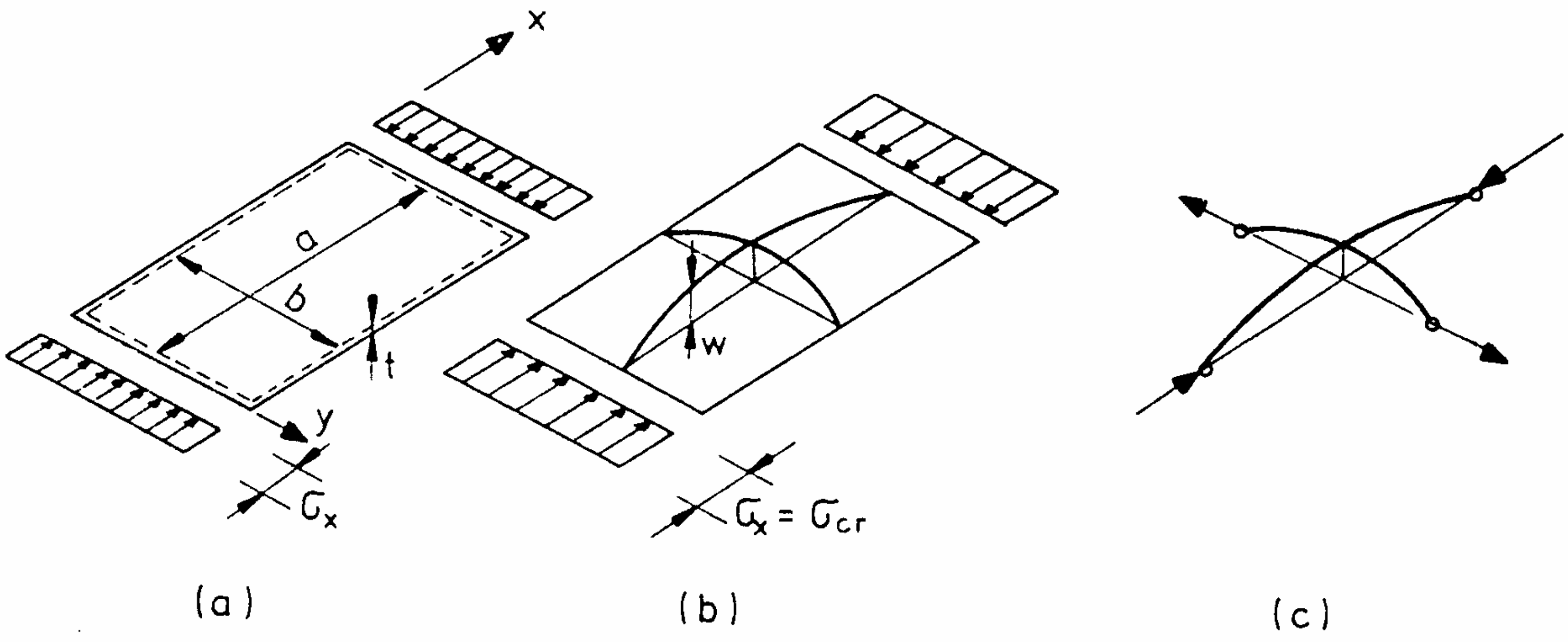
Készült az ERFP – DD2002 – HU – B – 01 szerzősésszámú projekt támogatásával

Chapter 6

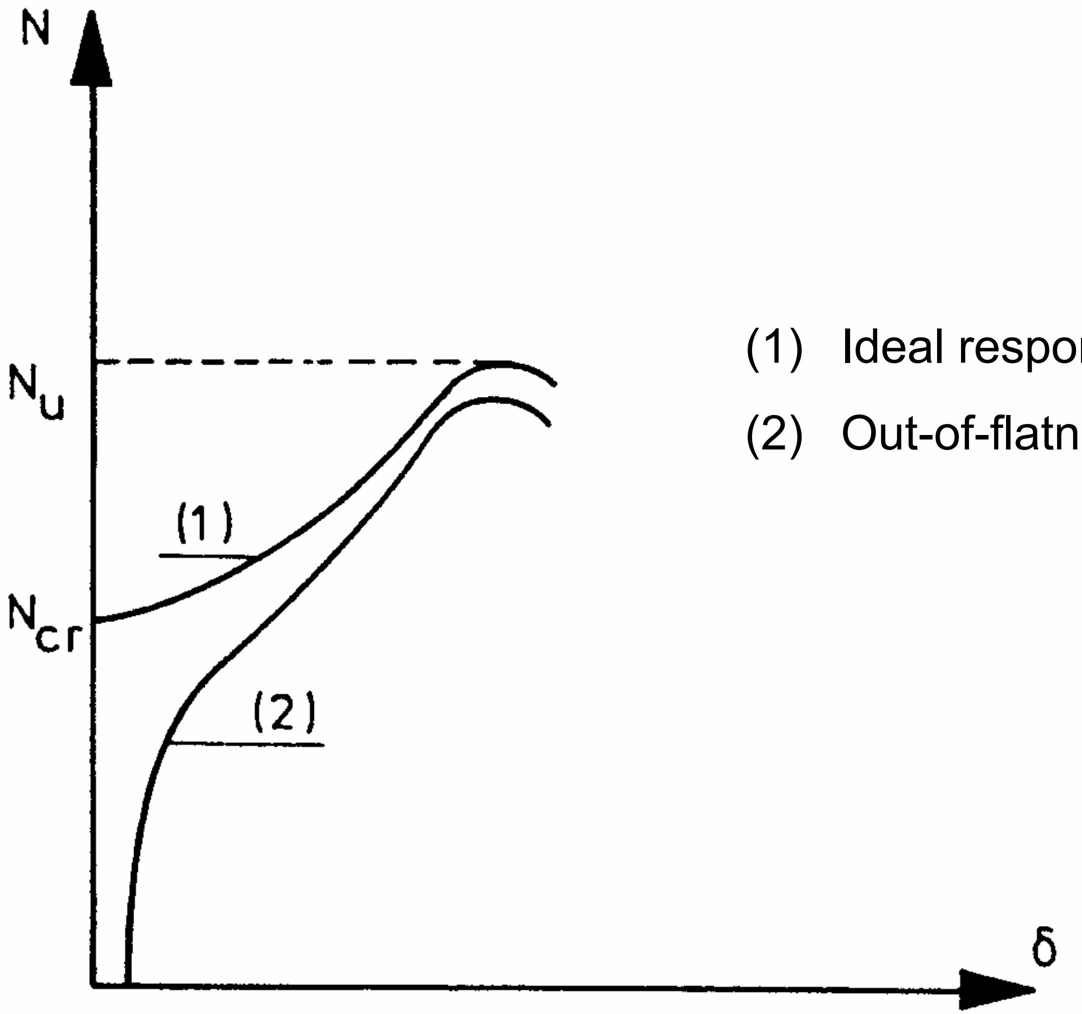
Elastic Critical Plate Buckling Loads

6.1. Introduction

[Szilárd,1974] [Chajes, 1974]



Buckling of an ideal plate



(1) Ideal response of perfect plate

(2) Out-of-flatness curve

Load-shortening curve

Elastic critical plate buckling stress of an unstiffened plate

$$\nabla^2\nabla^2 w \equiv \frac{\partial^4 w}{\partial x^4} + 2\frac{\partial^4 w}{\partial x^2 \partial y^2} + \frac{\partial^4 w}{\partial y^4}$$

$$= \frac{t}{D}\left[\sigma_x \frac{\partial^2 (w+w_0)}{\partial x^2} + 2\tau_{xy} \frac{\partial^2 (w+w_0)}{\partial x \partial y} + \sigma_y \frac{\partial^2 (w+w_0)}{\partial y^2}\right]$$

$D = Et^3/12(1-\nu^2)$	is the flexural stiffness of the plate
t	the thickness,
E	the Young modulus
ν	the Poisson ratio
$w(x,y)$	is the out-of-plane deflection of the plate

Pre- and postbuckling ranges of an imperfect plate

$$\nabla^2\nabla^2 w = \frac{t}{D}\left[\frac{\partial^2\phi}{\partial x^2}\frac{\partial^2(w+w_o)}{\partial y^2} - 2\frac{\partial^2\phi}{\partial x\partial y}\frac{\partial^2(w+w_o)}{\partial x\partial y} + \frac{\partial^2\phi}{\partial y^2}\frac{\partial^2(w+w_o)}{\partial x^2}\right] \quad \text{(a)}$$

$$\nabla^2\nabla^2\phi = E\left\{\left[\frac{\partial^2(w+w_o)}{\partial x\partial y}\right]^2 + \frac{\partial^2 w_o}{\partial x^2}\frac{\partial^2 w_o}{\partial y^2} - \left(\frac{\partial^2 w_o}{\partial x\partial y}\right)^2 - \frac{\partial^2(w+w_o)}{\partial x^2}\,\frac{\partial^2(w+w_o)}{\partial y^2}\right\} \quad \text{(b)}$$

$\phi(x,y)$ is the Airy stress function

$w_0(x,y)$ the initial out-of-flateness

Equations (a) and (b) reflect respectively *equilibrium* and *compatibility* conditions

6.2. Elastic Critical Plate Buckling Stresses

Euler reference stress σ_E

$$\sigma_{cr} = k_\sigma \sigma_E$$
$$\tau_{cr} = k_\tau \sigma_E$$

where

the reference stress writes
$$\sigma_E = \pi^2 D / b^2 t = \left[\pi^2 E / 12(1-\nu^2)\right](t/b)^2$$

i.e. for steel $(\nu = 0.3)$:

$$\sigma_E \approx 0.9E(t/b)^2 \approx 190.000(t/b)^2 \quad (in\ N/mm^2)$$

k_σ and k_τ are the buckling coefficients

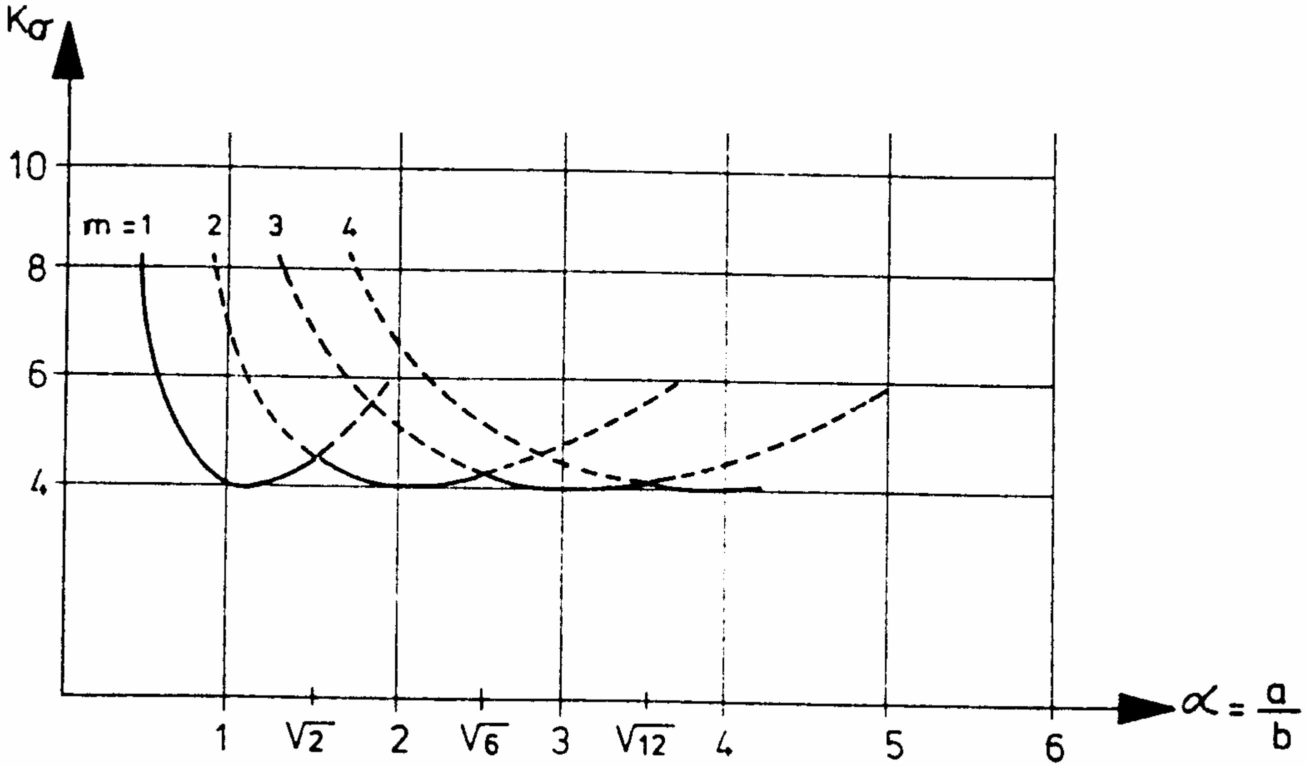
6.3. Plate Subject to Uniaxial Uniform Compression

Long *simply supported* plate subject to *uniaxial uniform compression*, the buckling coefficient is:

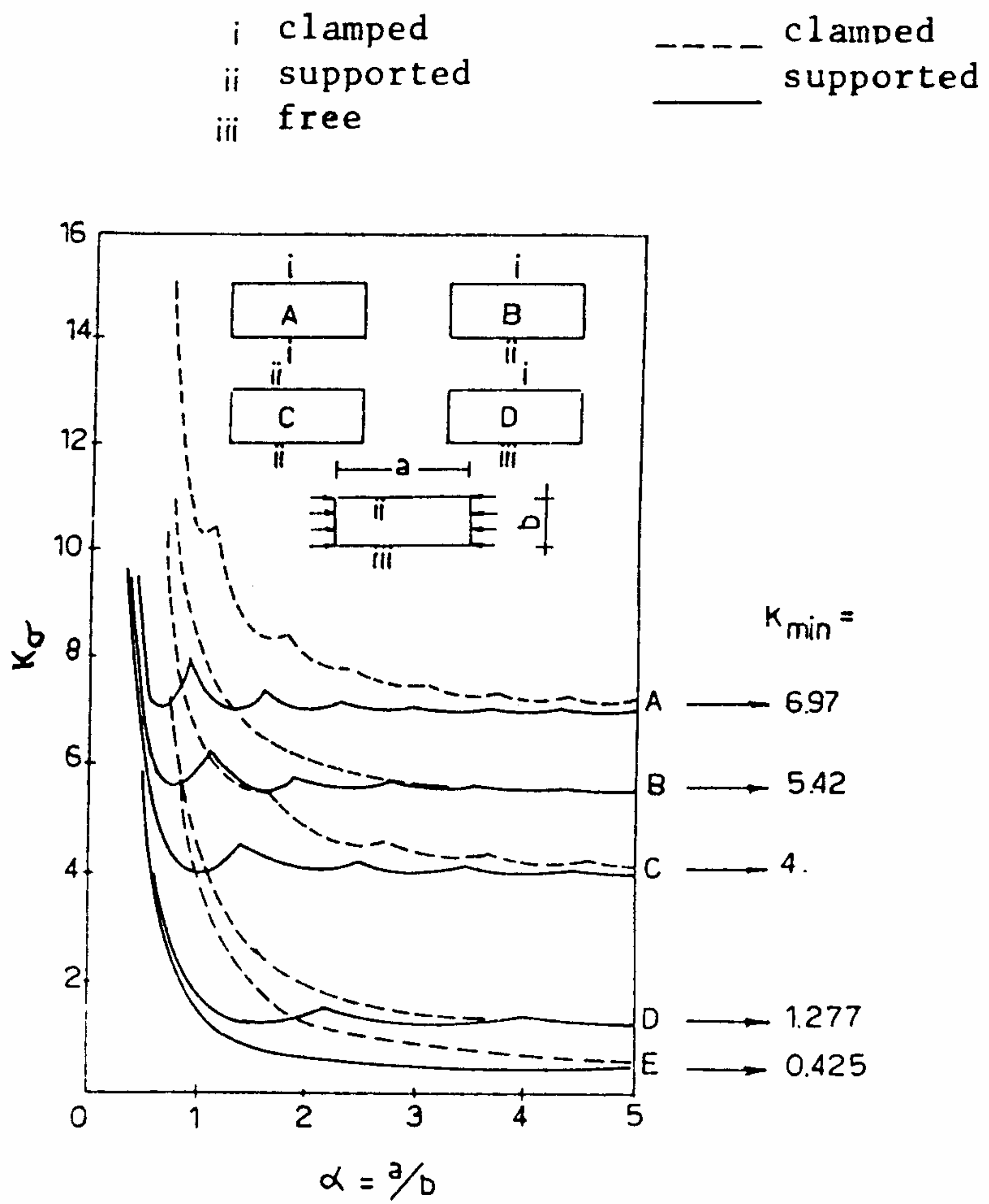
$$k_\sigma = (m/\alpha + n^2\alpha/m)^2$$

m is the number of half buckling waves in the direction of the compression

n is the number of such waves in the transverse direction



Buckling coefficient for uniform compression (simple supports)

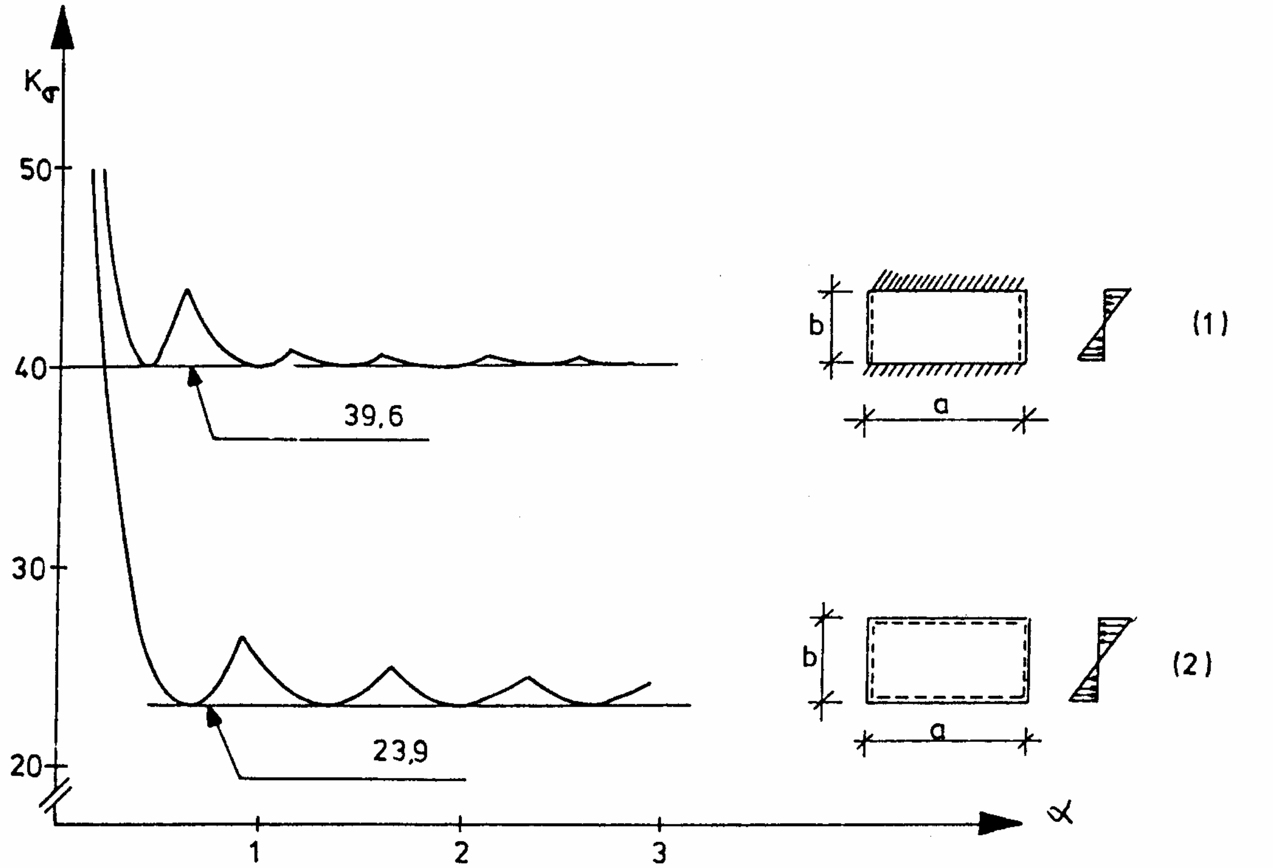


Influence of boundary conditions

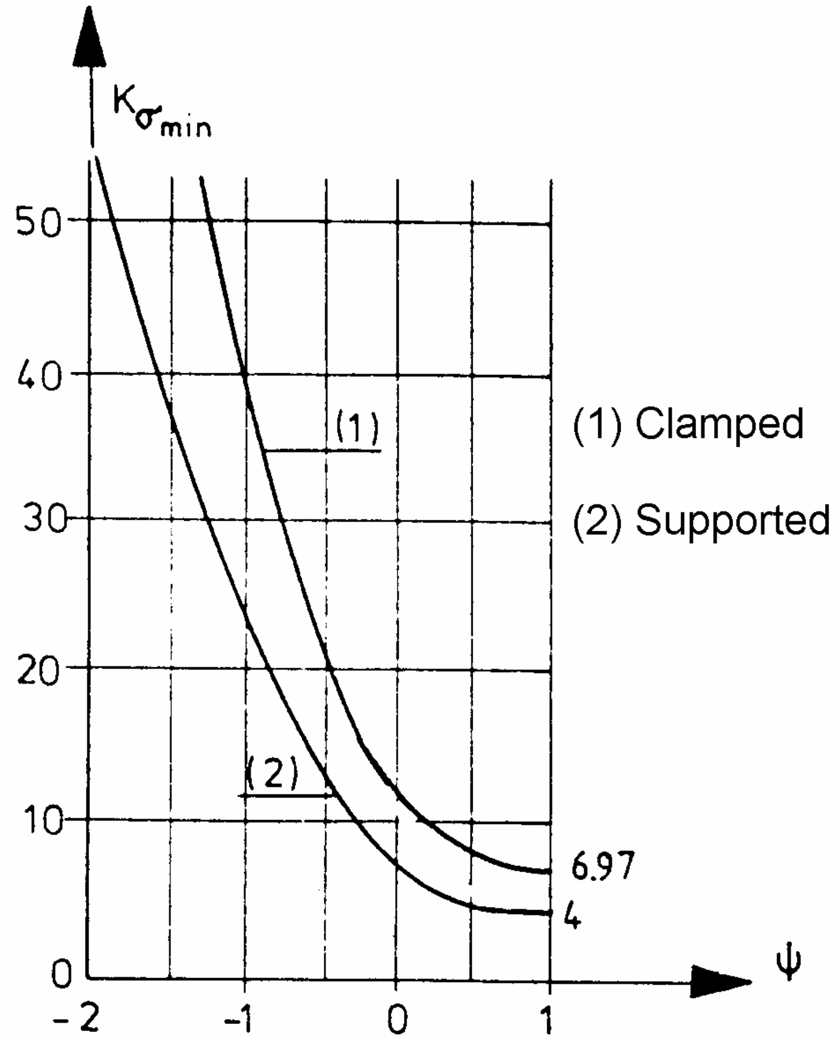
6.4. Plate Subject to a Linear Direct Stress Distribution

Two opposite edges of a rectangular plate are subjected to a *linear direct stress distribution*, the *direct stress ratio* is: $\psi = \sigma_2 / \sigma_1$

where σ_2 and σ_1 the minimum and maximum direct stresses.



Buckling coefficient for pure bending (ψ = -1)



Buckling coefficient for *pure bending*

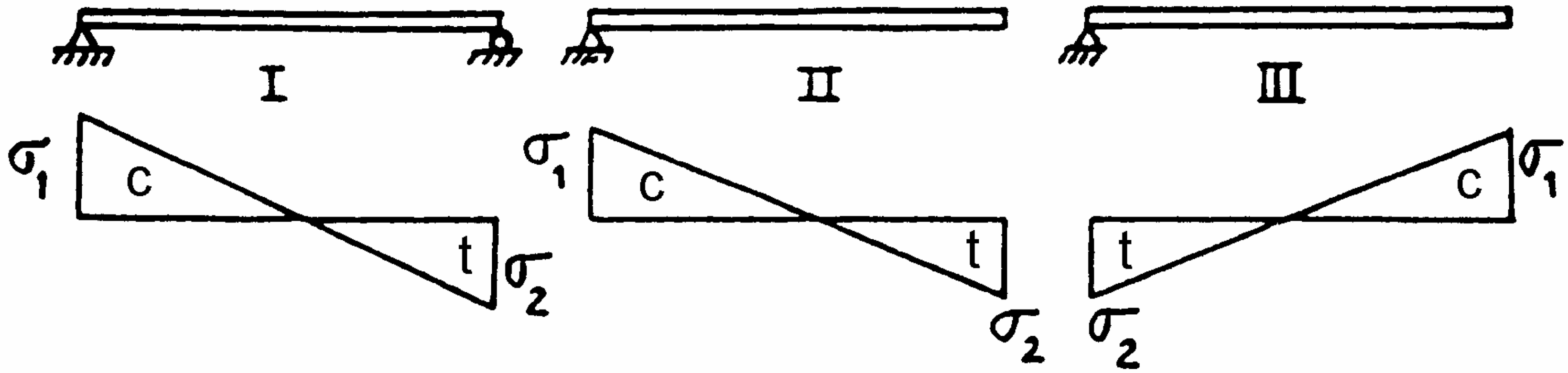
- for simply supported edges:

$$\alpha \leq 2/3 : k_\sigma = 15,87 + 1,87/\alpha^2 + 8,6\,\alpha^2$$
$$\alpha > 2/3 : k_\sigma = 23.9$$

- for longitudinal clamped edges:

$$\alpha \leq 0.475 : k_\sigma = 21,3 + 2/\alpha^2 + 42\,\alpha^2$$
$$\alpha > 0.475 : k_\sigma = 39.6$$

Influence of the direct stress ratio on $k_{\sigma,min}$



Compression is taken positive

$\psi = \sigma_2/\sigma_1$		$+1$	$1 > \psi > 0$	0	$0 > \psi > -1$	-1
k_σ	I	4.0	$\dfrac{8.2}{1.05+\psi}$	7.81	$7.81 - 6.29\psi + 9.78\psi^2$	23.9
	II	0.43	$\dfrac{0.578}{\psi + 0.34}$	1.70	$1.7 - 5\psi + 17.1\psi^2$	23.8
	III	0.43	$0.57 - 0.21\psi + 0.07\psi^2$	0.57	$0.57 - 0.21\psi + 0.07\psi^2$	0.85

Buckling coefficient k_σ for long plates

6.5. Plate Subject to a Pure Shear

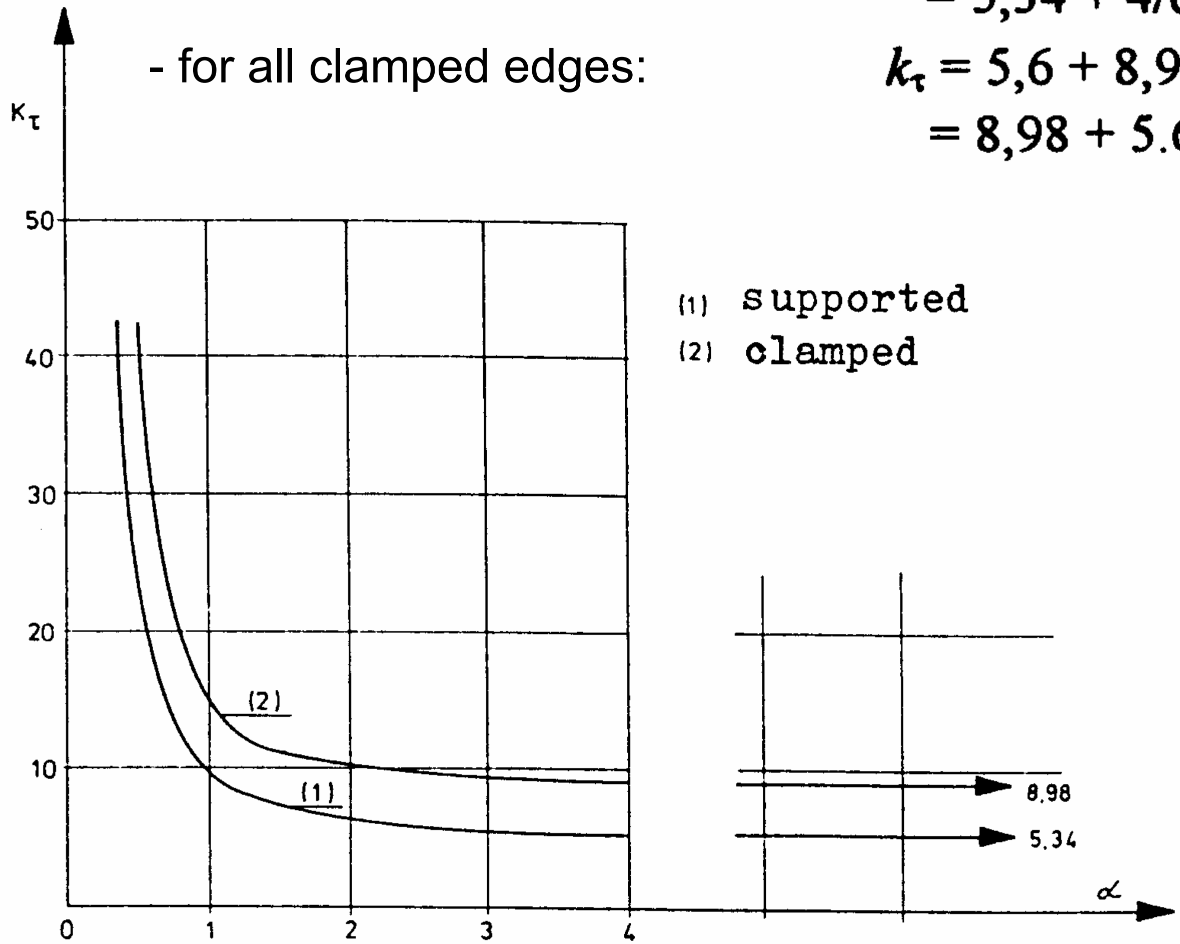
Thin rectangular plate is subject to a *pure shear*

- for all simply supported edges: $k_\tau = 4 + 5{,}34/\alpha^2 \quad (\alpha \leq 1)$

  $= 5{,}34 + 4/\alpha^2 \quad (\alpha \geq 1)$

- for all clamped edges: $k_\tau = 5{,}6 + 8{,}98/\alpha^2 \ (\alpha \leq 1)$

  $= 8{,}98 + 5.6/\alpha^2 \ (\alpha \geq 1)$



Buckling coefficient k_τ for pure shear

6.6. Plate Subject to Combined Shear and Direct Stresses

Rectangular plate is subject to a *combine shear and direct stresses*

(a) $\sigma^o_{cr} = k_\sigma \, \sigma_E$ and $\tau^o_{cr} = k_\tau \, \sigma_E$:the individual critical stresses which should cause plate buckling, when acting *seperately*;

(b) σ_{cr} and τ_{cr} :the critical stresses which should produce buckling, when acting *coincidently*

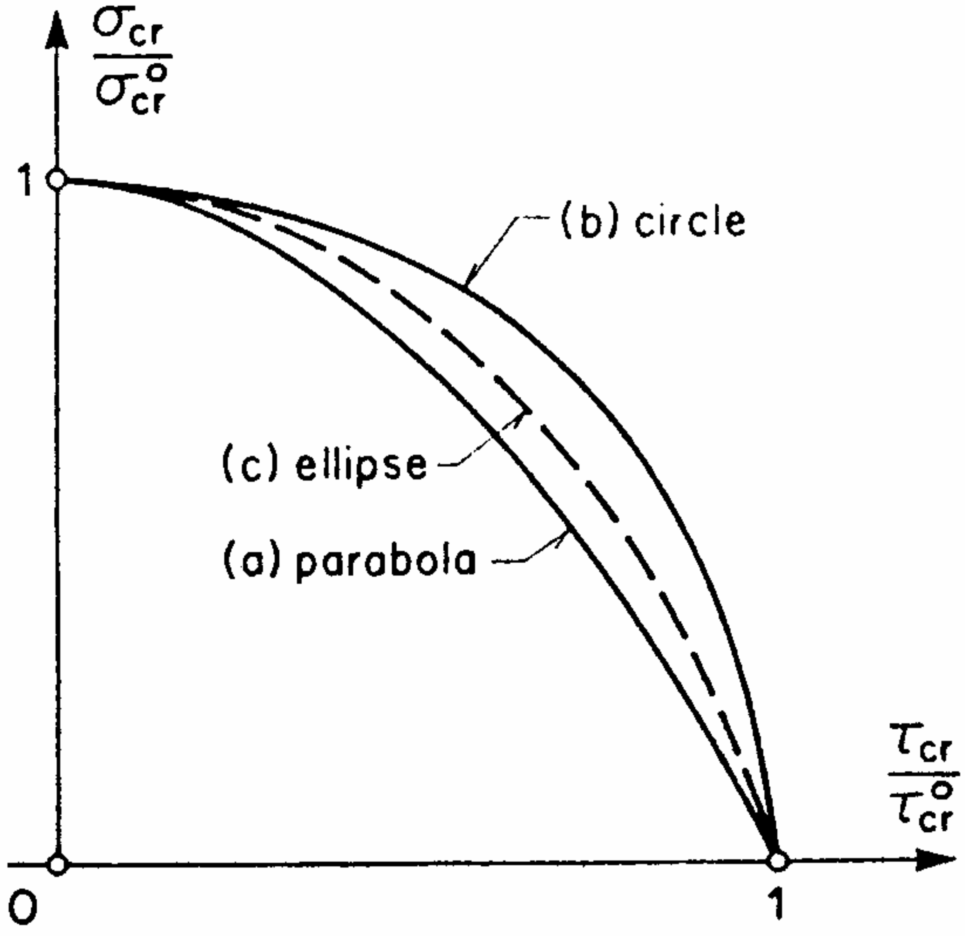
The stresses (a) may be computed based on above Sections 6.4 and 6.5. The stresses (b) are drawn from an interaction relation, which is just an engineering approach and is most often only an approximate. For instance, such an interaction formula is:

$$0.25(1+\psi)(\sigma_{cr} / \sigma^o_{cr}) + \sqrt{\left[0.25(3-\psi)\left(\sigma_{cr} / \sigma^o_{cr}\right)\right]^2 + (\tau_{cr} / \tau^o_{cr})^2} = 1 \quad (1)$$

Designating σ_c and σ_b as the respective components of axial force and pure bending of the direct stress distribution, an alternative interaction equation is:

$$\left(\sigma_{c,cr} / \sigma^o_{c,cr}\right) + (\sigma_{b,cr} / \sigma^o_{b,cr})^2 + (\tau_{cr} / \tau^o_{cr})^2 = 1 \quad (2)$$

In normalized coordinates $(y = \sigma_{cr} / \sigma_{cr}^{o}, x = \tau_{cr} / \tau_{cr}^{o})$,the interaction curve (1) is an ellipse which is centered on the vertical axis and is symmetrical with respect to this axis. Both curves (1) and (2) become a parabola when combined shear and uniaxial uniform compression or a circle when combined shear and pure bending.



Normalized interaction buckling curves

The real interaction curves depend on the aspect ratio; however above approximates, which do not depend on this factor, may be considered as quite satisfactory for practice purposes.

Of course both values σ_{cr} and τ_{cr} cannot be drawn from a single interaction equation. A second relation is necessary; therefore it is generally assumed that the loading is proportional, so that :

$$\sigma_{cr} / \tau_{cr} = \sigma / \tau = \bar{s}$$

The relevant ratio $\bar{s} = \sigma / \tau$ is presumably known for a reference state, for instance, in service conditions.