Készült az ERFP – DD2002 – HU – B – 01 szerzősésszámú projekt támogatásával

C h a p t e r   4

Concepts and Applications of Imperfect Steel Frames

4.1. Introduction

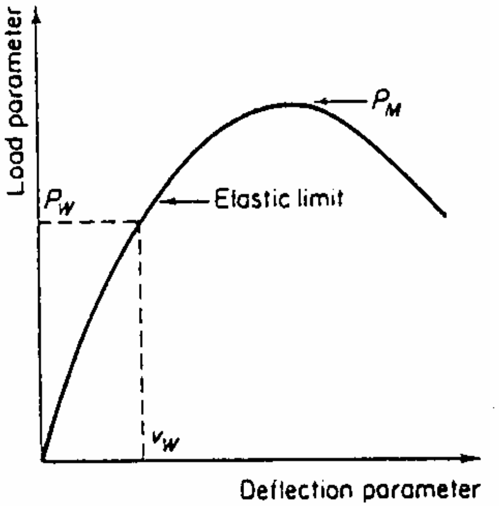


Figure 1

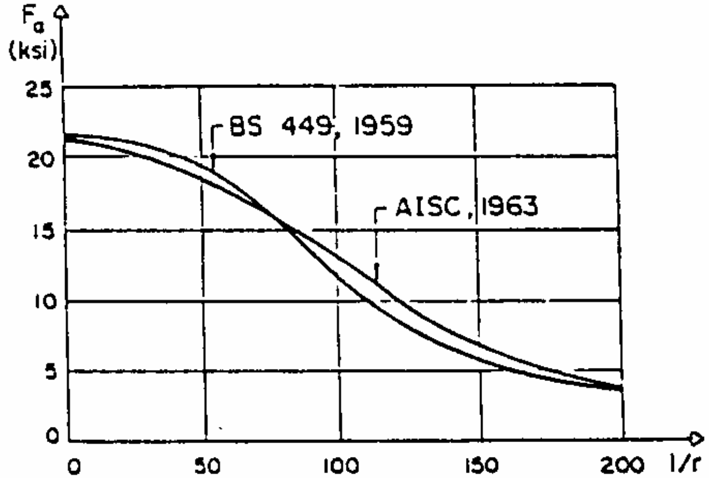


Figure 2

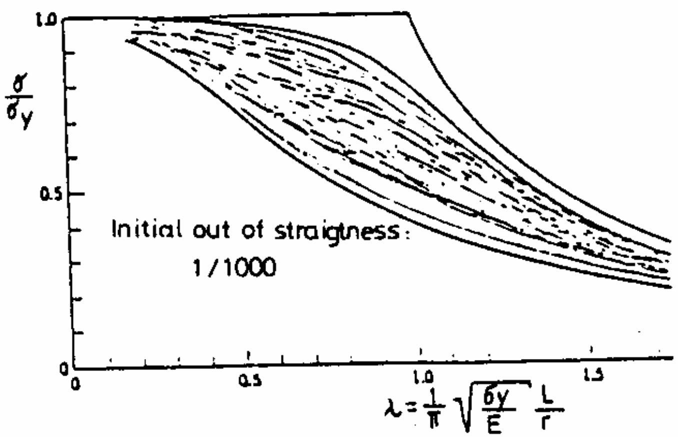


Figure 3

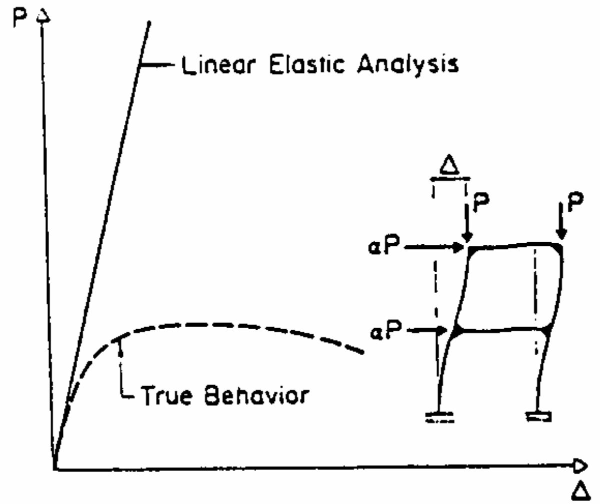


Figure 4

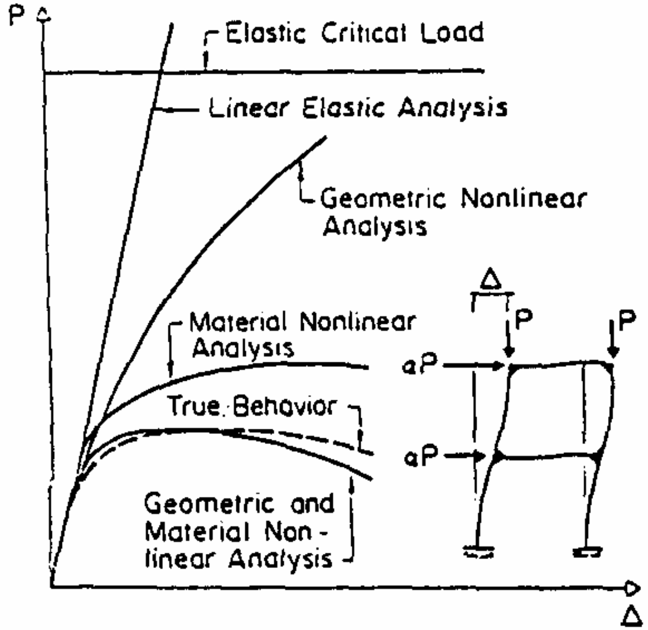


Figure 5

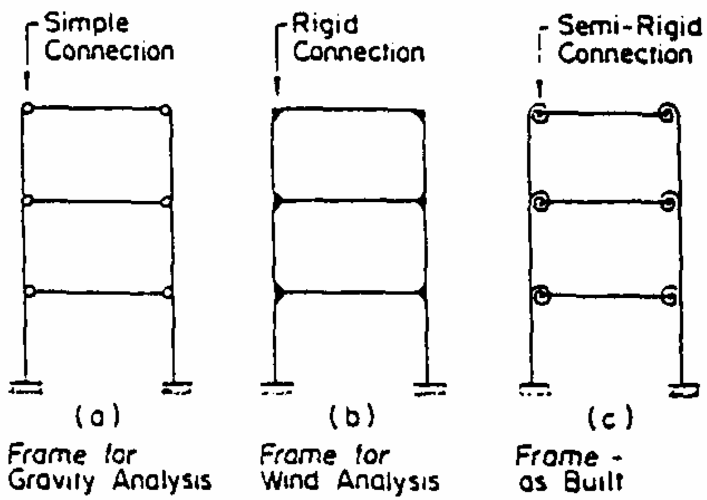


Figure 6

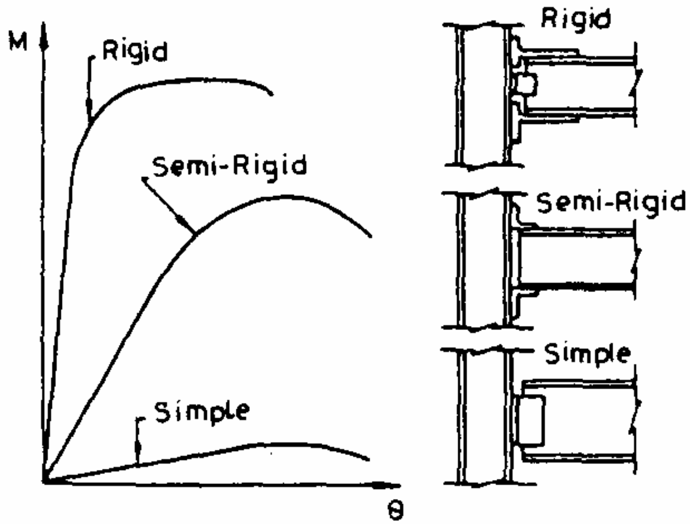


Figure 7

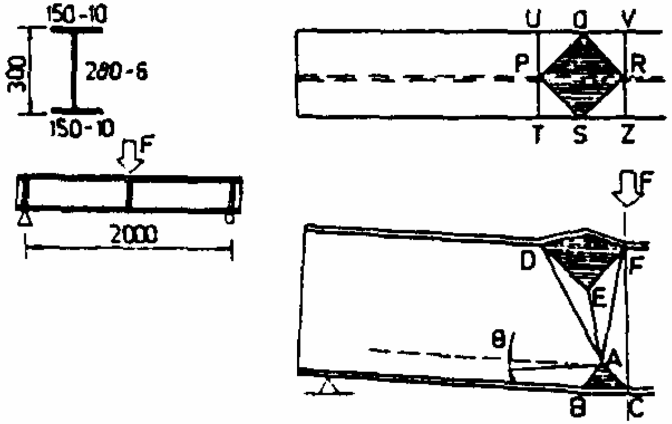


Figure 8

4.2. Some Aspects of the Stability Criterion

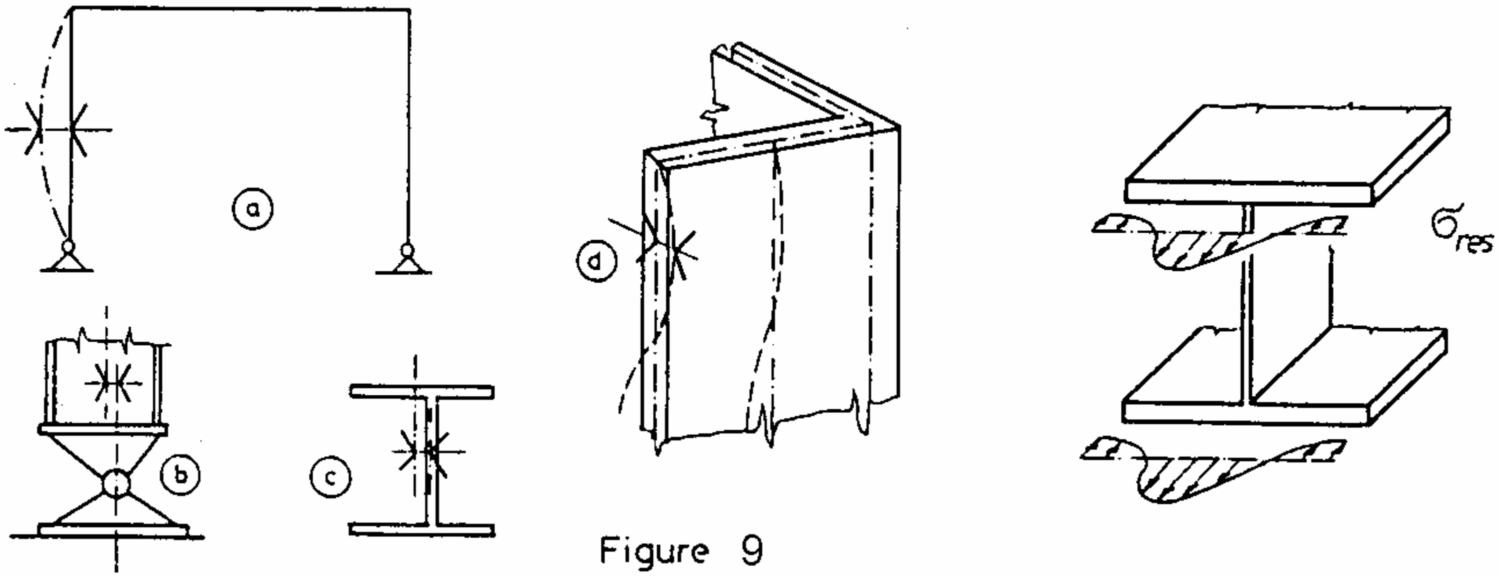


Figure 9

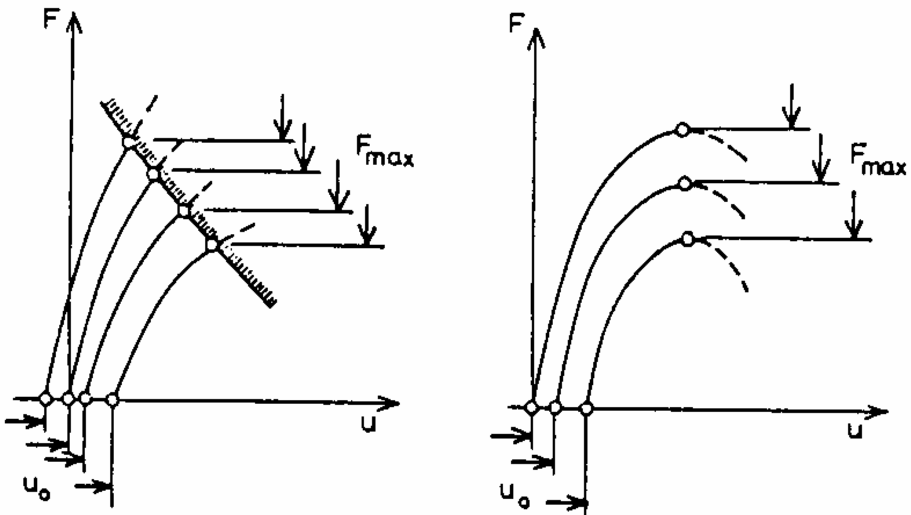


Figure 10

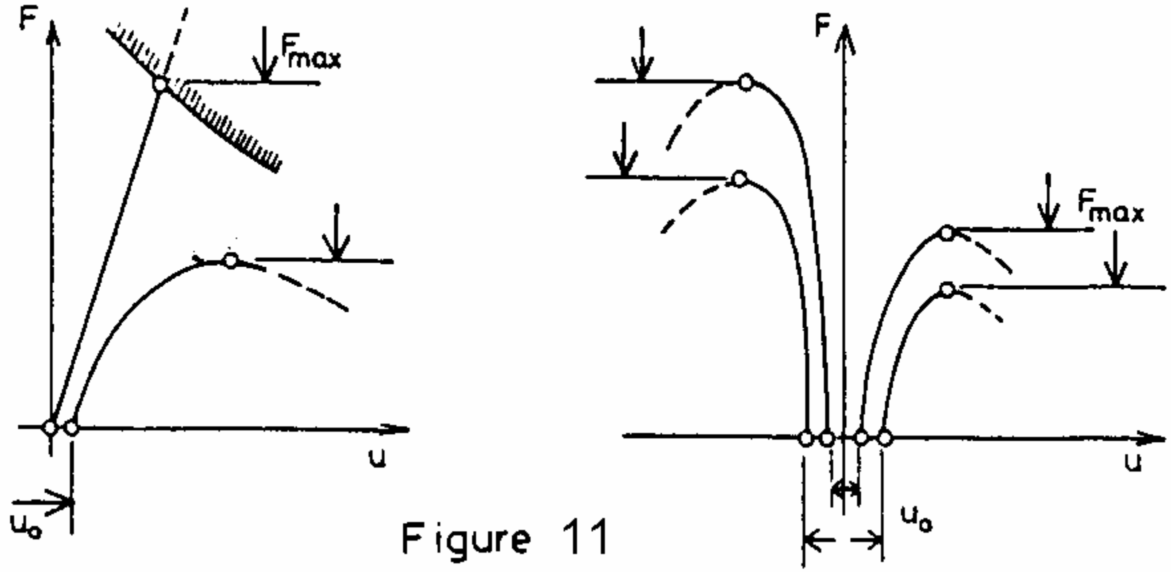


Figure 11

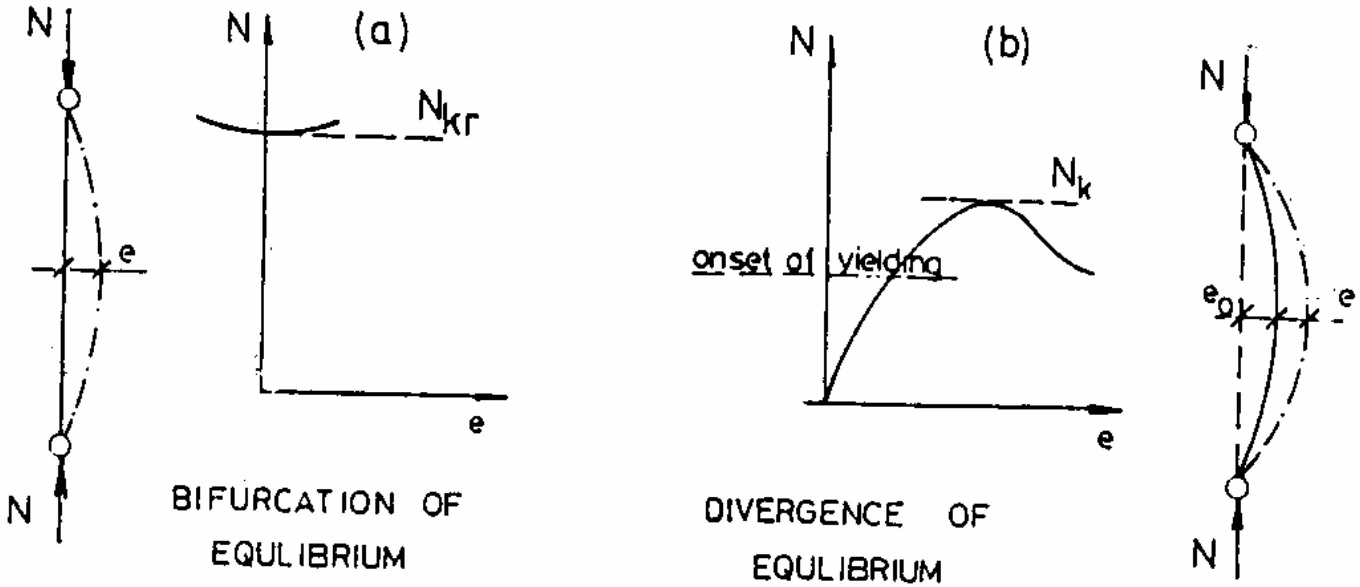
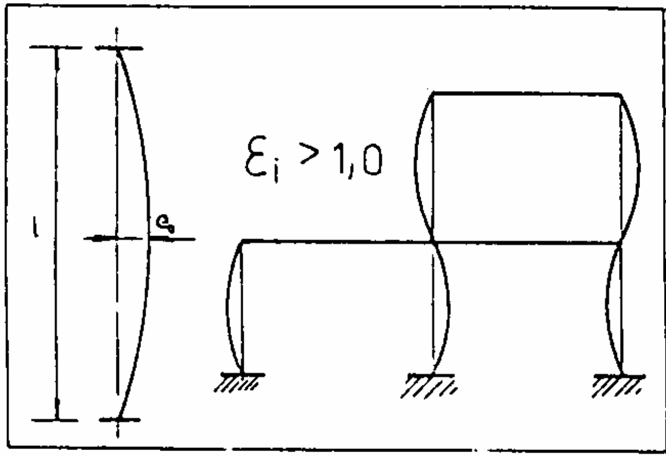


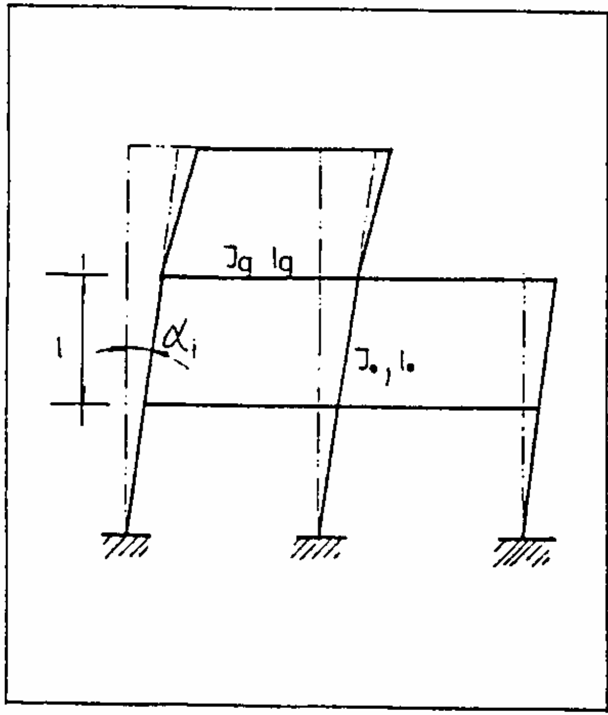
Figure 12

4.3. Taking the Initial (Real and Fictious) Imperfections



Buckling curve (MSZ 15024/1-85)	Initial imperfection e_o
a	$\frac{l}{500}$
b	$\frac{l}{250}$
c	$\frac{l}{200}$

Table I.



Range of application:

1.) $$\varepsilon_i = l_i \sqrt{\frac{N_i}{E \cdot I_i}} \leq 1{,}6$$

2.) $$\nu_j = \frac{\Sigma J_g / l_g}{\Sigma J_o / l_o} \geq 1{,}0$$

$$\alpha_i = \frac{1}{150} \cdot \gamma \qquad \gamma = \frac{1}{2}\left(1 + \frac{1}{n-1}\right)$$

n = number of columns in one storey in the plane

Table II.

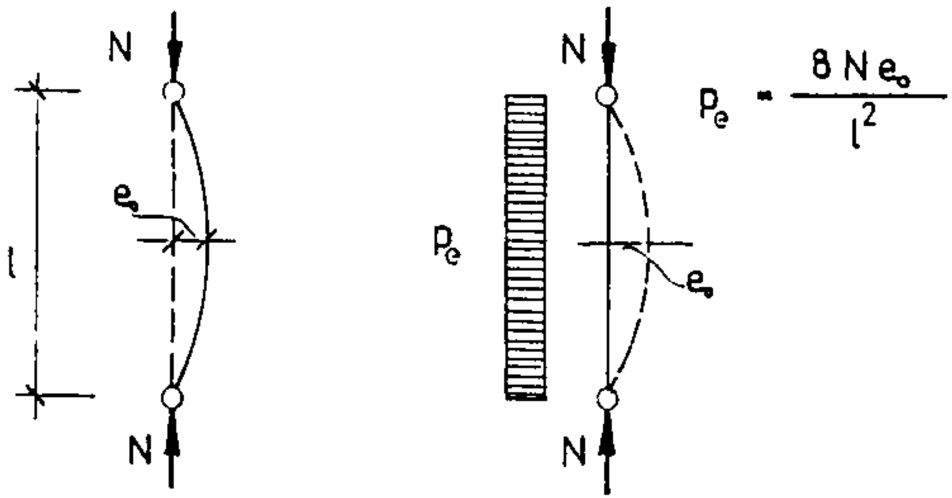


Figure 13

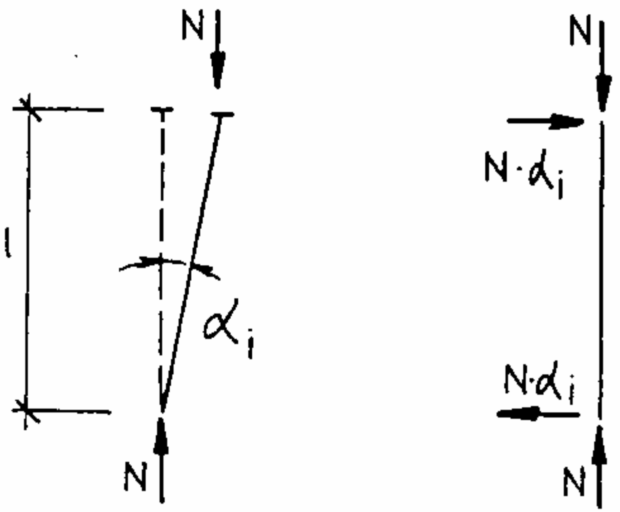


Figure 14

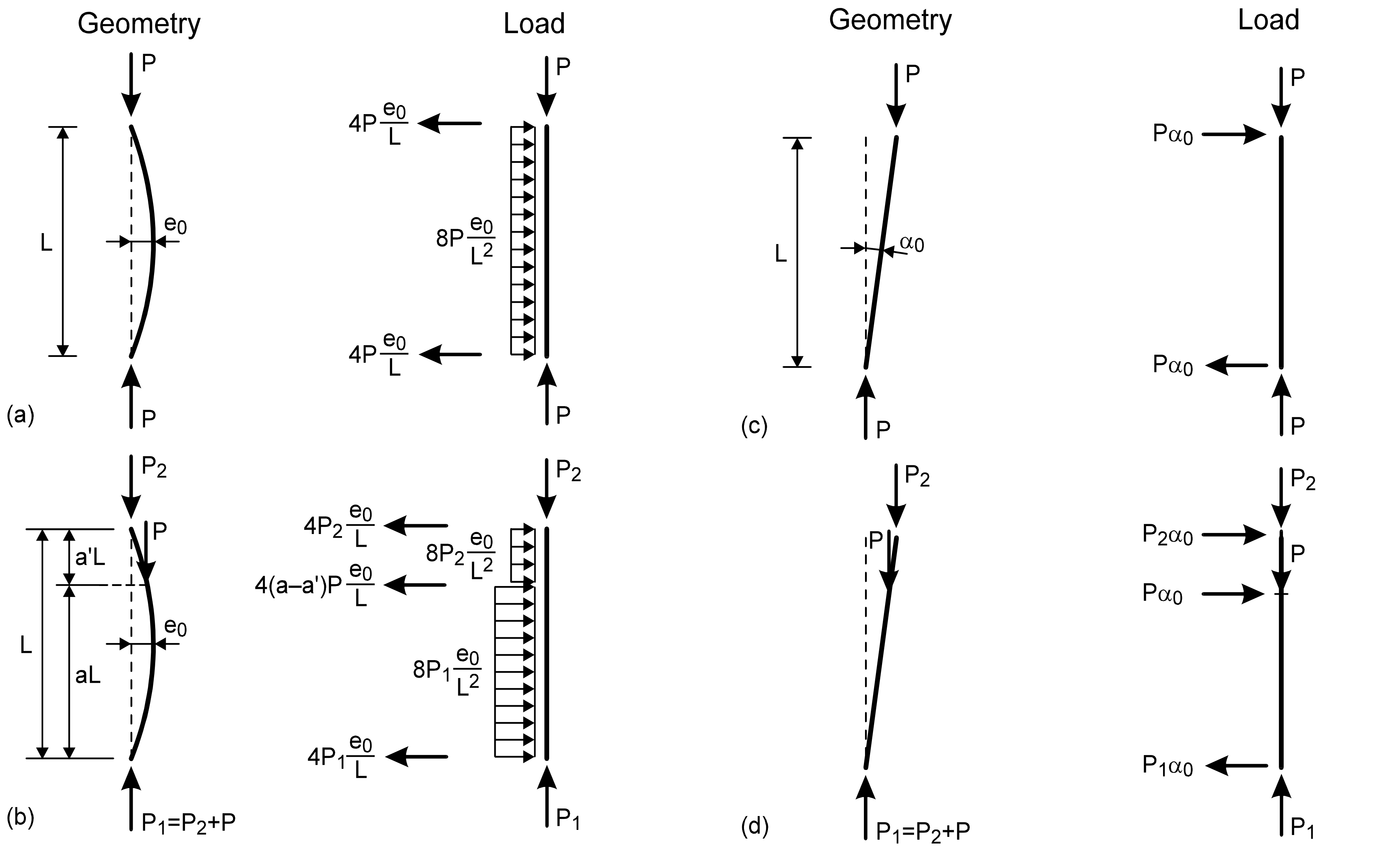


Figure 15: Geometry- and load-type imperfections for columns

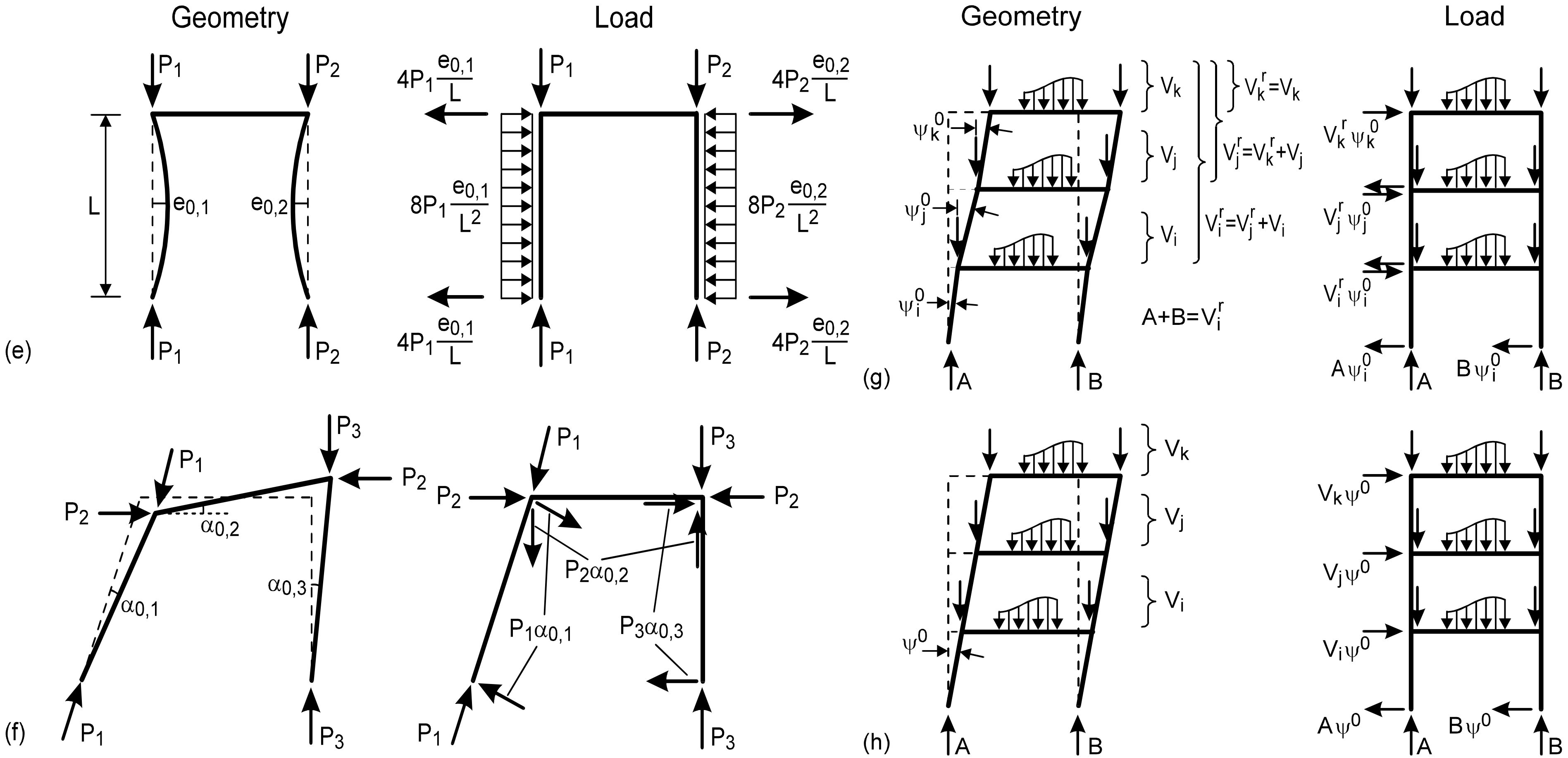


Figure 16: Geometry- and load-type imperfections for frames

4.4. Simplification Possibilities of Second-Order Theory in Computer Calculation of Sway Frames

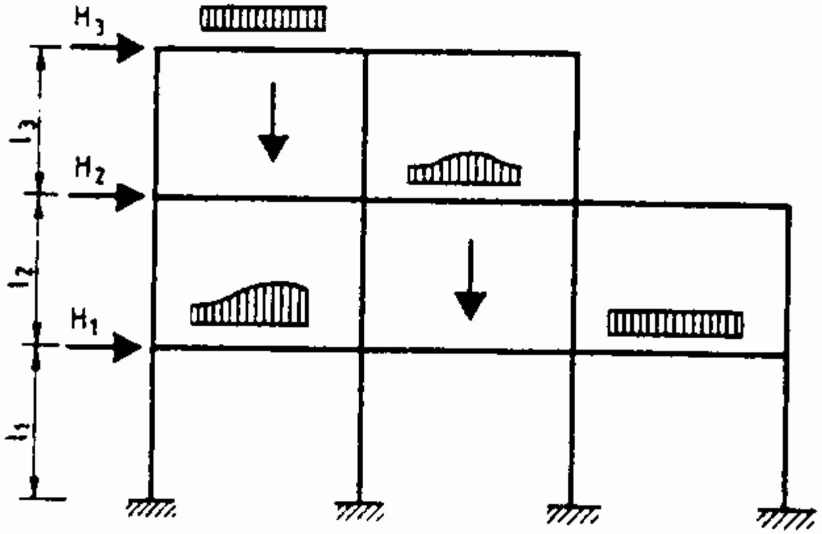


Figure 17

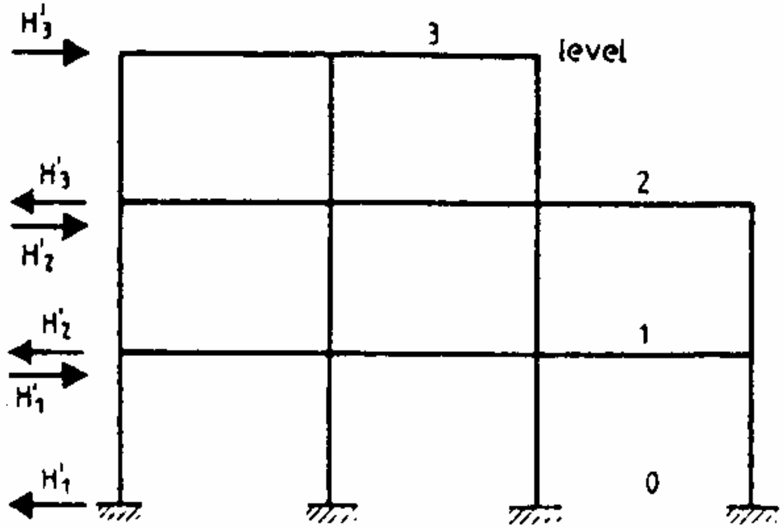


Figure 18

$$H'_j = \alpha_i \cdot V_j \text{ and } V_j = \sum N_{j,i}$$

$$\Delta\alpha_j = \frac{\delta_j}{\ell_j}$$

Afterward the increment of "shifting" force

$$\Delta H_j = V_j \cdot \Delta\alpha_j$$

can be determined and the value of $\Delta H_j / Q_j$, where Q_j is the original shifting force

$$Q_j = \sum_j^m H_i + H_j'$$

In the formula of Q_j summation includes the horizontal forces above level "j", m is the number of levels.

If condition

$$\frac{\Delta H_j}{Q_j} \leq 0.1$$

is met on each level of the frame then internal forces calculated by first–order theory, but taken the initial out–of–straightness of columns into consideration are considerably accurate.

If condition

$$0.1 < \frac{\Delta H_j}{Q_j} \leq 0.25$$

is met on each level of the frame and the distribution of internal forces due second-order effect is similar to the one of the first-order theory, then internal forces can be calculated by first-order analysis, increased by

$$Q_j' = \frac{1}{1 - \frac{\Delta H_j}{Q_j}} \cdot Q_j$$

shifting force on each level.

4.5. Second-Order Theory in Manual Calculations; Stability Functions

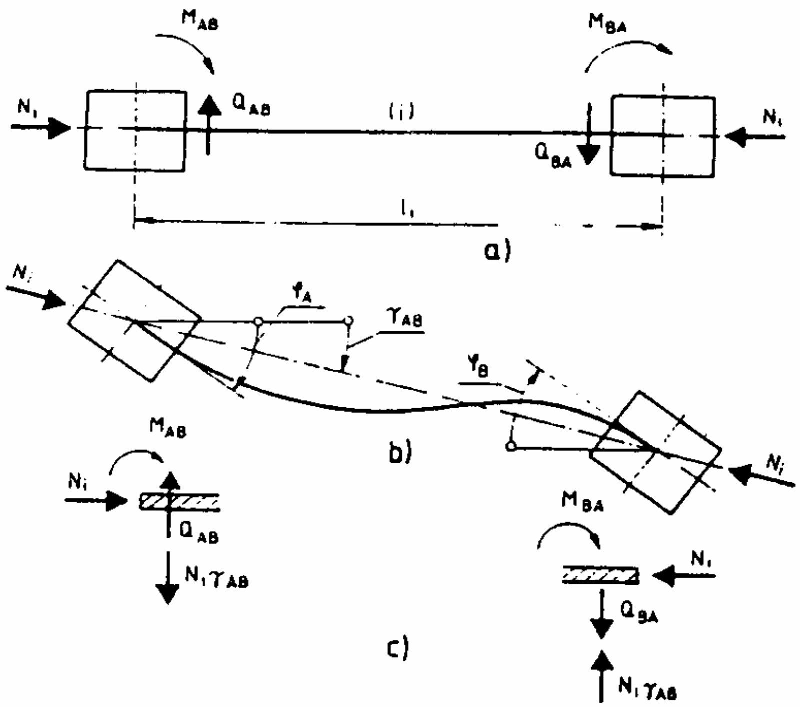


Figure 19

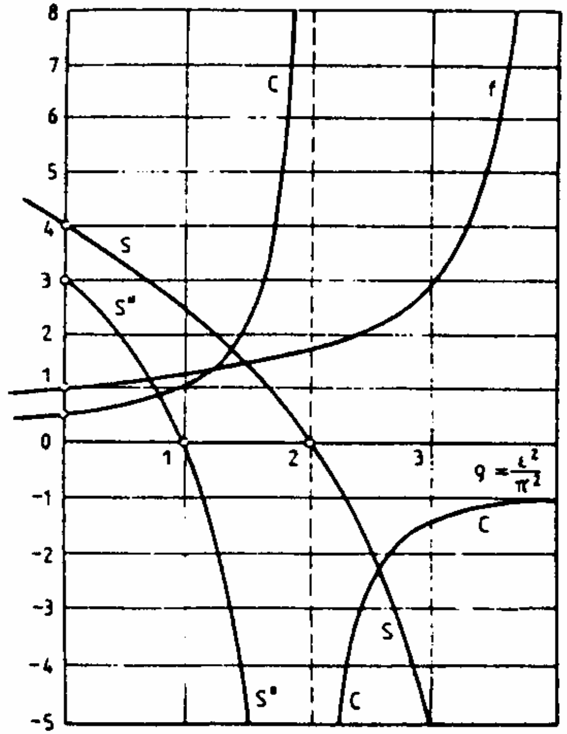


Figure 20

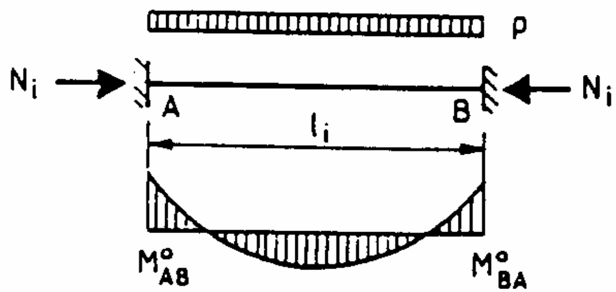


Figure 21

Horne, Merchant, 1965

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$$M_{AB} = \frac{E \cdot I_i}{\ell_i}\left[s_i\varphi_A + s_i c_i \varphi_B - s_i(1+c_i)\gamma_{AB}\right]$$

$$M_{BA} = \frac{E \cdot I_i}{\ell_i}\left[s_i\varphi_B + s_i c_i \varphi_A - s_i(1+c_i)\gamma_{AB}\right]$$

Coefficients s_i and c_i, the so–called stability functions, can be determined from the following formulas

$$s_i = \frac{(1-\varepsilon_i \cot\varepsilon_i)\frac{\varepsilon_i}{2}}{\tan\frac{\varepsilon_i}{2} - \frac{\varepsilon_i}{2}} \qquad c_i = \frac{\epsilon_i - \sin\varepsilon_i}{\sin\varepsilon_i - \varepsilon_i\cos\varepsilon_i}$$

$$\text{and} \qquad \varepsilon_i = \ell_i\sqrt{\frac{N_i}{E \cdot I_i}}$$

If one end (B) is pinned, then

$$M_{BA} = 0 \qquad \text{and} \qquad M_{BA} = \frac{E \cdot I_i}{\ell_i} s_i'' \, (\varphi_A - \gamma_{AB});$$

$$s_i'' = s_i \cdot (1 - c_i^2)$$

Values of stability functions, s_i, c_i and s_i'' are given in Figure 2.10 in the function of $\rho_i = \frac{\varepsilon_i^2}{\pi^2}$.

When the joint equilibrium is written it should be considered that besides moments M_{AB} and M_{BA}, compression force N_i and shear forces

$$Q_{AB} = -Q_{BA} = -\frac{M_{AB} + M_{BA}}{\ell_i}$$

the additional shear forces of

$$Q_{AB} = -Q_{BA} = -N_i \cdot \gamma_{AB} = N_i \cdot \frac{e_B}{\ell_i}$$

act on bar ends.

The end moments M_{AB} and M_{BA}, of bar with both ends fixed should be determined by taking the compression force into consideration. The followings are the calculation formulae of the principal case.

Values for distributed load are also given in Figure.

$$M^0_{AB} = -M^0_{BA} = f_i \cdot \frac{p \cdot \ell_i^2}{12}$$

$$f_i = \frac{12}{\varepsilon_i^2} \cdot \left(1 - \frac{\varepsilon_i}{2} \cdot \cot \frac{\varepsilon_i}{2}\right)$$

Superposition of loads for second-order analysis holds only if compression forces for different loading cases are equal. With the approximation on the safety side the compression forces belonging to the most unfavorable cases can be assumed for each loading cases.

4.6. Examples

4.6.1 Example #1

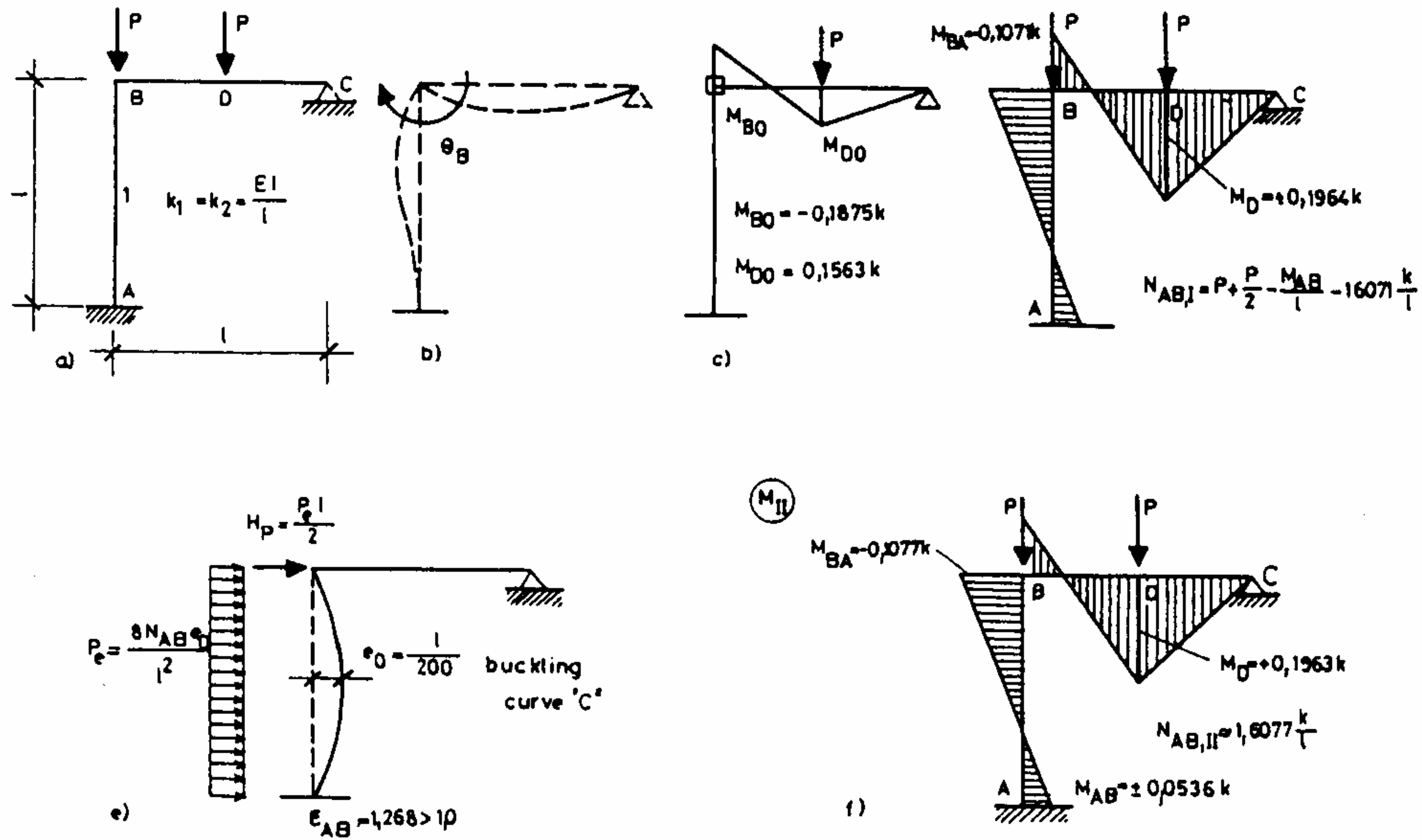


Figure 22

a) Calculation of framework by disassembling to elements (beams, columns).

The equilibrium equation:

$$(s_1 k_1 + s_2'' k_2) \cdot \Theta_B = M_B$$

During first–order analysis we neglect the effect of axial forces, therefore the equilibrium equation and its solutions are:

$$(4+3) \cdot k \cdot \Theta_B = 0.1875 \cdot k$$

$$\Theta_B = 0.0268$$

Figure shows the distribution of the first–order moments.

The bending moment calculated by first–order analysis shall be modified. In order to do this, the effective length of the column must be determined. The factor showing the growth of eccentricity is (MSZ 15024-85)

$$\nu = 0.5 + \frac{1}{6\mu + 4} = 0.618 \qquad \mu = \frac{0.75 \cdot I_2 \cdot \ell_i}{I_1 \cdot \ell_g} = 0.75$$

$$\psi = \frac{1}{1 - \frac{N}{N_E}} = \frac{1}{1 - \frac{N}{A\sigma_H}\left[\frac{\lambda}{\lambda_E}\right]^2} = \frac{1}{1 - \frac{N\nu^2}{\pi^2}} = 1.066$$

Load–bearing capacity of column AB by taking the maximum internal forces into consideration:

$$\frac{N_I}{N_H} + \frac{\psi M_I}{M_H} = 1 \qquad \psi \cdot M_I = 1.066 \cdot 0.1071\, k = 0.1142 \cdot k$$

b) Calculation of plane framework (statical skeleton). The initial inaccuracies taken into consideration at planar statical skeletons make possible that, beside strength analysis carried out by second–order analysis, only plate element shall be checked for plate buckling and the beam shall be checked in the plane perpendicular to it.

Following are the initial inaccuracies .

$$N_{AB} = 1.607\frac{k}{\ell} \qquad \varepsilon_{AB} = 1.268$$

$$p_e = \frac{8N_{AB}e_0}{\ell^2} = \frac{8 \cdot 1.607\ k}{200\ \ell^2} = 0.0643\frac{k}{\ell^2}$$

$$H_P = \frac{p_e \ell}{2} = 0.0321\frac{k}{\ell}$$

$$\sum H = H_P = 0.0321\frac{k}{\ell} \approx 0.$$

$$\rho = \frac{\epsilon^2}{\pi^2} = 0.163 \qquad s = 3.7890 \qquad f = 1.0279 \qquad c = 0.5439$$

$$M_{AB,P} = f \cdot \frac{p_e\ \ell^2}{12} = 1.0279\frac{0.0643}{12} \cdot k = 0.00551k$$

$$(3.7890 + 3) \cdot k \cdot \Theta_B = 0.193k$$

$$\Theta_B = 0.0248 \qquad M_{BA} = 0.1077k$$

Load–bearing capacity analysis of column AB: $\frac{N_{AB,I}}{N_{AB,II}} = \frac{1.6971}{1.6077} \approx 1.00$

$$\frac{\psi M_{AB,I}}{M_{AB,II}} = \frac{0.1142}{0.1077} = 1.06$$

4.6.2 Example #2

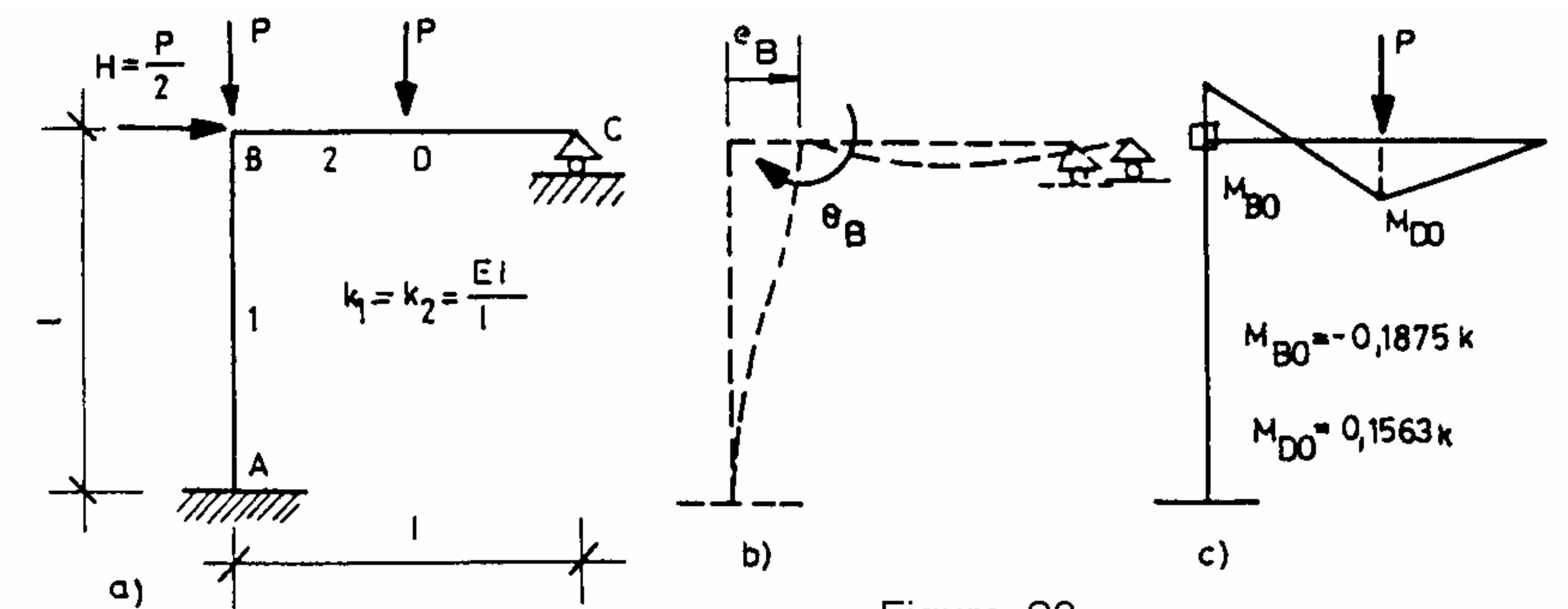


Figure 23

Analysis of Sway Steel Structure

a) Calculation of the structure by disassembling to elements (beams, columns)

The equilibrium equations are:

$$\begin{bmatrix} (s_1+s_2)k & \vdots & -s_1(1+c_1)\frac{k}{\ell} \\ \cdots & \cdots & \cdots \\ -s_1(1+c_1)\frac{k}{\ell} & \vdots & \frac{2s_1(1+c_1)}{m_1}\frac{k}{\ell} \end{bmatrix} = \begin{bmatrix} \Theta_B \\ \cdots \\ e_B \end{bmatrix} = \begin{bmatrix} M_B \\ \cdots \\ H_B \end{bmatrix}$$

During the first–order analysis we neglect the effect of axial forces, therefore the equilibrium equations and their solution are:

$$\begin{bmatrix} (4+3)k & \vdots & -6\frac{k}{\ell} \\ \cdots & \cdots & \cdots \\ -6\frac{k}{\ell} & \vdots & 12\frac{k}{\ell} \end{bmatrix} = \begin{bmatrix} \Theta_B \\ \cdots \\ e_B \end{bmatrix} = \begin{bmatrix} 0.18575k \\ \cdots \\ 0.5\frac{k}{l} \end{bmatrix}$$

$$\Theta_B = 0.1094 \qquad e_B = 0.0964\ell$$

The bending moment calculated by first–order analysis, shall be modified, therefore the effective length of the column shall be determined. The ψ factor showing the increment of the eccentricity is:

$$\nu = \sqrt{\frac{1+m}{2}} \cdot \sqrt{1+0.35(c+6\alpha)-0.017(c+6\alpha)^2}$$

$$\text{if} \qquad m=1 \qquad c=2 \qquad \alpha\approx 0 \qquad \nu = 1.29$$

$$\psi = \frac{1}{1-\dfrac{N}{N_E}} = \frac{1}{\dfrac{N_{AB}}{\frac{\pi^2 EI}{(\nu\ell)^2}}} = \frac{1}{1-\dfrac{1.36N^2}{\pi^2}} = 1.298.$$

Load–bearing capacity of column AB by taking the maximum internal forces into consideration

$$\frac{N_I}{N_H} + \frac{\psi M_I}{M_H} = 1; \qquad \psi \cdot M_I = 1.298 \cdot 0.359k = 0.466k$$

b) Calculation of plane framework (statical skeleton). The initial inaccuracies taken into consideration at planar statical skeletons, make possible that, beside strength analysis carried out by second–order analysis, only plate elements shall be checked in the plane perpendicular to it.

Following are (Figure 2.13e) the initial inaccuracies according to Section 2.22:

$$N_{AB} = 1.36\frac{k}{\ell} \qquad \varepsilon_{AB} = 1.36$$

$$H_\alpha = N_{AB} \cdot \alpha_0 = 1.36\frac{k}{\ell}\frac{1}{150} = 0.00905\frac{k}{\ell}$$

$$\sum H = H + H_\alpha = 0.5091\frac{k}{\ell}$$

$$\rho = \frac{\varepsilon^2}{\pi^2} = 0.138; \qquad s = 3.515; \qquad c = 0.537;$$

$$m = 1.1314; \qquad s(1+c) = 5.863;$$

$$\begin{bmatrix} 6.815k & \vdots & -5.863\frac{k}{\ell} \\ \cdots & \cdots & \cdots \\ -5.863\frac{k}{\ell} & \vdots & 10.363\frac{k}{\ell} \end{bmatrix} = \begin{bmatrix} \Theta_B \\ \cdots \\ e_B \end{bmatrix} = \begin{bmatrix} 0.1875k \\ \cdots \\ .5091\frac{k}{l} \end{bmatrix}$$

$$\Theta_B = 0.1395 \qquad e_B = 0.1251\ell$$

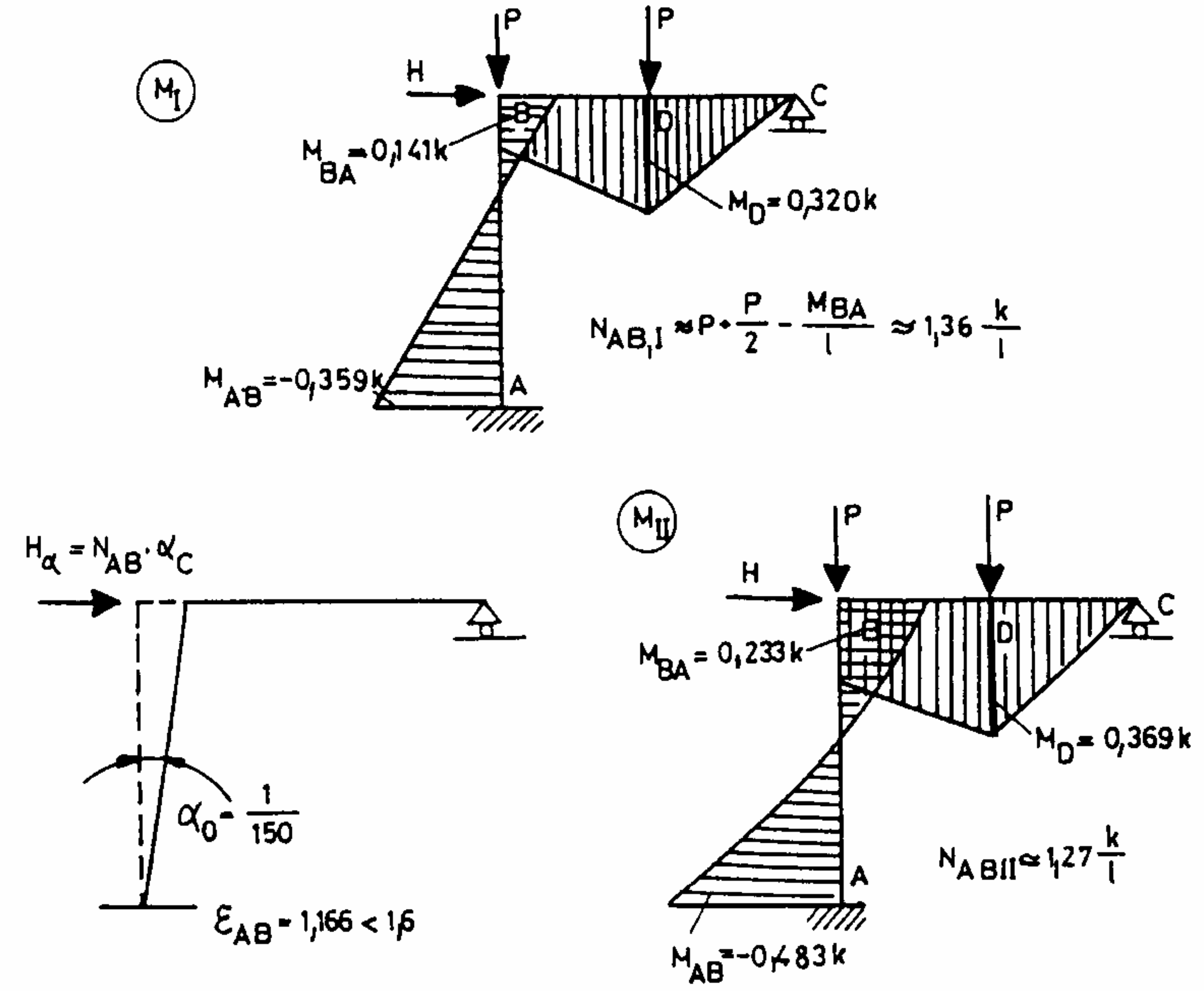
Load–bearing capacity analysis of column AB: $$\frac{N_{AB,I}}{N_{AB,II}} = \frac{1.36}{1.27} = 1.07$$
$$\frac{\psi M_{AB,I}}{M_{AB,II}} = \frac{0.466}{0.483} = 0.96$$

Analyzing the effect of simplification possibilities
$$H_j = 0.50906\frac{k}{\ell}$$
$\delta_j = 0.0977\ell$ calculated by first–order analysis is horizontal deflection, therefore $\Delta\alpha_j = 0.0977$ is the deflection increment.

Increment of the displacing force is:
$$\Delta H_j = N_{AB} \cdot \Delta\alpha_j = 0.13$$
its value related to the original displacing force is:
$$\frac{\Delta H_j}{Q_j} = \frac{0.13}{0.5} = 0.26 \approx 0.25$$

According to the foregoings it is sufficient to calculate the internal forces by first–order analysis, but magnified, by levels, with the displacing force
$$\overline{Q}_j = \frac{1}{1 - \frac{\Delta H_j}{Q_j}} Q_j = 0.688\frac{k}{\ell}$$
Solution of the equilibrium equations $$e_B = 0.1237\ell$$
$$\Theta_B = 0.1329$$

$\overline{M}_{AB} = -0.4765k$ in the most loaded section of column AB, which is 2% lower than that of calculated by analysis b).



M_I
P
P
H
B
D
C
M_BA = 0,141k
M_D = 0,320k
N_AB,I ≈ P + P/2 − M_BA/l ≈ 1,36 k/l
M_AB = −0,359k
A
H_α = N_AB · α_C
α_0 = 1/150
ε_AB = 1,166 < 1,6
M_II
P
P
H
B
D
C
M_BA = 0,233k
M_D = 0,369k
N_ABII ≈ 1,27 k/l
A
M_AB = −0,483k

Figure 24