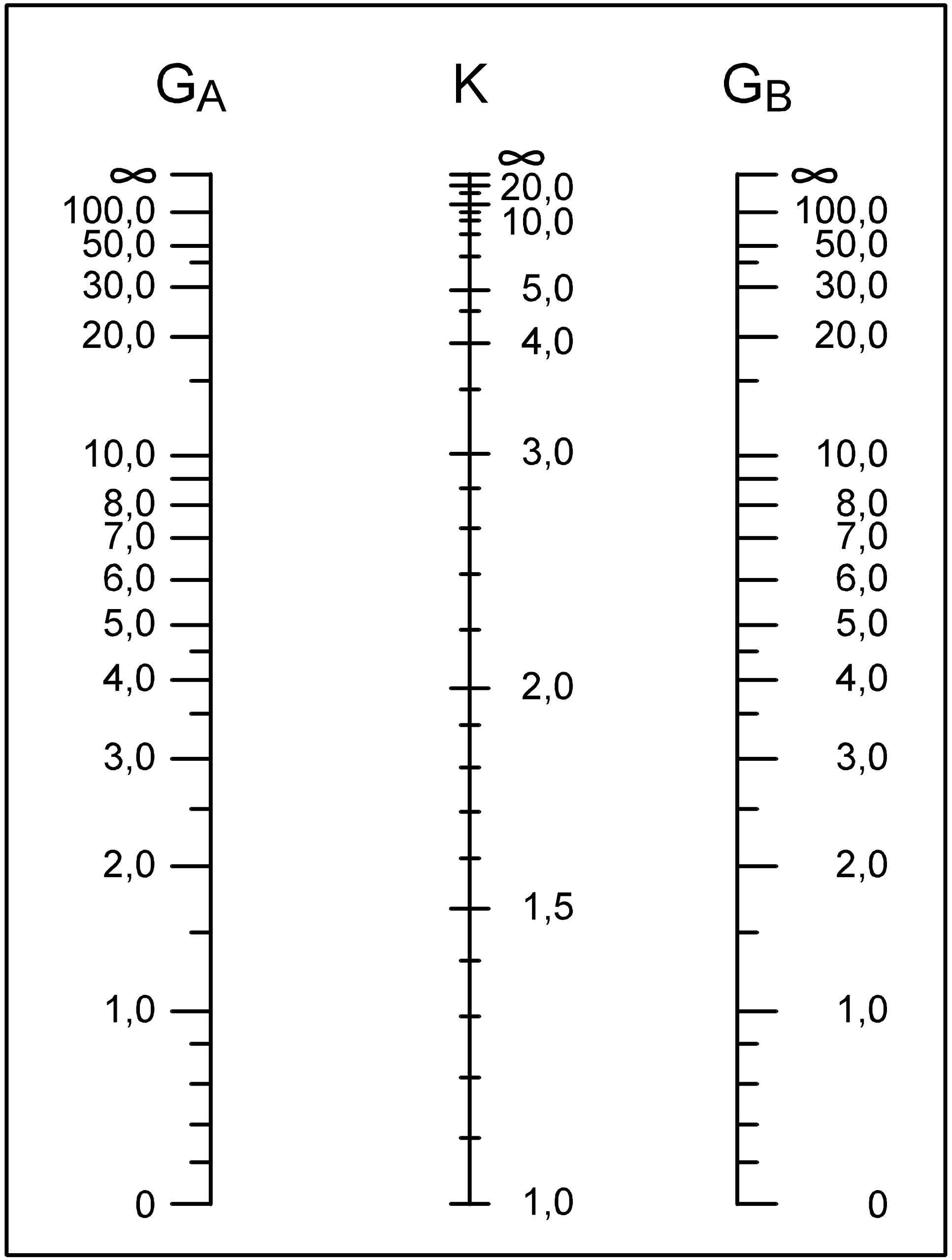
Matrix equation of equilibrium:

$$\begin{bmatrix} s_i + \frac{6}{G'_A} & s_i \cdot c_i & -s_i \cdot (1 + c_i) \\ s_i \cdot c_i & s_i + \frac{6}{G'_B} & -s_i \cdot (1 + c_i) \\ -\frac{6}{G'_A} & -\frac{6}{G'_B} & (k_{c2} \cdot L_{c2})^2 \end{bmatrix} \cdot \begin{bmatrix} \theta_A \\ \theta_B \\ \Delta / L_{c2} \end{bmatrix} = \mathbf{0}$$

$$G'_A = \frac{\sum_{i(A)} \frac{EI_{ci}}{L_{ci}}}{\sum_{i(A)} \alpha_{k,bi} \cdot \frac{EI_{bi}}{L_{bi}}} \qquad G'_B = \frac{\sum_{i(B)} \frac{EI_{ci}}{L_{ci}}}{\sum_{i(B)} \alpha_{k,bi} \cdot \frac{EI_{bi}}{L_{bi}}}$$

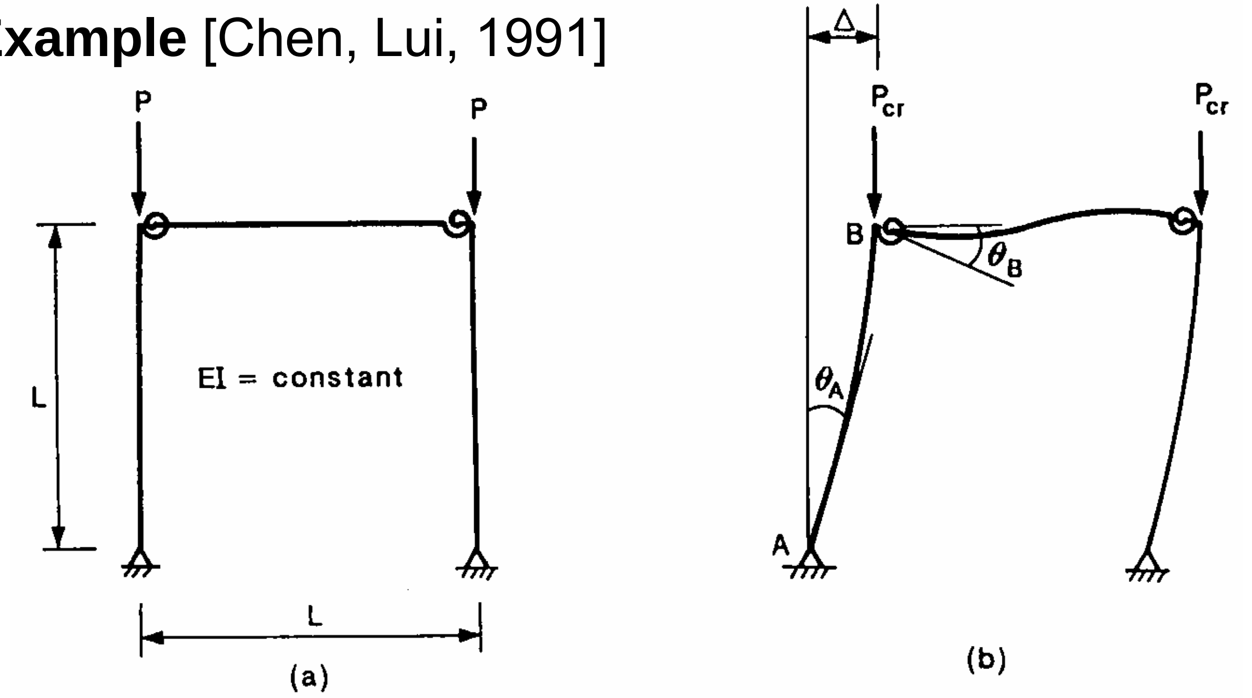
From the condition of non-trivial solution existence:

$$\frac{G'_A \cdot G'_B \cdot \left(\frac{\pi}{\nu}\right)^2 - 36}{6 \cdot (G'_A + G'_B)} - \frac{\frac{\pi}{\nu}}{\mathrm{tg}\frac{\pi}{\nu}} = 0$$



Nomogram to Determine the Effective Length Factor for Unbraced Frames

3.4.3 An Illustrative Example [Chen, Lui, 1991]



Buckling of a pinned-based portal frame

$$\begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} = \frac{EI}{L} \begin{bmatrix} 4\phi_3 & 2\phi_4 & -\frac{6}{L}\phi_2 \\ & 4\phi_3 + s_{ii}^* + s_{ij}^* & -\frac{6}{L}\phi_2 \\ \text{sym.} & & \frac{12}{L^2}\phi_1 \end{bmatrix} \begin{pmatrix} \theta_A \\ \theta_B \\ \Delta \end{pmatrix}$$

or, symbolically

$$\boldsymbol{R} = \boldsymbol{0} = \boldsymbol{K}\,\boldsymbol{D}$$

For a nontrivial solution, we must have

$$\det|\boldsymbol{K}| = 0$$

By assuming $EI/LR_{ki} = 0.1$ where R_{ki} is the initial stiffness of the connections, the critical load can be obtained by trial and error as $1.56\ EIL^2$.

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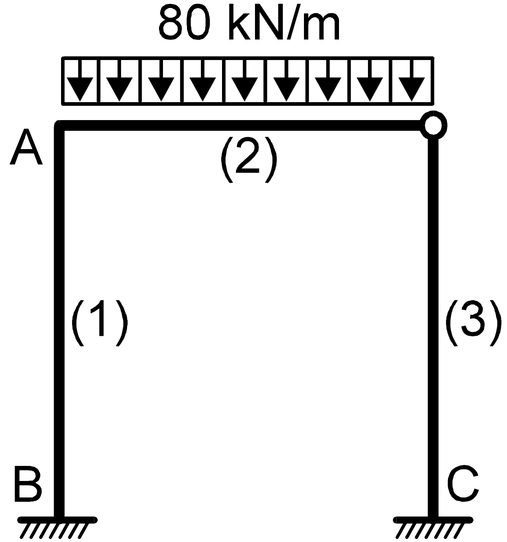
3.5. Examples for Use of Stability Functions

3.5.1 Second-Order Bending Moments

(a) Determine in detail the equilibrium equations for the frame:



$$M_A = (s_1 \cdot k_1 + s_2'' \cdot k_2) \cdot \theta_A - s_1 \cdot (1 + c_1) \cdot \frac{k_1}{l_1} \cdot \Delta = \frac{q_2 \cdot l_2^2}{8};$$

$$V = -s_1 \cdot (1 + c_1) \cdot \frac{k_1}{l_1} \cdot \theta_A + \left[\frac{2s_1 \cdot (1 + c_1)}{m_1} \cdot \frac{k_1}{l_1^2} + (s_3'' - \pi^2 \cdot \rho_3) \cdot \frac{k_3}{l_3^2} \right] \cdot \Delta = 0.$$

$$l_1 = l_2 = l_3 = 4\,\text{m}$$

$$EI_1 = 3200\,\text{kNm}^2$$

$$EI_2 = 6000\,\text{kNm}^2$$

$$EI_3 = 1920\,\text{kNm}^2$$

(b) Show the condition of the normal forces:

$$N_1 = \frac{5}{8} \cdot q \cdot l_1 = 200\text{kN}; \quad P_{E1} = \frac{\pi^2 \cdot EI_1}{l_1^2} = 2000\text{kN}; \quad \rho_1 = \frac{N_1}{P_{E1}} = 0{,}1;$$

$$N_1 \approx 0; \qquad \rho_2 = 0;$$

$$N_3 = \frac{3}{8} \cdot q \cdot l_3 = 120\text{kN}; \quad P_{E3} = \frac{\pi^2 \cdot EI_3}{l_3^2} = 1200\text{kN}; \quad \rho_3 = \frac{N_3}{P_{E3}} = 0{,}1;$$

$$k_1 = \frac{EI_1}{l_1} = 800\text{kNm}; \quad k_2 = \frac{EI_2}{l_2} = 1500\text{kNm}; \quad k_3 = \frac{EI_3}{l_3} = 480\text{kNm}.$$

(c) Define the displacements: $\Delta = 0.0607$ m; $\theta_A = 0.0305.$

(d) Define the internal forces at the bar ends:

$$M_{1A} = M_{2A} = s_1 \cdot k_1 \cdot \theta_A - s_1 \cdot (1 + c_1) \cdot \frac{k_1}{l_1} \cdot \Delta = 22.75 \text{ kNm;}$$

$$M_{1B} = s_1 \cdot c_1 \cdot k_1 \cdot \theta_A - s_1 \cdot (1 + c_1) \cdot \frac{k_1}{l_1} \cdot \Delta = -21.65 \text{ kNm;}$$

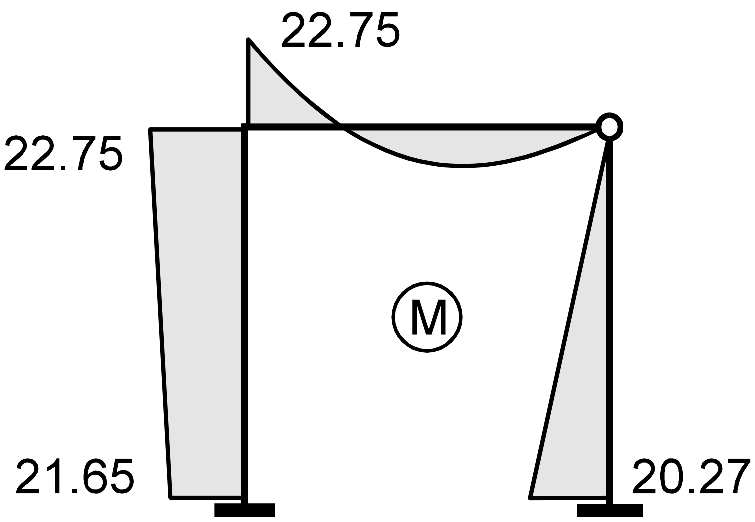
$$M_{3C} = -s_3'' \cdot \frac{k_3}{l_3} \cdot \Delta = -20.27 \text{ kNm.}$$

$$N_2 = V_3 = (s_3'' - \pi^2 \cdot \rho_3) \cdot \frac{k_3}{l_3^2} \cdot \Delta = 3.28 \text{ kN}$$

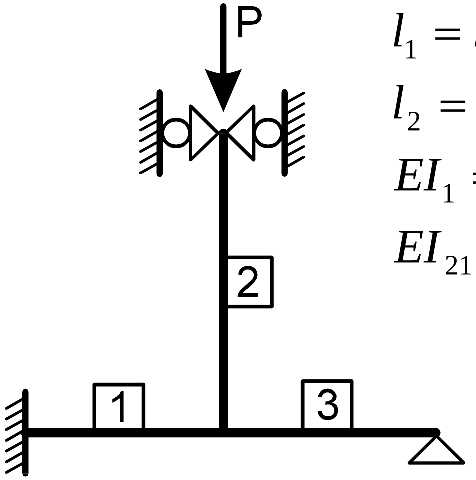
(compression)

$\rho_2 = 0{,}001 \approx 0$: O.K.

(e) Sketch the figures of the internal forces:



3.5.2 Critical Force

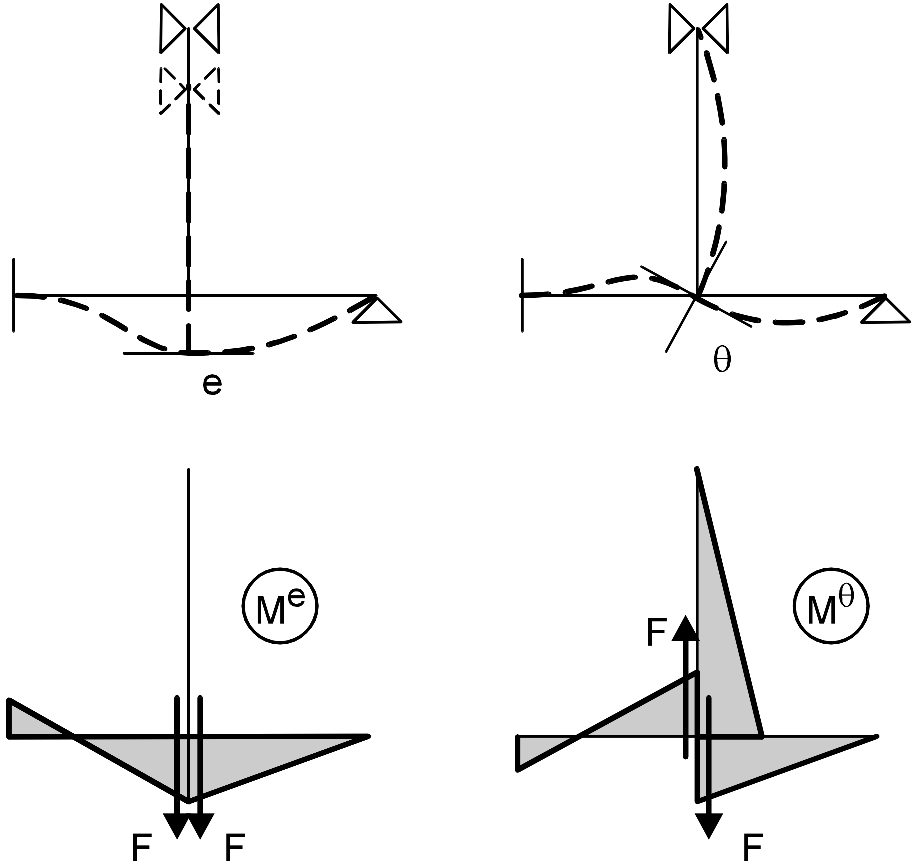


$$l_1 = l_3 = 2\,\text{m}$$

$$l_2 = 4\,\text{m}$$

$$EI_1 = EI_3 = 200\,\text{kNm}^2$$

$$EI_{21} = 600\,\text{kNm}^2$$



(a) Determine in detail the equilibrium equations for the frame:

$$\left[\begin{array}{c:c} \dfrac{2s_1(1+c_1)}{m_1}\dfrac{k_1}{l_1}\dfrac{1}{l_1}+(2s_3''-\pi^2\rho_3)\dfrac{k_3}{l_3}\dfrac{1}{l_3} & -s_1(1+c_1)\dfrac{k_1}{l_1}+s_3''\dfrac{k_3}{l_3} \\ \hdashline -s_1(1+c_1)\dfrac{k_1}{l_1}+s_3''\dfrac{k_3}{l_3} & s_1k_1+s_3''k_3+s_2''k_2 \end{array}\right]\begin{bmatrix} e \\ - \\ \theta \end{bmatrix}=\begin{bmatrix} 0 \\ - \\ 0 \end{bmatrix}$$

(b) Show the condition of the normal forces:

$$\rho_1 = \rho_3 \approx 0$$

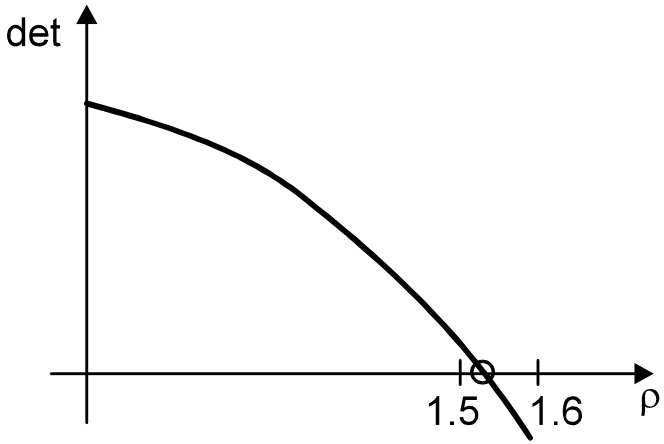
(c) Define the critical force:

$$\det\left|\begin{array}{c|c} 375 & -150 \\ \hline -150 & 700+150s_2'' \end{array}\right| = 0$$

$$\rho_2 = 1.5 \rightarrow s_2'' = -4.215$$
$$\det = +116.25$$

$$\rho_2 = 1.6 \rightarrow s_2'' = -6.032$$
$$\det = -3972.0$$

$$\boxed{\rho_2 = 1.503}$$



$$P_{cr,2} = \rho_2 \frac{\pi^2 \cdot EI_2}{l_2^2} = 1.503 \cdot \frac{\pi^2 \cdot 600}{16} = 556.0 \text{ kN}$$

(d) Calculate the effective length factor for column #2:

$$\rho_2 = \frac{1}{\nu_2^2} \longrightarrow \nu_2 = \frac{1}{\sqrt{\rho_2}} = 0.816$$