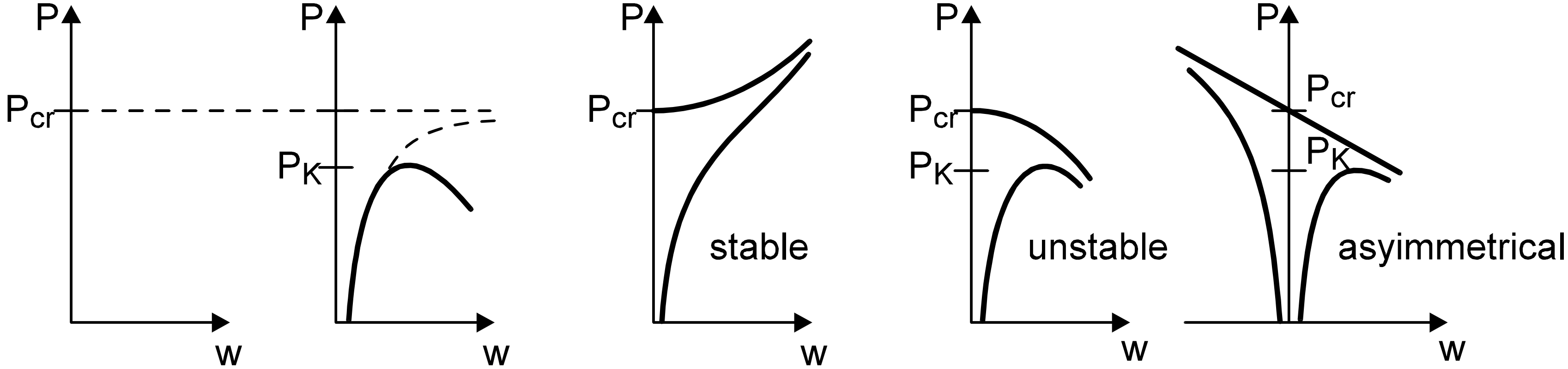
Készült az ERFP – DD2002 – HU – B – 01 szerzősésszámú projekt támogatásával

Chapter 7

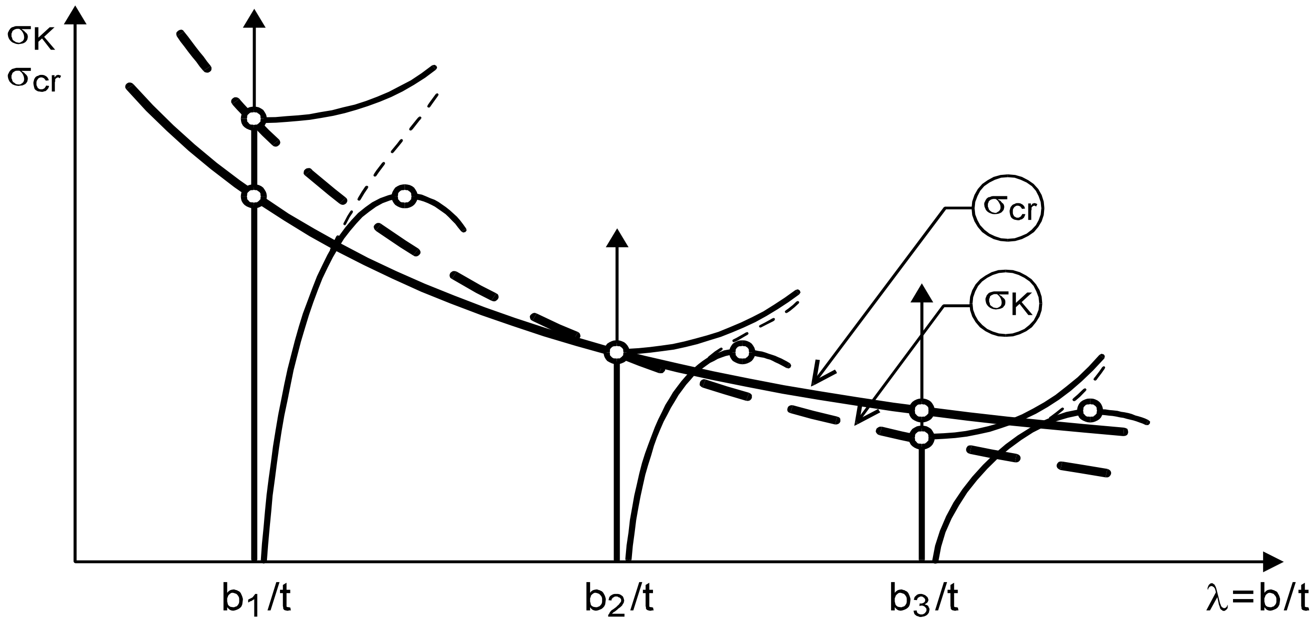
Behaviour of Plate Elements of Steel Frames

7.1. Introduction: Background and Object

7.1.1 Critical and Post-Critical Behaviour



The non-linear examination of occurrence of bifurcation



Behaviour of thick and thin plates

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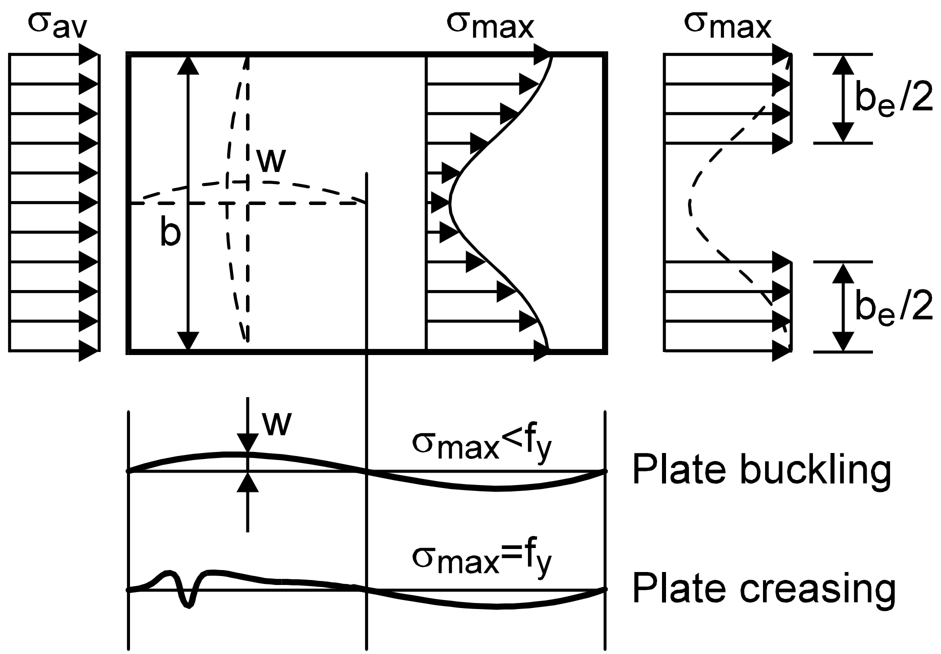
Karman's non-linear buckling equations [1910]:

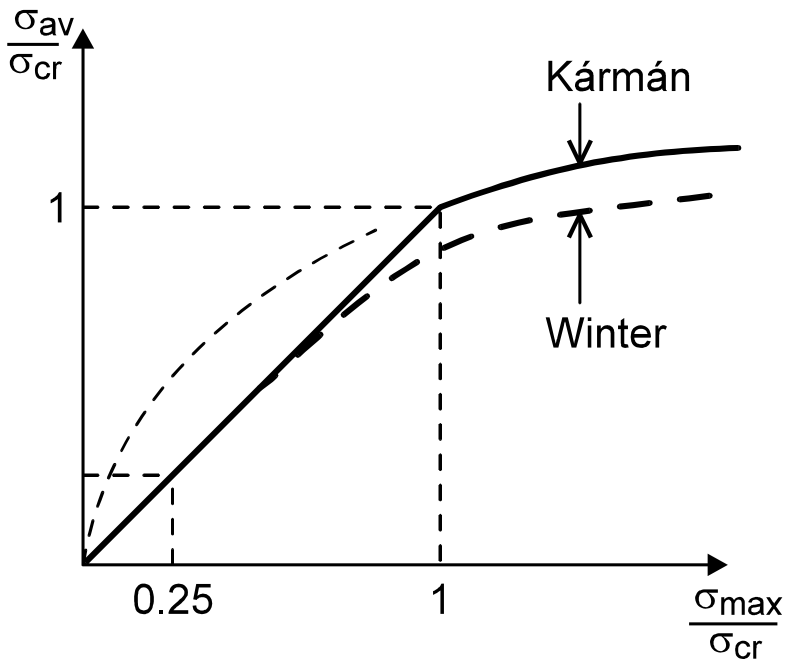
$$\Delta\Delta w = \frac{t}{D} \cdot \left[\Phi_{yy} \cdot w_{xx} + \Phi_{xx} \cdot w_{yy} - 2\Phi_{xy} \cdot w_{xy}\right];$$

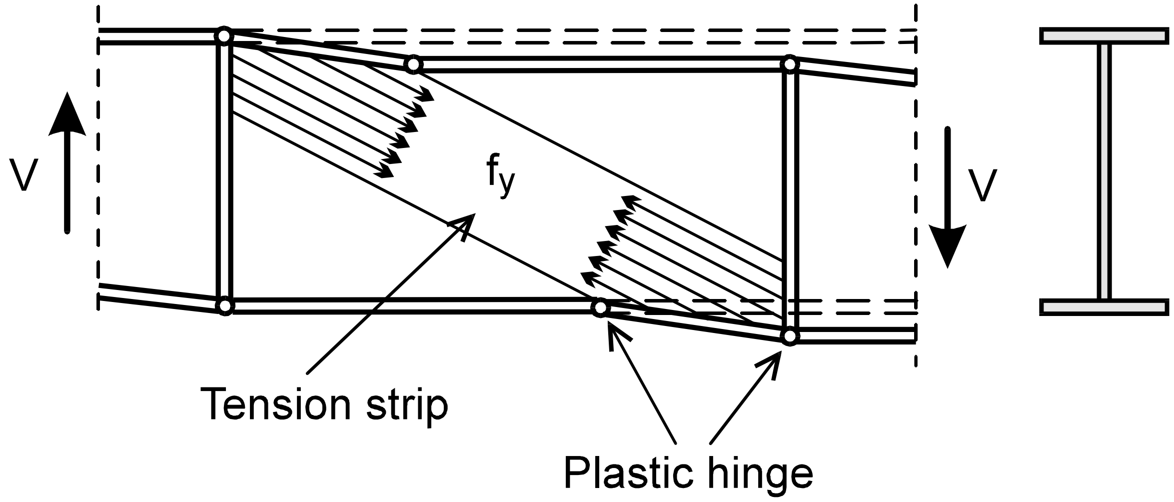
$$\Delta\Delta\Phi = E \cdot \left[w_{xy}^2 - w_{xx} \cdot w_{yy}\right],$$

Notion of post-critical "effective width" [Karman, 1932] [Wagner, 1922] [Winter, 1947]:

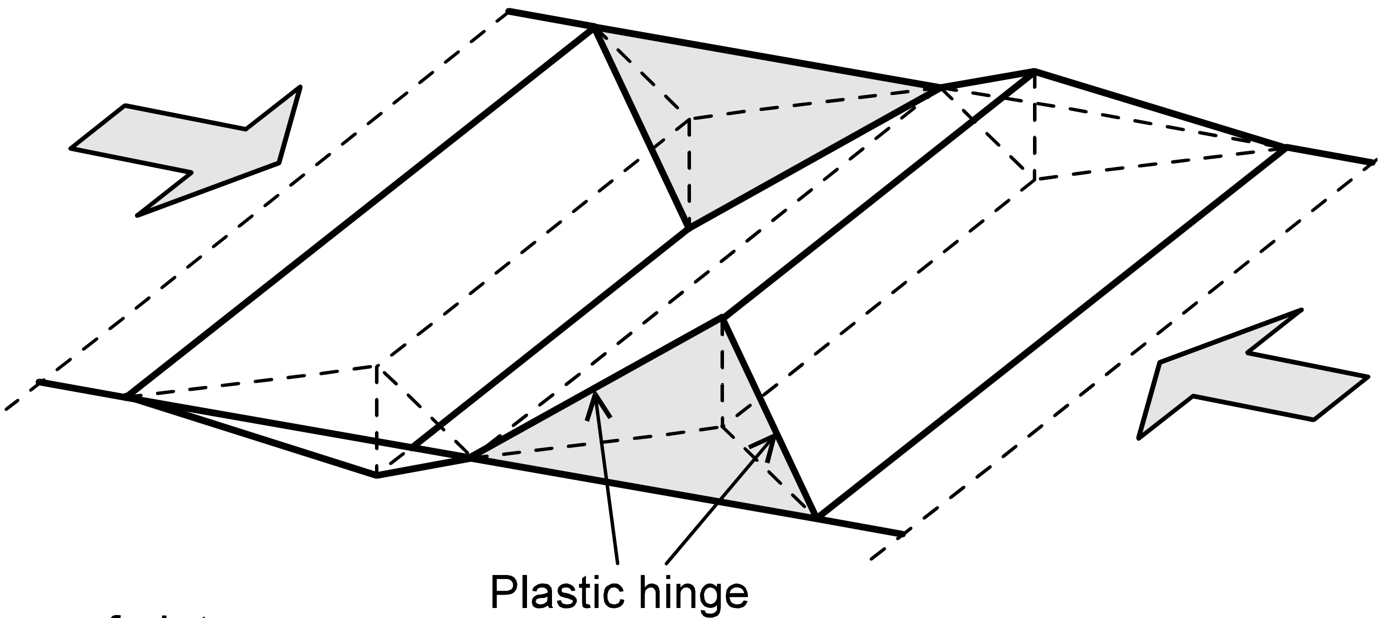
$$\frac{b_e}{b} = \sqrt{\frac{\sigma_{cr}}{\sigma_{max}}} \qquad \frac{b_e}{b} = \sqrt{\frac{\sigma_{cr}}{\sigma_{max}}} \cdot \left(1 - 0.25 \cdot \sqrt{\frac{\sigma_{cr}}{\sigma_{max}}}\right)$$





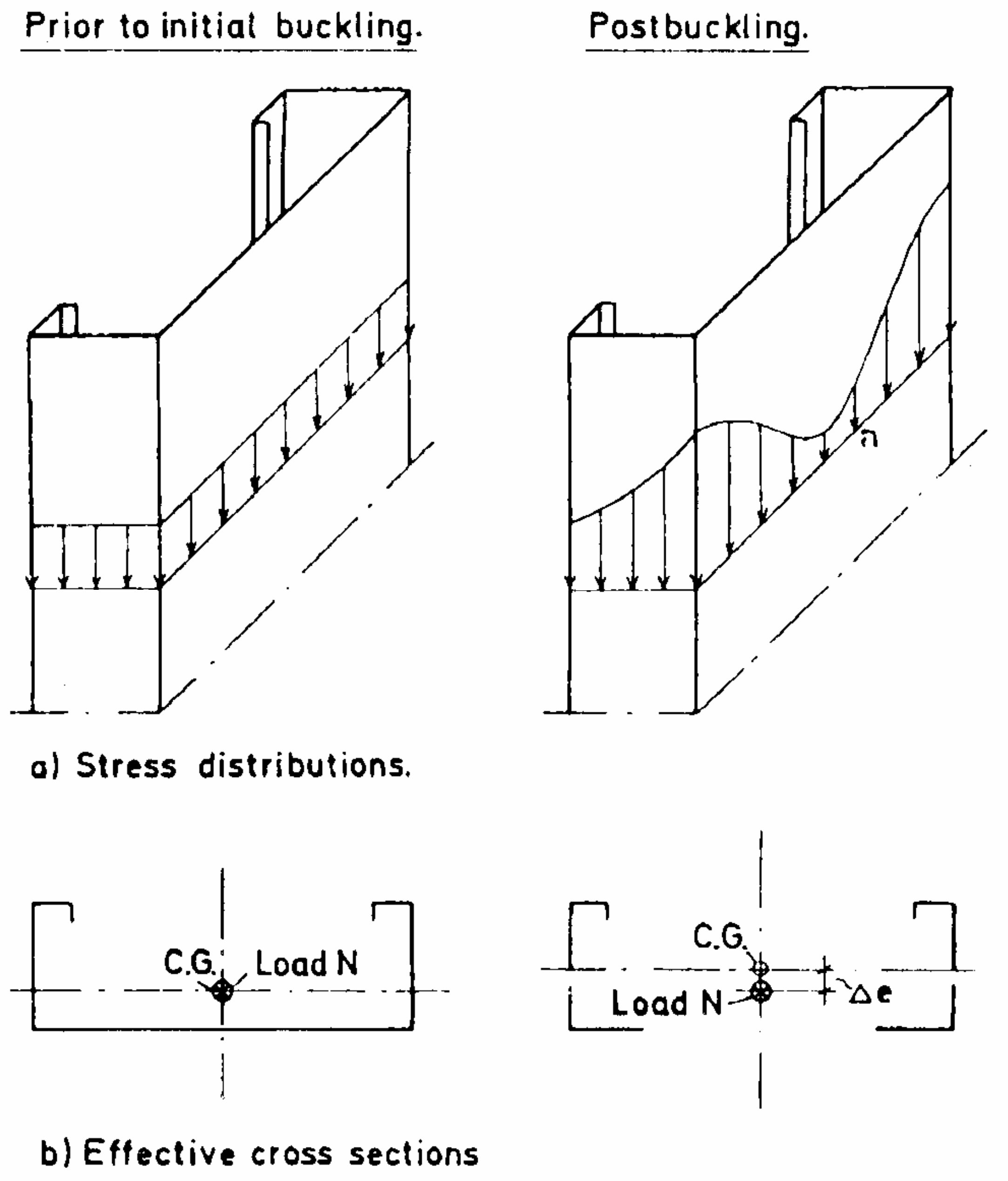


Tension strip model

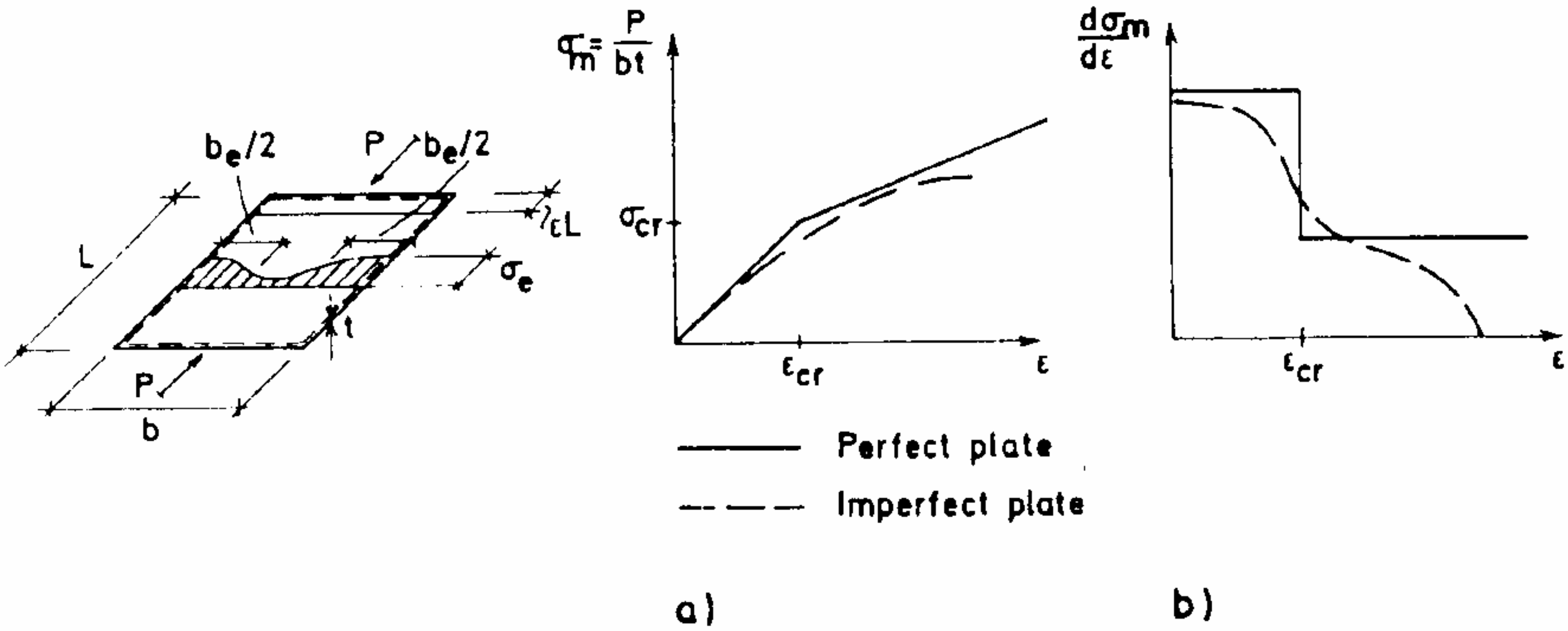


Yield mechanism of plates

7.1.2 Interaction Between Local and Overall Buckling



Stress distributions and effective cross-sections before and after initial local buckling



(a) Load-shortening curve for a compressed plate element
(b) Corresponding tangent stiffness

$$P = \int_0^b \sigma(y) \cdot t \cdot dy = \sigma_{av} \cdot b \cdot t = \sigma_{\max} \cdot b_e \cdot t$$

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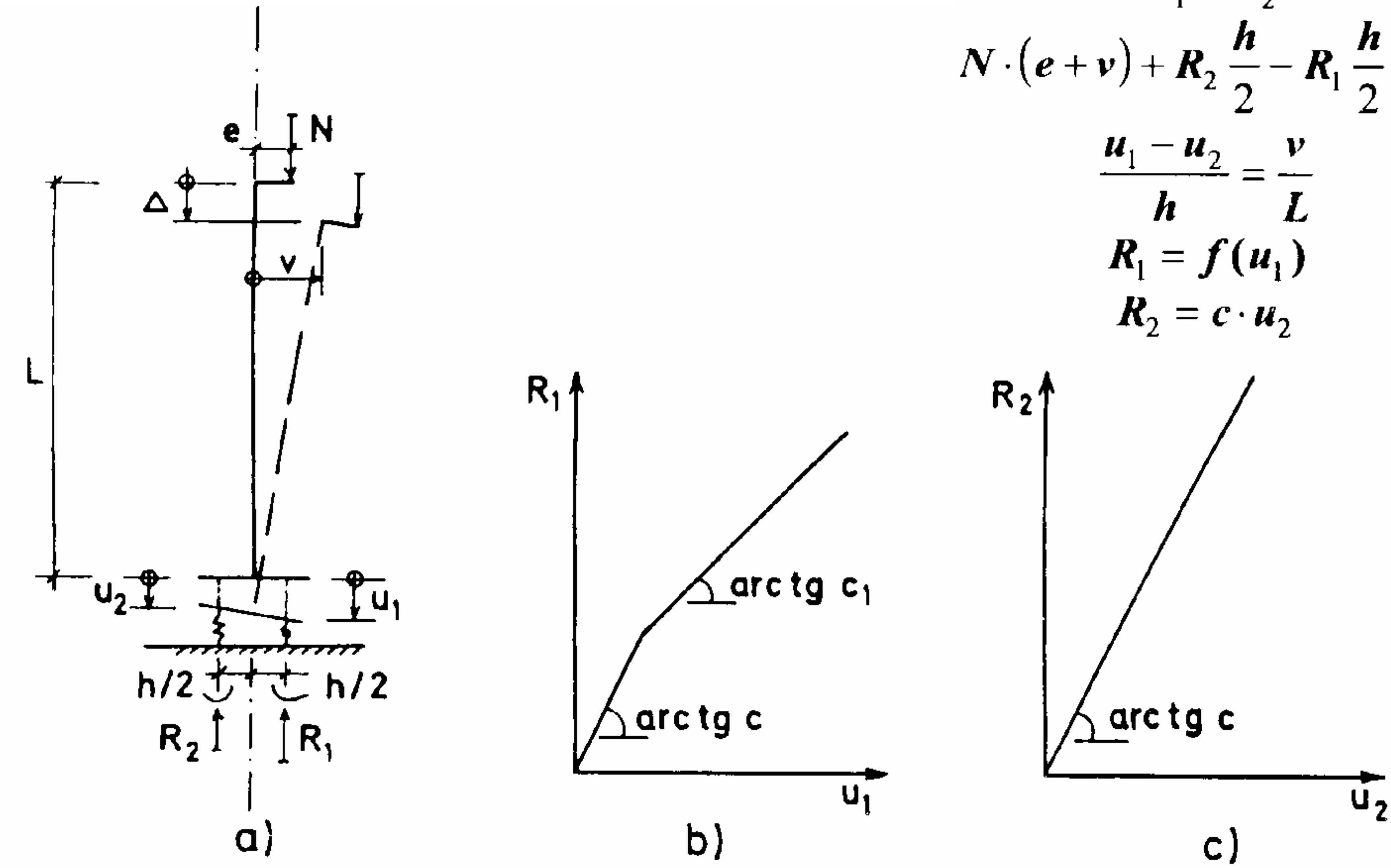
$$N - R_1 - R_2 = 0$$

$$N \cdot (e + v) + R_2 \frac{h}{2} - R_1 \frac{h}{2} = 0$$

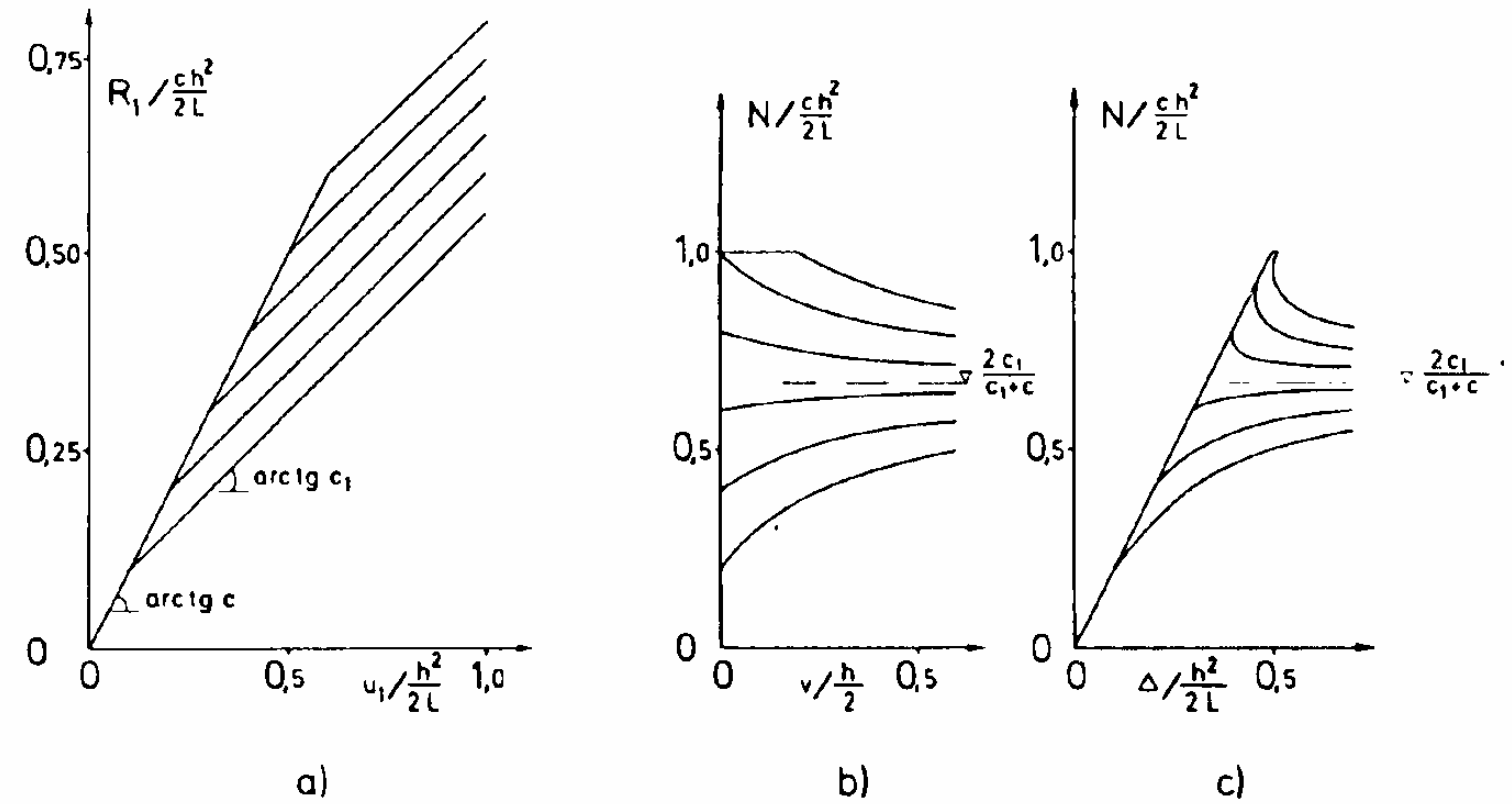
$$\frac{u_1 - u_2}{h} = \frac{v}{L}$$

$$R_1 = f(u_1)$$

$$R_2 = c \cdot u_2$$



Spring model for study of the interaction between local and overall buckling
(a) System sketch, notation; (b) Spring characteristics



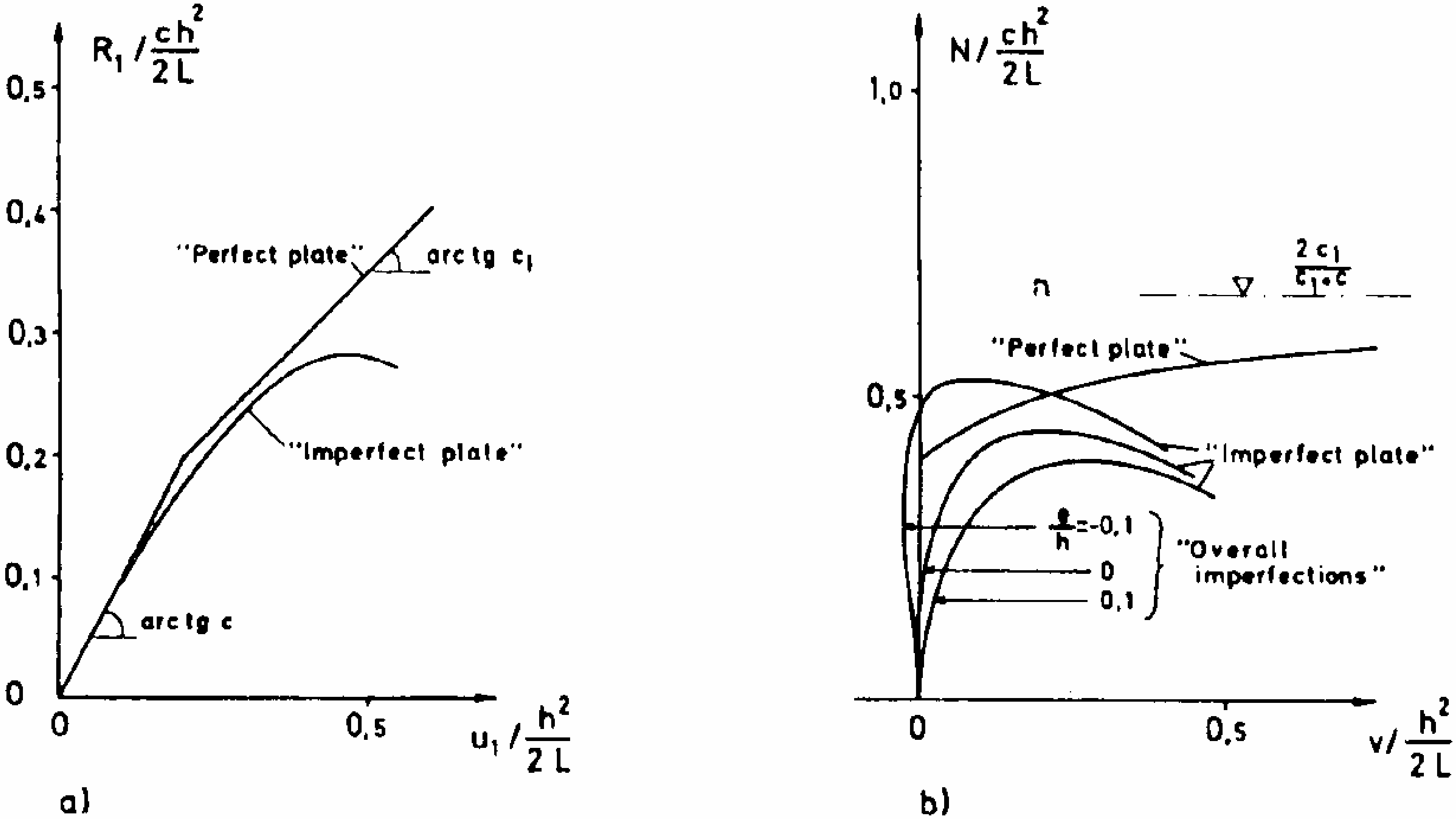
$$\Delta = \frac{u_1 + u_2}{2} + \frac{L \cdot (u_1 - u_2)^2}{2h^2}$$

$$N = \frac{c}{2}\frac{h^2}{L}$$

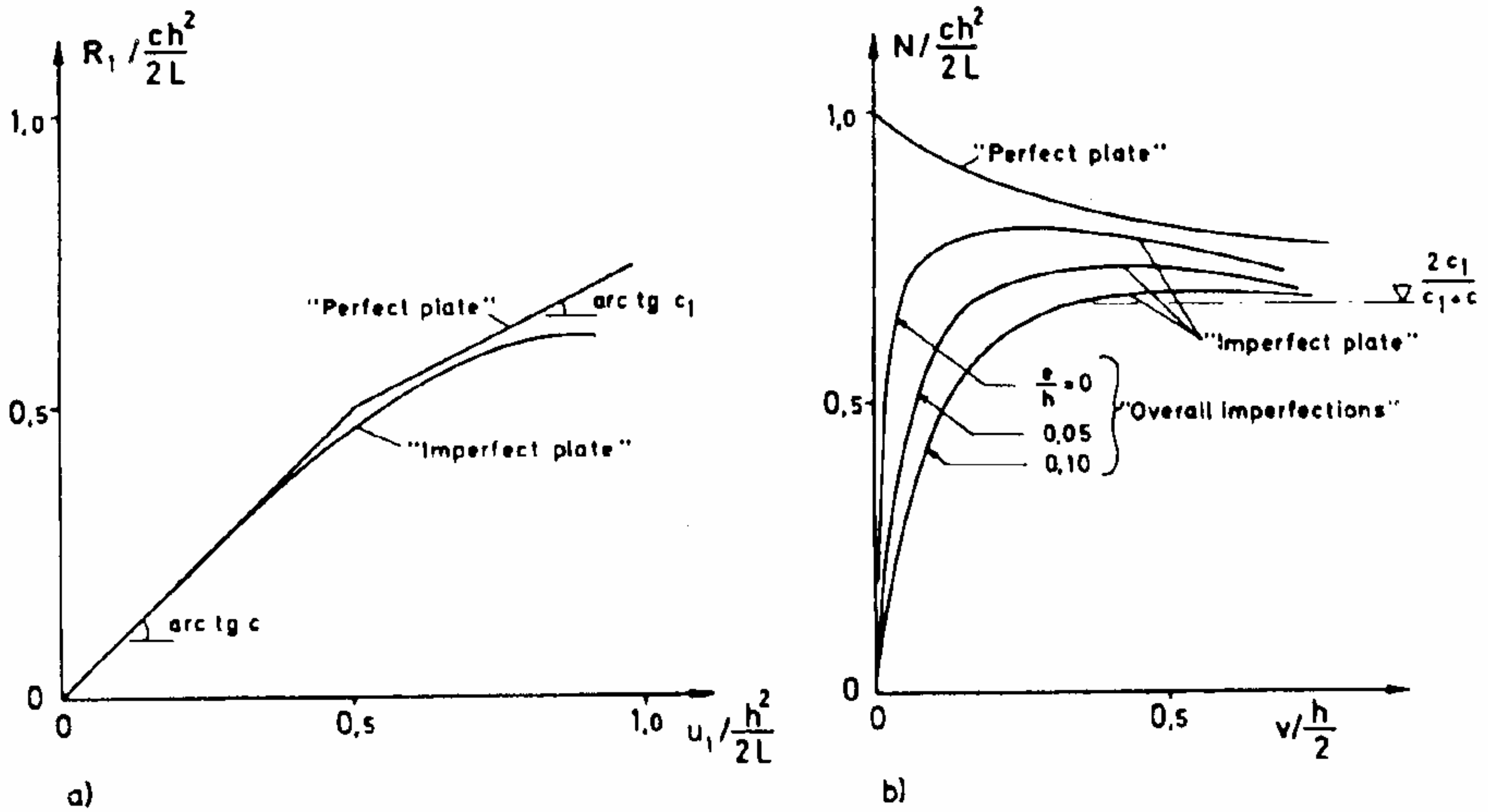
$$\frac{N}{\frac{c}{2}\frac{h^2}{L}} \quad (\text{see } \frac{N}{N_E})$$

$$\frac{2c_1}{c_1 + c} \cdot \frac{c}{2} \cdot \frac{h^2}{L}$$

(a) Characteristics of spring;
(b) Load-deflection curve **N - v**;
(c) Load-displacement **N - Δ**



Examples of load–deflection curves when there are imperfections



Examples of load–deflection curves when there are imperfections

7.2. Plastic Buckling of Plates Based on the Deformation and Flow Theories

7.2.1 Literature Survey [Iványi, Skaloud, 1995]

Differential equation of thin plates originally derived by *Saint-Venant*:

$$D\left(\frac{\partial^4 w}{\partial x^4}+2\frac{\partial^4 w}{\partial x^2 \partial y^2}+\frac{\partial^4 w}{\partial x^4}\right)+t\cdot\left(\sigma_x\frac{\partial^2 w}{\partial x^2}+2\tau_{xy}\frac{\partial^2 w}{\partial x \partial y}+\sigma_y\frac{\partial^2 w}{\partial x^2}\right)=0$$

$D=\dfrac{E\cdot t^3}{12\cdot(1-\nu^2)}$	Flexural rigidity of the plate
w	Deflection of the plate
σ_x, σ_y	Normal stress components in the Cartesian coordinates
τ_{xy}	Shear stress in the Cartesian coordinates
E	Young's modulus
t	Thickness of the plate
ν	Poisson's ration

Extension of theory of plates by *Bleich*:

$$D\left(\alpha_t\frac{\partial^4 w}{\partial x^4}+2\sqrt{\alpha_t}\frac{\partial^4 w}{\partial x^2 \partial y^2}+\frac{\partial^4 w}{\partial x^4}\right)+t\sigma_x\frac{\partial^2 w}{\partial x^2}=0$$

E_t Tangent modulus

$$\alpha_t=\frac{E_t}{E}$$

Assumptions by *Chwalla, Ros and Eichenger*:

$$D\alpha_r\left(\frac{\partial^4 w}{\partial x^4}+2\frac{\partial^4 w}{\partial x^2\partial y^2}+\frac{\partial^4 w}{\partial x^4}\right)+t\sigma_x\frac{\partial^2 w}{\partial x^2}=0$$

E_r Reduced modulus

$$\alpha_r=\frac{E_r}{E}$$

Deformation theory by *Hencky*

Flow-theory by *Prandtl-Reuss*

Theory by *Drucker*

7.2.2 Elastic, Elastic-Plastic and Plastic Buckling of Plates

7.2.2.1 Stress-Strain Relationship in the Elastic and the Plastic Ranges

$$\varepsilon_x=\frac{1}{E}\cdot(\sigma_x-\nu(\sigma_y+\sigma_z))$$

$$\gamma_{xy}=\frac{1}{G}\cdot\tau_{xy}=\frac{2\cdot(1+\nu)}{E}\cdot\tau_{xy}$$

$$\varepsilon_y=\frac{1}{E}\cdot(\sigma_y-\nu(\sigma_x+\sigma_z))$$

$$\gamma_{xz}=\frac{1}{G}\cdot\tau_{xz}=\frac{2\cdot(1+\nu)}{E}\cdot\tau_{xz}$$

$$\varepsilon_z=\frac{1}{E}\cdot(\sigma_z-\nu(\sigma_x+\sigma_y))$$

$$\gamma_{yz}=\frac{1}{G}\cdot\tau_{yz}=\frac{2\cdot(1+\nu)}{E}\cdot\tau_{yz}$$

Thin plates – plane stress problem:

$$\varepsilon_x = \frac{1}{E} \cdot (\sigma_x - \nu \cdot \sigma_y) \qquad \varepsilon_y = \frac{1}{E} \cdot (\sigma_y - \nu \cdot \sigma_x) \qquad \gamma_{xy} = \frac{1}{G} \cdot \tau_{xy} = \frac{2 \cdot (1+\nu)}{E} \cdot \tau_{xy}$$

<u>In plastic range</u> (*based on deformation theory and flow-theory*):

– Proportional loading – secant modulus (E_S):

$$\frac{\sigma_x - \nu \cdot \sigma_y}{\varepsilon_x} = \frac{\sigma_y - \nu \cdot \sigma_x}{\varepsilon_y} = \frac{\tau_{xy}}{2(1+\nu)\gamma_{xy}} = \frac{\sigma_i}{\varepsilon_i} = E_s$$

– Process of loading:

$$\varepsilon_x = \frac{1}{E_S} \cdot (\sigma_x - \nu \cdot \sigma_y) \qquad \varepsilon_y = \frac{1}{E_S} \cdot (\sigma_y - \nu \cdot \sigma_x) \qquad \gamma_{xy} = \frac{2 \cdot (1+\nu)}{E_S} \cdot \tau_{xy}$$

– Process of unloading:

$$\dot{\varepsilon}_x = \frac{1}{E} \cdot (\dot{\sigma}_x - \nu \cdot \dot{\sigma}_y) \qquad \dot{\varepsilon}_y = \frac{1}{E} \cdot (\dot{\sigma}_y - \nu \cdot \dot{\sigma}_x) \qquad \dot{\gamma}_{xy} = \frac{2 \cdot (1+\nu)}{E} \cdot \dot{\tau}_{xy}$$

7.2.2.2 Potential Energy in the Plate

Strain energy per unit volume:

$$\mathrm{d}W(x,y,z)=\int_0^{\varepsilon_x}\sigma_x d\varepsilon_x+\int_0^{\varepsilon_y}\sigma_y d\varepsilon_y+\int_0^{\gamma_{xy}}\tau_{xy}d\gamma_{xy}$$

In elastic region:

$$V=\iint\frac{D}{2}\left(\left(\frac{\partial^2 w}{\partial x^2}\right)^2+2(1-\nu)\left(\frac{\partial^2 w}{\partial x\partial y}\right)^2+2\nu\left(\frac{\partial^2 w}{\partial x^2}\right)\left(\frac{\partial^2 w}{\partial y^2}\right)+\left(\frac{\partial^2 w}{\partial y^2}\right)^2\right)dxdy-$$

$$-\iint\frac{t}{2}\left(\sigma_x\left(\frac{\partial w}{\partial x}\right)^2+2\tau\left(\frac{\partial w}{\partial x}\right)\left(\frac{\partial w}{\partial y}\right)+\sigma_y\left(\frac{\partial w}{\partial y}\right)^2\right)dxdy$$

In plastic region (*Stowell's equation*):

(based on deformation theory)

$$V=\iint\frac{D_d}{2}\left(c_1\left(\frac{\partial^2 w}{\partial x^2}\right)^2-c_2\left(\frac{\partial^2 w}{\partial x^2}\right)\left(\frac{\partial^2 w}{\partial x\partial y}\right)+c_3'\left(\frac{\partial^2 w}{\partial x\partial y}\right)^2+\right.$$

$$\left.+c_3''\left(\frac{\partial^2 w}{\partial x^2}\right)\left(\frac{\partial^2 w}{\partial y^2}\right)-c_4\left(\frac{\partial^2 w}{\partial y^2}\right)\left(\frac{\partial^2 w}{\partial x\partial y}\right)+c_5\left(\frac{\partial^2 w}{\partial y^2}\right)^2\right)dxdy-$$

$$-\iint\frac{t}{2}\left(\sigma_x\left(\frac{\partial w}{\partial x}\right)^2+2\tau\left(\frac{\partial w}{\partial x}\right)\left(\frac{\partial w}{\partial y}\right)+\sigma_y\left(\frac{\partial w}{\partial y}\right)^2\right)dxdy$$

$$D_d=\frac{E_S\cdot t^3}{12\cdot(1-\nu^2)}$$

In plastic region (*Pearson's* considerations, *Handelman-Prager equation*):

(based on flow-theory)

$$V = \iint \frac{D_f}{2}\left(c_1 \frac{\partial^2 w}{\partial x^2} + c_3' \frac{\partial^2 w}{\partial x \partial y} + c_3'' \left(\frac{\partial^2 w}{\partial x^2}\right)\left(\frac{\partial^2 w}{\partial y^2}\right) +_5 \left(\frac{\partial^2 w}{\partial y^2}\right)^2 \right) dxdy -$$

$$- \iint \frac{t}{2}\left(\sigma_x \left(\frac{\partial w}{\partial x}\right)^2 + 2\tau \left(\frac{\partial w}{\partial x}\right)\left(\frac{\partial w}{\partial y}\right) + \sigma_y \left(\frac{\partial w}{\partial y}\right)^2 \right) dxdy$$

$$D_f = \frac{E \cdot t^3}{12 \cdot (1 - \nu^2)}$$

7.2.2.3 Theorem of Minimum Potential Energy and Equilibrium Differential Equation

$$c_1 \frac{\partial^4 w}{\partial x^4} - c_2 \frac{\partial^4 w}{\partial x^3 \partial y} + c_3 \frac{\partial^4 w}{\partial x^2 \partial y^2} - c_4 \frac{\partial^4 w}{\partial x \partial y^3} + c_5 \frac{\partial^4 w}{\partial y^4} = -\frac{1}{D_d}\left(\sigma_x \left(\frac{\partial^2 w}{\partial x^2}\right) + 2\tau \left(\frac{\partial^2 w}{\partial x \partial y}\right) + \sigma_y \left(\frac{\partial^2 w}{\partial y^2}\right)\right)$$

7.2.3 Theoretical Derivations

(a) Definitions

$$\sigma_i = \sqrt{\sigma_x^2 + \sigma_y^2 - \sigma_x\sigma_y + 3\,\tau^2}$$

$$e_i = \frac{2}{\sqrt{3}}\sqrt{\varepsilon_x^2 + \varepsilon_y^2 + \varepsilon_x\varepsilon_y + \frac{\gamma^2}{4}}$$

where

σ_x	is the stress in the **x** direction
ε_x	is the strain in the **x** direction
σ_y	is the stress in the **y** direction
ε_y	is the strain in the **y** direction
τ	is the shear stress

γ is the shear strain

$$\varepsilon_x = \frac{\sigma_x - \frac{1}{2}\sigma_y}{E_{sec}} = \frac{S_x}{E_{sec}} \qquad \gamma = \frac{3\,\tau}{E_{sec}}$$

$$\varepsilon_y = \frac{\sigma_y - \frac{1}{2}\sigma_x}{E_{sec}} = \frac{S_y}{E_{sec}} \qquad e_i = \frac{\sigma_i}{E_{sec}}$$

(b) Variations of Strain and Stress

$$\delta\varepsilon_x = \varepsilon_1 - z\,\chi_1$$

$$\delta\varepsilon_y = \varepsilon_2 - z\,\chi_2$$

$$\delta\gamma = 2\,\varepsilon_3 - 2\,z\,\chi_3$$

$$\delta T_x = \int_{-\frac{t}{2}}^{2} \delta\sigma_x\, dz \qquad \delta M_x = \int_{-\frac{t}{2}}^{\frac{t}{2}} \delta\sigma_x\, z\, dz$$

$$\delta T_y = \int_{-\frac{t}{2}}^{\frac{t}{2}} \delta\sigma_y\, dz \qquad \delta M_y = \int_{-\frac{t}{2}}^{\frac{t}{2}} \delta\sigma_y\, z\, dz$$

$$\delta T_{xy} = \int_{-\frac{t}{2}}^{\frac{t}{2}} \delta\tau\, dz \qquad \delta M_{xy} = \int_{-\frac{t}{2}}^{\frac{t}{2}} \delta\tau\, z\, dz$$

© Equation of Equilibrium

$$\frac{\partial^2(\delta M_y)}{\partial x^2} + 2\frac{\partial^2(\delta M_{xy})}{\partial x\,\partial y} + \frac{\partial^2(\delta M_y)}{\partial y^2} = t\left(\sigma_x \frac{\partial^2 w}{\partial x^2} + \sigma_y \frac{\partial^2 w}{\partial y^2} + 2\tau \frac{\partial^2 w}{\partial x\,\partial y}\right)$$

$$\nabla^4 w = -\frac{t}{D}\left(\sigma_x \frac{\partial^2 w}{\partial x^2} + \sigma_y \frac{\partial^2 w}{\partial y^2} + 2\tau \frac{\partial^2 w}{\partial x\partial y}\right) \qquad D = \frac{Et^3}{9}$$

(d) Energy Integrals

$$\iint \Big(\frac{D'}{2} \Big\{ C_1 \Big(\frac{\partial^2 w}{\partial x^2}\Big)^2 - C_2 \frac{\partial^2 w}{\partial x^2}\frac{\partial^2 w}{\partial x \partial y} + C_3 \Big[\Big(\frac{\partial^2 w}{\partial x \partial y}\Big)^2 + \frac{\partial^2 w}{\partial x^2}\frac{\partial^2 w}{\partial y^2}\Big] - C_1 \frac{\partial^2 w}{\partial x \partial y}\frac{\partial^2 w}{\partial y^2} + C_5 \Big(\frac{\partial^2 w}{\partial y^2}\Big)^2 \Big\} - \frac{t\,\sigma_i}{2}\Big[\frac{\sigma_x}{\sigma_i}\Big(\frac{\partial w}{\partial x}\Big)^2 + \frac{2\tau}{\sigma_i}\frac{\partial w}{\partial x}\frac{\partial w}{\partial y} + \frac{\sigma_y}{\sigma_i}\Big(\frac{\partial w}{\partial y}\Big)^2\Big]\Big)\, dx\, dy$$

the strain energy $\quad \dfrac{e}{2}\dfrac{D}{b}\displaystyle\int \Big[\Big(\frac{\partial w}{\partial y}\Big)_{y=y_0}\Big]^2 dx$

(e) Critical Stress in Plastic Region

the critical stress intensity in the plastic region $(\sigma_i)_{nl}$ is

$$(\sigma_i)_{pl} = \frac{\Big\{\iint C_1\Big(\frac{\partial^2 w}{\partial x^2}\Big)^2 - C_2\frac{\partial^2 w}{\partial x^2}\frac{\partial^2 w}{\partial x \partial y} + C_3\Big[\Big(\frac{\partial^2 w}{\partial x \partial y}\Big)^2 + \frac{\partial^2 w}{\partial x^2}\frac{\partial^2 w}{\partial y^2}\Big] - C_4\frac{\partial^2 w}{\partial x \partial y}\frac{\partial^2 w}{\partial y^2} + C_5\Big(\frac{\partial^2 w}{\partial y^2}\Big)^2\Big\}\, dx\, dy + \frac{e}{b}\int\Big[\Big(\frac{\partial w}{\partial y}\Big)_{y=y_0}\Big]^2 dx}{\iint\Big[\frac{\sigma_x}{\sigma_i}\Big(\frac{\partial w}{\partial x}\Big)^2 + 2\frac{\tau}{\sigma_i}\frac{\partial w}{\partial x}\frac{\partial w}{\partial y} + \frac{\sigma_y}{\sigma_i}\Big(\frac{\partial w}{\partial y}\Big)^2\Big]\, dx\, dy}$$

the critical stress intensity in the elastic region $(\sigma_i)_{el}$ is as follows:

$$(\sigma_i)_{el} = \frac{D}{t}\,\frac{\iint\Big[\Big(\frac{\partial^2 w}{\partial x^2}\Big)^2 + \Big(\frac{\partial^2 w}{\partial x \partial y}\Big)^2 + \frac{\partial^2 w}{\partial x^2}\frac{\partial^2 w}{\partial y^2} + \Big(\frac{\partial^2 w}{\partial y^2}\Big)^2\Big]\, dx\, dy + \frac{e}{b}\int\Big[\Big(\frac{\partial w}{\partial y}\Big)_{y=y_0}\Big]^2 dx}{\iint\Big[\frac{\sigma_x}{\sigma_i}\Big(\frac{\partial w}{\partial x}\Big)^2 + 2\frac{\tau}{\sigma_i}\frac{\partial w}{\partial x}\frac{\partial w}{\partial y} + \frac{\sigma_y}{\sigma_i}\Big(\frac{\partial w}{\partial y}\Big)^2\Big]\, dx\, dy}$$

7.3. Local Buckling and Flexural-Torsional Buckling with Regard to Residual Stresses

7.3.1 Introduction

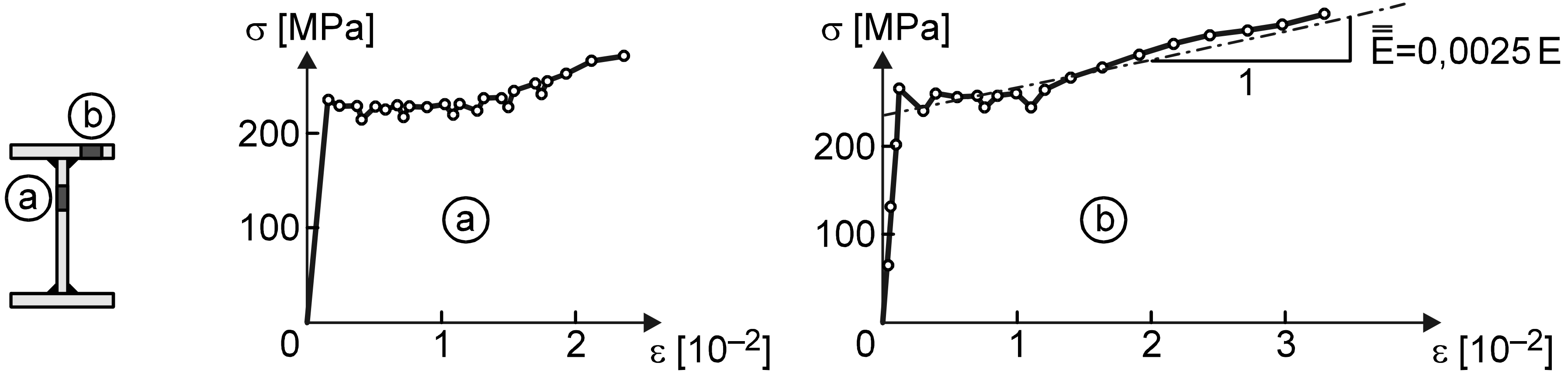
7.3.2 Experimental Investigations

(a) Plastic material characteristics of structural steel;

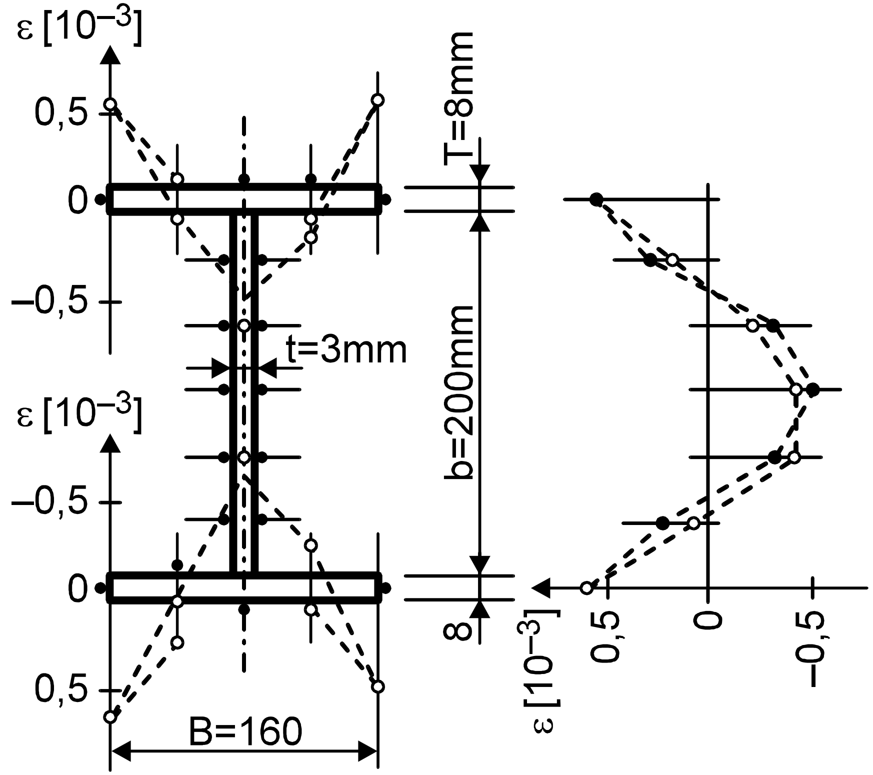
(b) Residual deformations;

(c) Load-carrying capacities of members affected by compression, by bending and by eccentric compression,

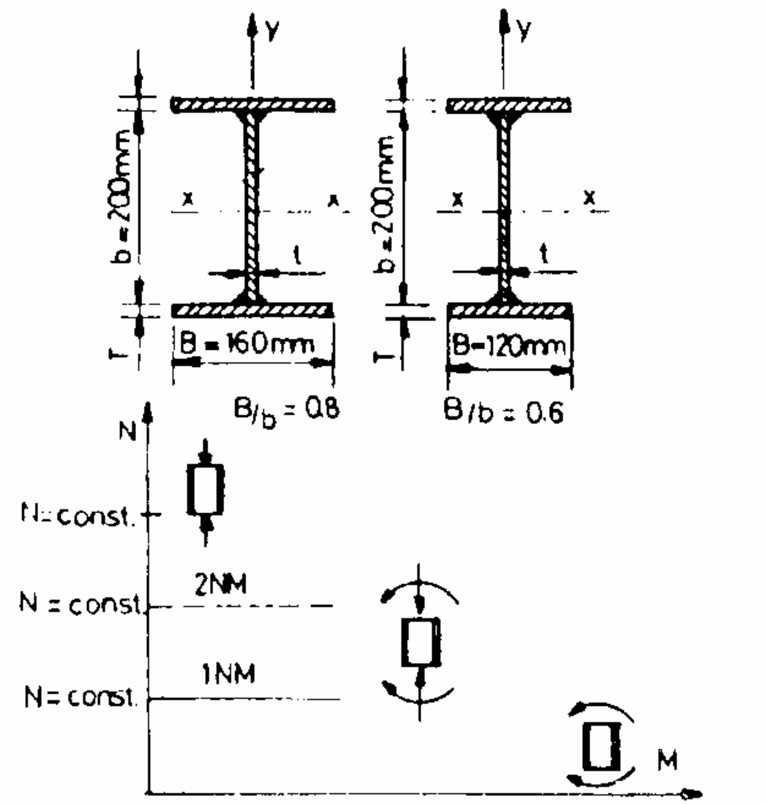
(a) Plastic Material Characteristics of Steel



(b) Residual Deformations



(c) Load-Carrying Capacities



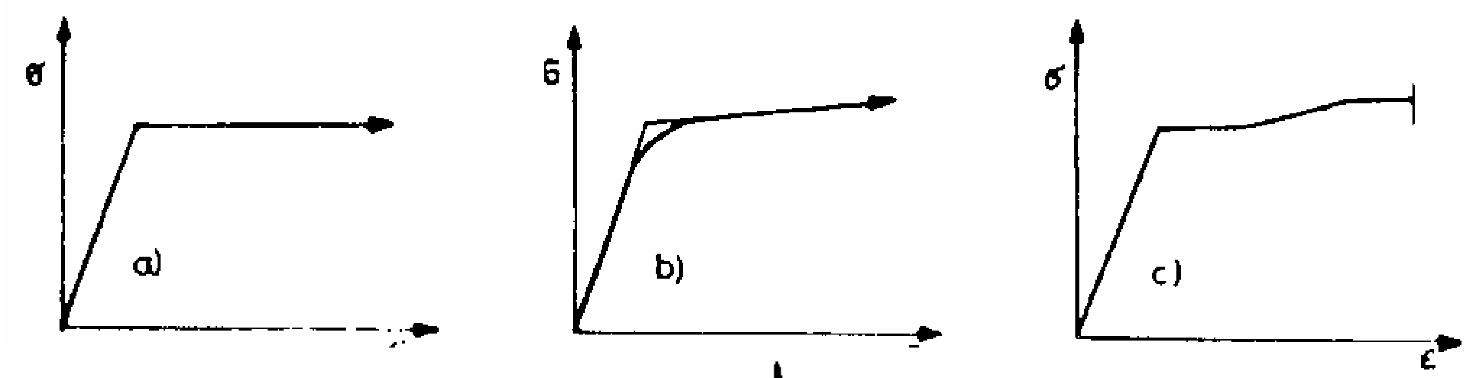
b/t	B/T
40	16
	20
	26.6
	32
50	16
	20
	26.6
	32
67	16
	20
	26.6
	32
80	16
	20
	26.6
	32

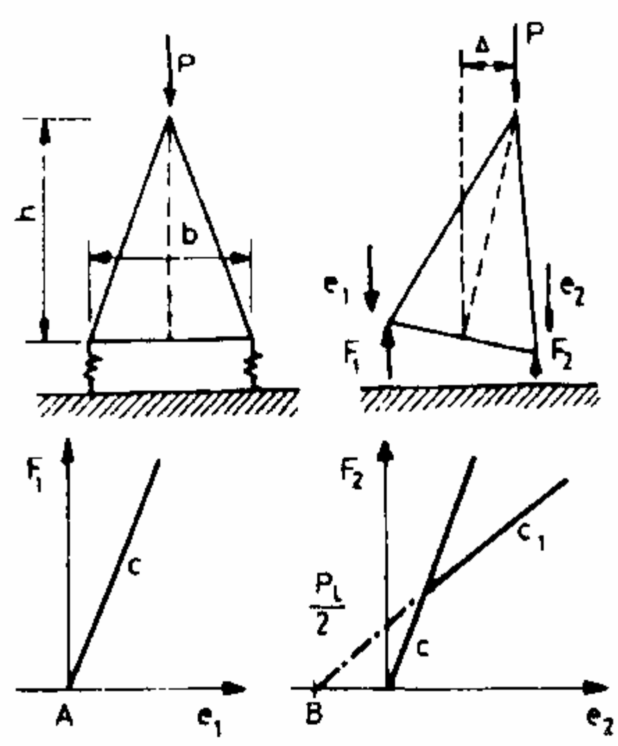
Variation of geometrical properties and loading conditions during the test program

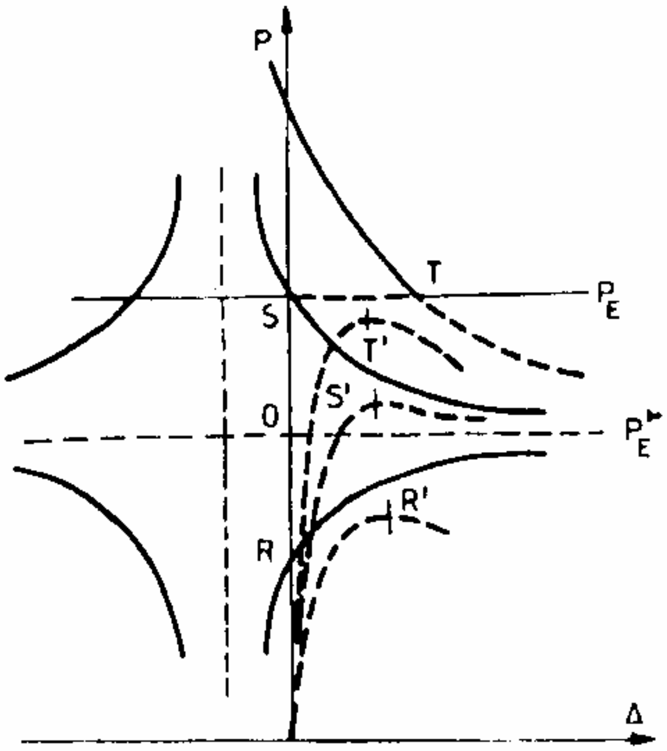
7.3.3 Phenomena and Effects

Simplified models for material:

(a) elastic - ideally plastic
(b)(c) elastic - strain hardening







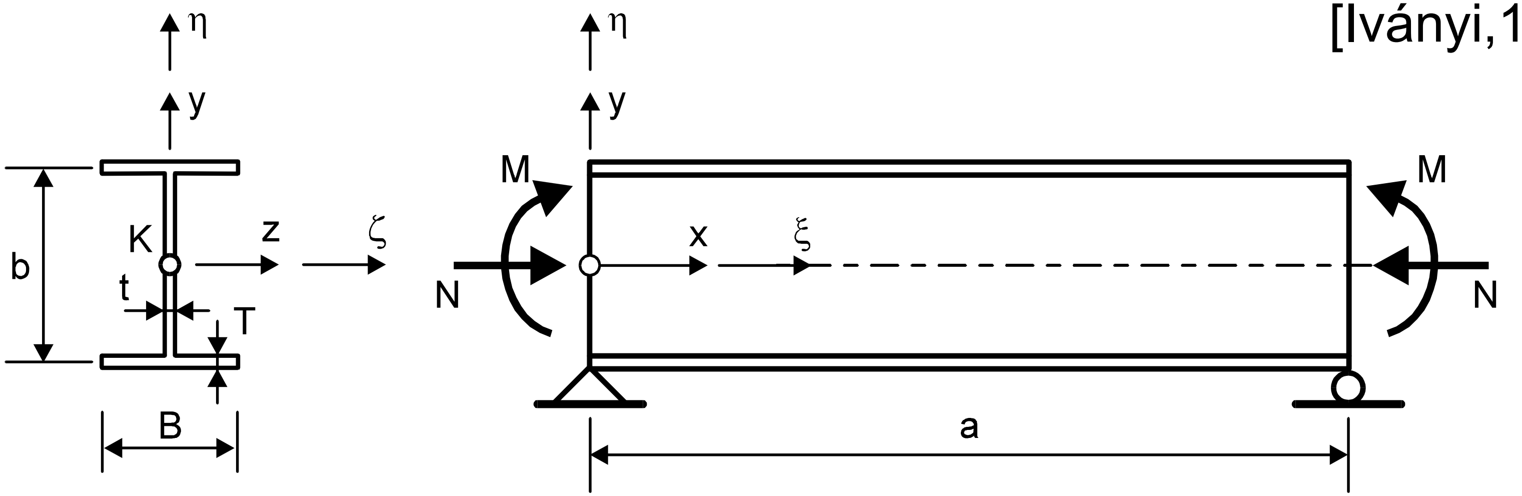
So-called Shanley-model illustrates the interaction between plate and lateral-torsional buckling

$$P_E = \frac{b^2}{2h} c$$

$$P_E^* = \frac{b^2}{2h} \frac{2cc_1}{c + c_1}$$

7.3.4 Interaction Between Plate and Lateral-Torsional Buckling

[Iványi,1983]

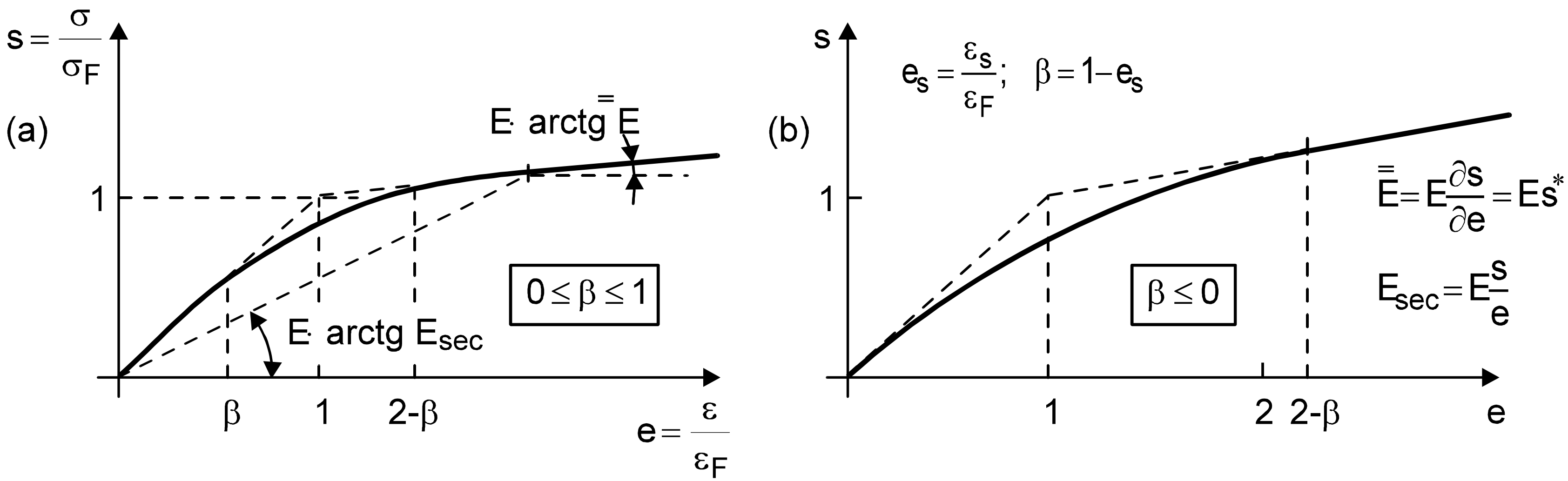


$$\xi = \frac{x}{a}$$

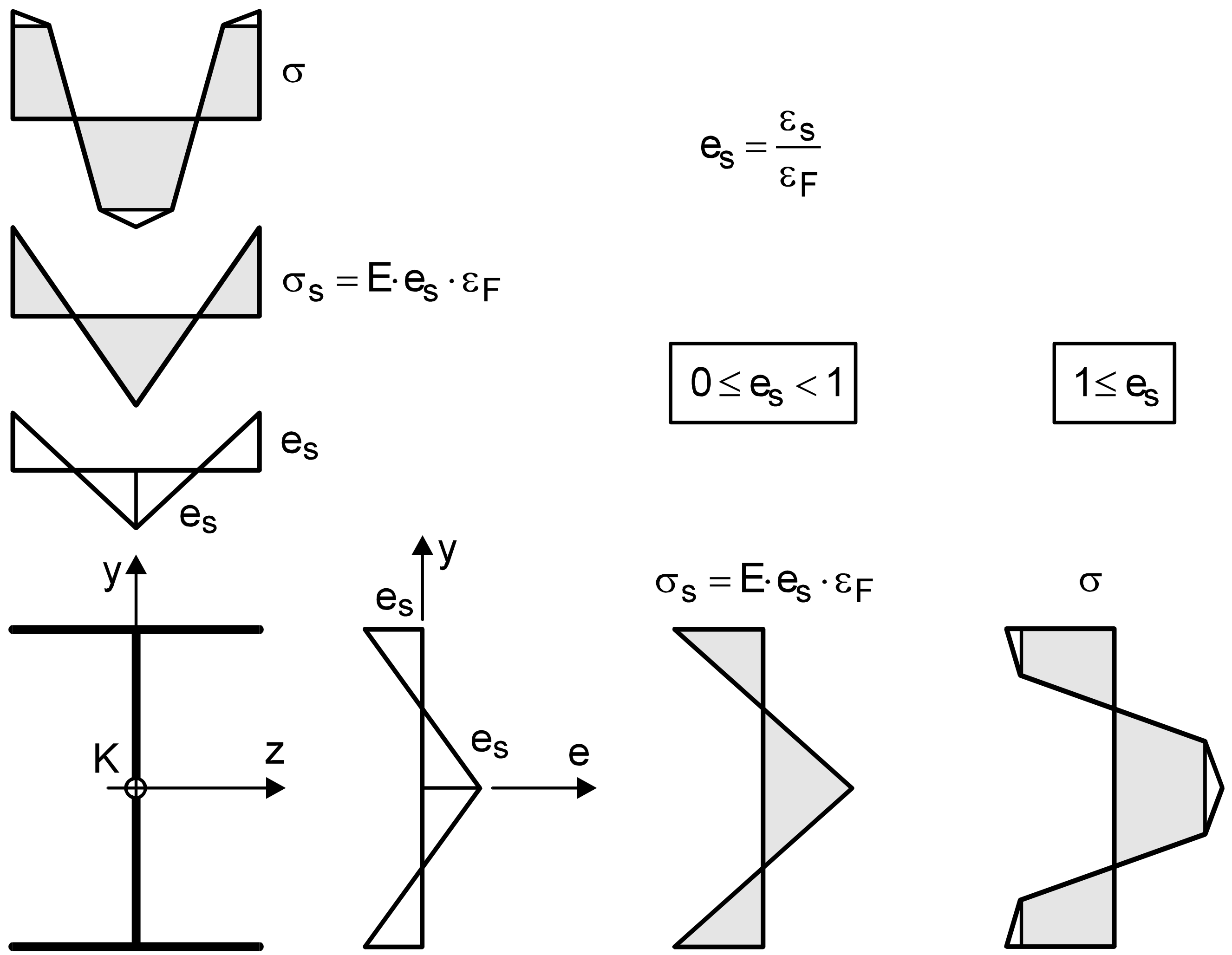
$$\eta = \frac{2y}{b}$$

$$\zeta = \frac{2z}{B}$$

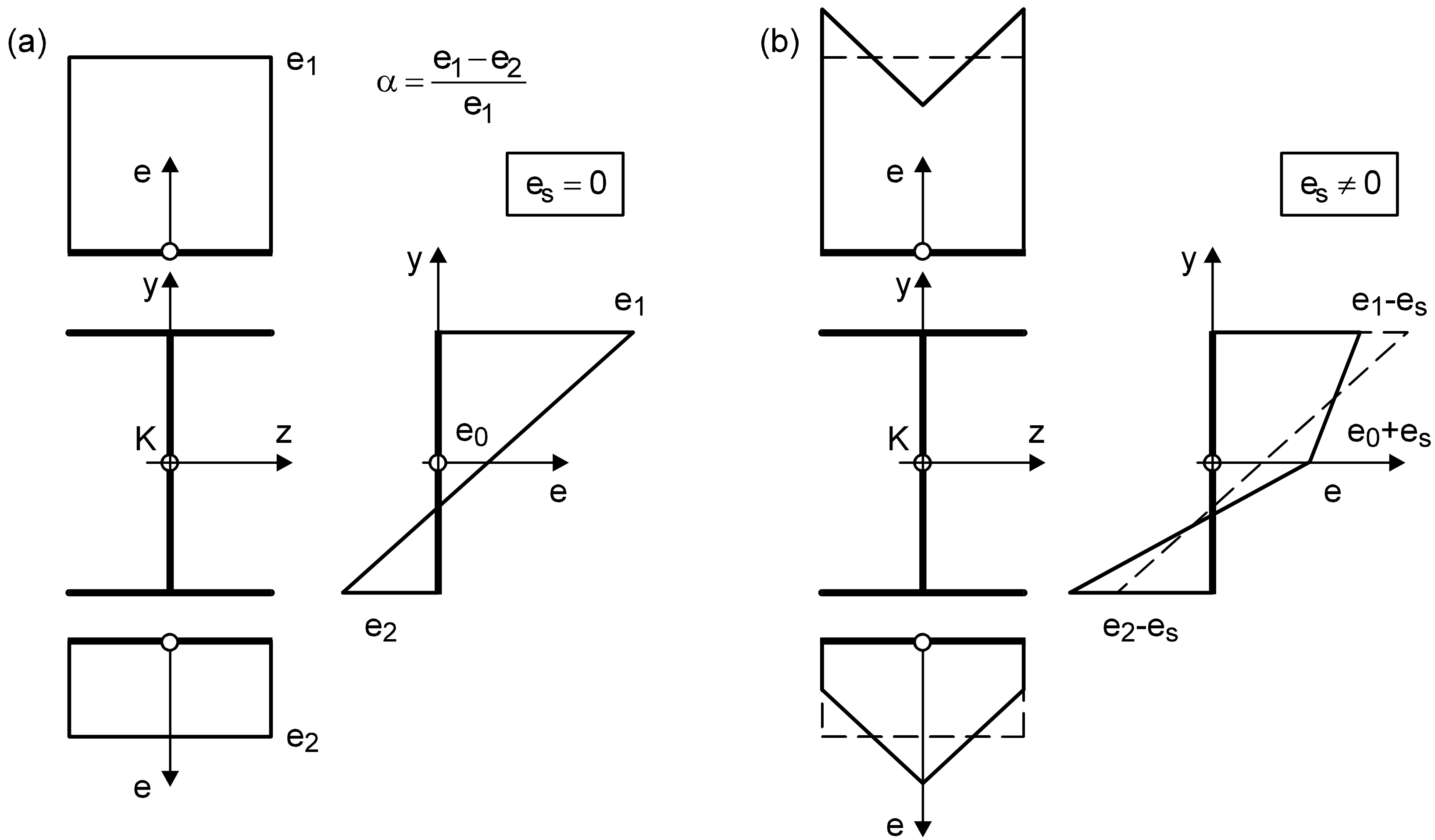
General geometry of tested members



Elastic-strain hardening material model, if residual stresses are lower (a) or higher (b) than yield stress



Supposed distribution of residual strains (stresses) due to technology processes over the cross-section



Strain distributions (a) from external loading,
(b) resultant of (a) and residual strains

Potential energy of the web:

$$U_w = \frac{1}{2}\int_0^a \int_{-b/2}^{+b/2} \left\{ D'\left[C_1 (w'')^2 + (\dot{w}')^2 + (w'')(\ddot{w}) + (\ddot{w})^2 \right] \right\} \mathrm{d}\,x\,\mathrm{d}\,y$$

$$V_w = -\frac{1}{2}\int_0^a \int_{-b/2}^{+b/2} t\sigma_w (w')^2 \,\mathrm{d}\,x\,\mathrm{d}\,y,$$

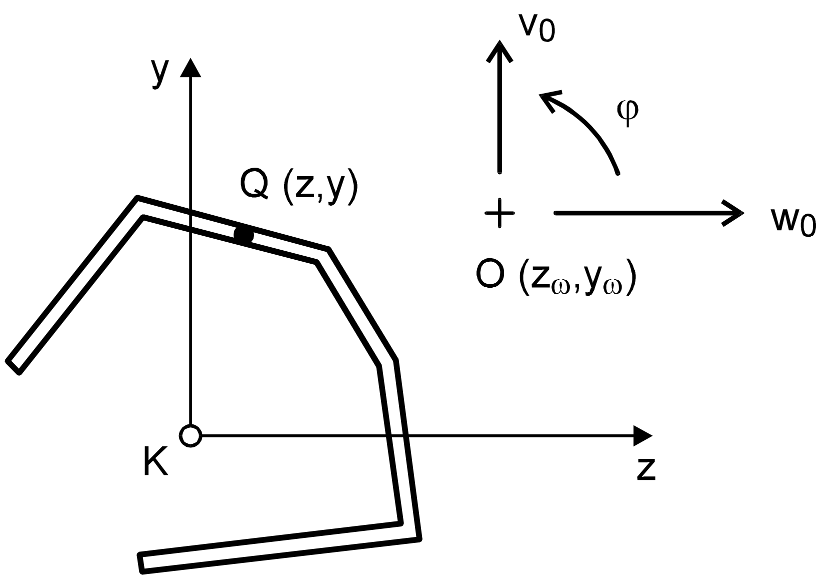
Potential energy of the flange:

$$U_f = \frac{1}{2}\int_0^a \int_{A_f} E_{\sec}\left[\omega_{01}^2 (\varphi'')^2 + (w_0'')^2 z^2 \right] \mathrm{d}\,A_f\,\mathrm{d}\,x + \frac{1}{2}\int_0^a (\varphi')^2 \int_{-B/2}^{B/2} G_{\sec}\frac{T^3}{3}\,\mathrm{d}\,z\,\mathrm{d}\,x$$

$$V_f = \frac{1}{2}\int_{A_f} (\sigma_{f1} - \sigma_{f2}) \int_0^a \left[(w_k')^2 - 2yw_k'\varphi' + (y^2 + z^2)(\varphi')^2 \right] \mathrm{d}\,A_f\,\mathrm{d}\,x$$

Overall potential energy of the member:

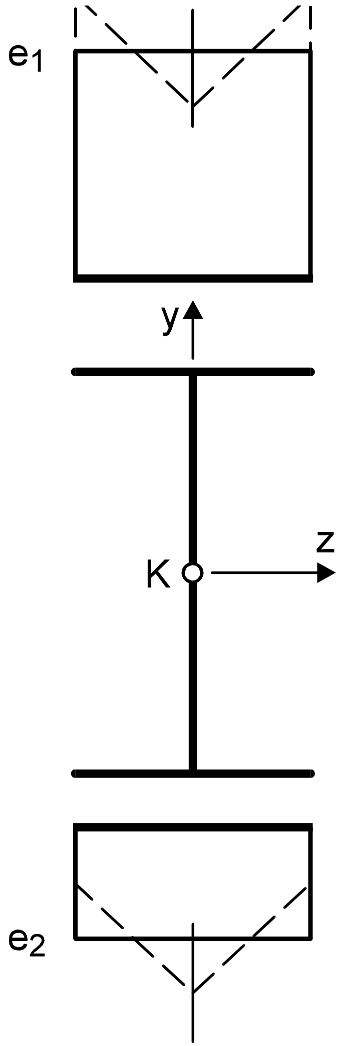
$$\Pi = U_w + V_w + U_f + V_f$$

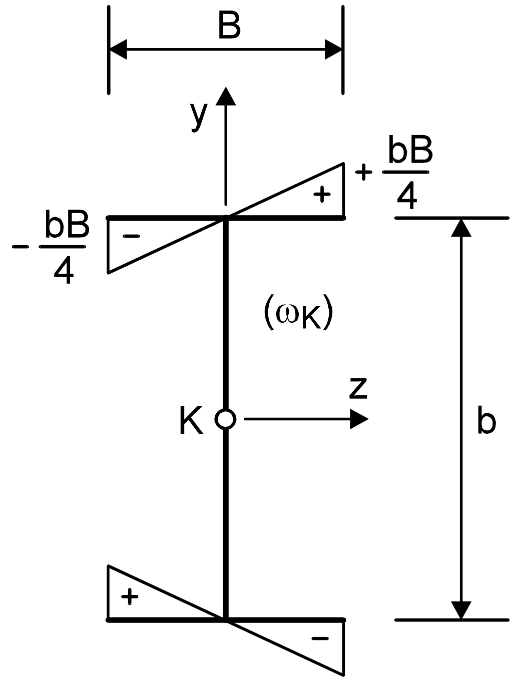


$$w_Q = w_0 + \varphi(y_\omega - y) = w_K - \varphi \cdot y$$

$$v_Q = v_0 - \varphi(z_\omega - z) = v_K + \varphi \cdot z$$

Relation between the displacements of centroid (K) and torsion centre (O)





$$\omega_0 = \omega_K - y_\omega \cdot z + z_\omega \cdot y + \overline{\omega}$$

$$z_\omega = 0$$

$$\overline{\omega} = 0$$

Illustration for torsion centre migration in the plastic range

For a doubly symmetric cross-section in the elastic range, centroid and torsion centre are coincident

In the plastic range these two points do not coincide!

Let the deformed shape be: $\boxed{w(x,y) = X(x)\cdot Y(y)}$

$$Y(y) = \sum_p A_{pq} \sin\frac{q\pi}{b}\left(y + \frac{b}{2}\right) + C_p \frac{y}{b} + D_p$$

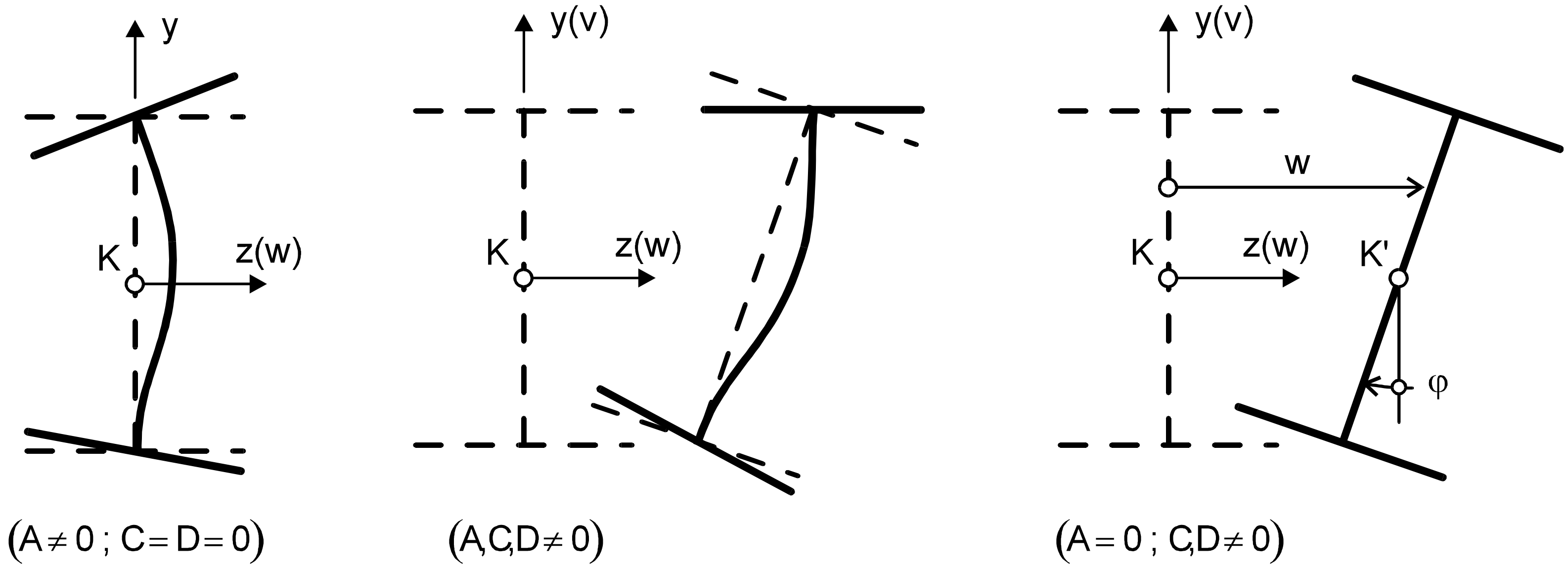
For different boundary conditions:

– pinned ends for twisting:

$$X(x) = \sum_p \sin\frac{p\pi x}{a}$$

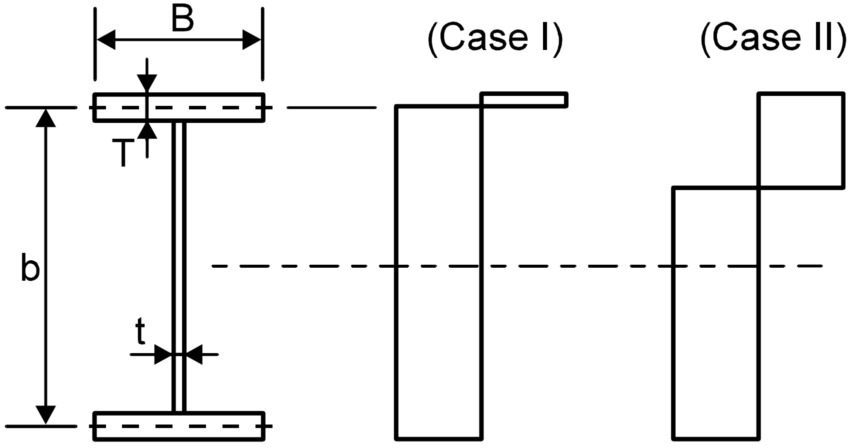
– fixed ends for twisting:

$$X(x) = \frac{1}{2}\sum_p \left(\cos\frac{(p-1)\pi x}{a} - \cos\frac{(p+1)\pi x}{a}\right)$$



Deformations of the cross-section in cases of (a) web plate buckling; (b) combined web plate buckling and lateral-torsional buckling; (c) lateral-torsional buckling

7.3.5 Comparison of the Results of Theoretical and Experimental Investigations [Iványi, 1983]



$$\eta^* = \frac{\sigma_{Fg}}{\sigma_{F\check{s}}}, \quad A_{\check{s}} = BT, \; A_g = bt$$

$$A_r = 2A_{\check{s}} + \eta^* A_g$$

$$\alpha_{\check{s}} = \frac{A_{\check{s}}}{A_r}, \; \alpha_g = \eta^* \frac{A_g}{A_r}, \; (2\alpha_{\check{s}} + \alpha_g = 1)$$

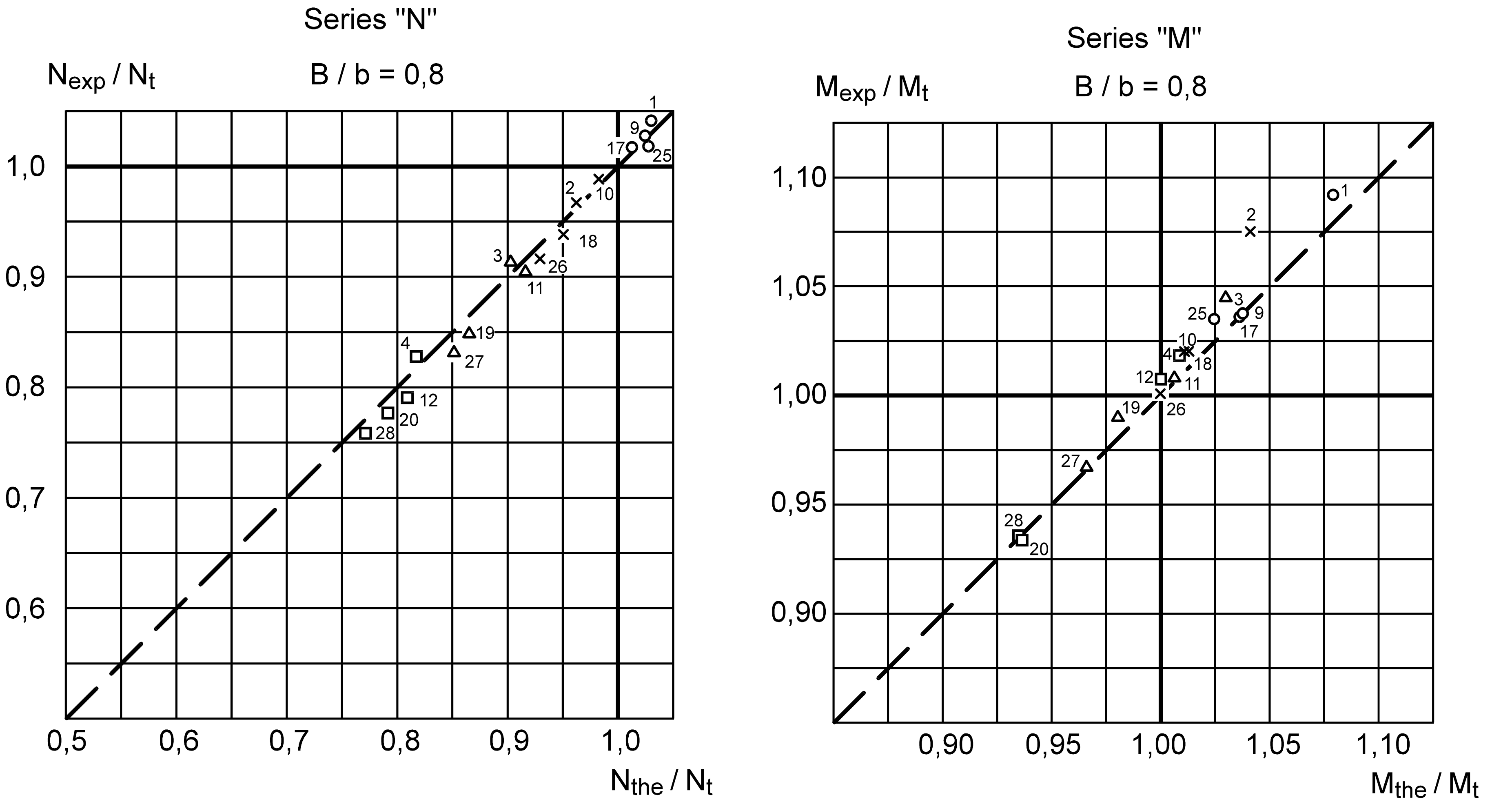
Normal force: $N_t = A_r \cdot \sigma_{F\check{s}}$ Bending moment: $M_t = \frac{2-\alpha_g}{2} N_t \frac{b}{2}$

Case I (neutral axis in the flange):

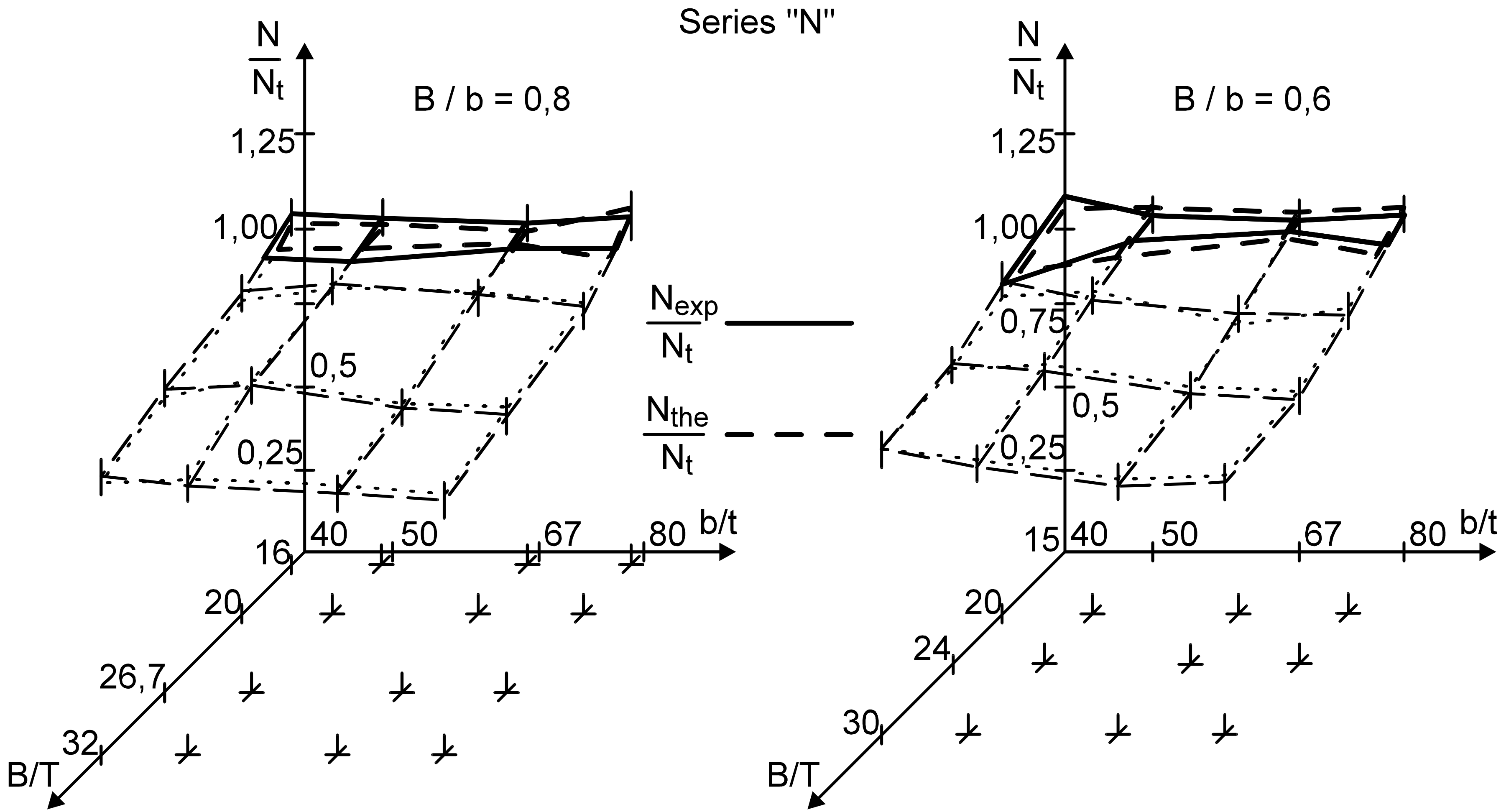
$$\alpha_g \le \frac{N}{N_t} \le 1 \qquad M_{tN} = (N_t - N)\frac{b}{2}$$

Case II (neutral axis in the web):

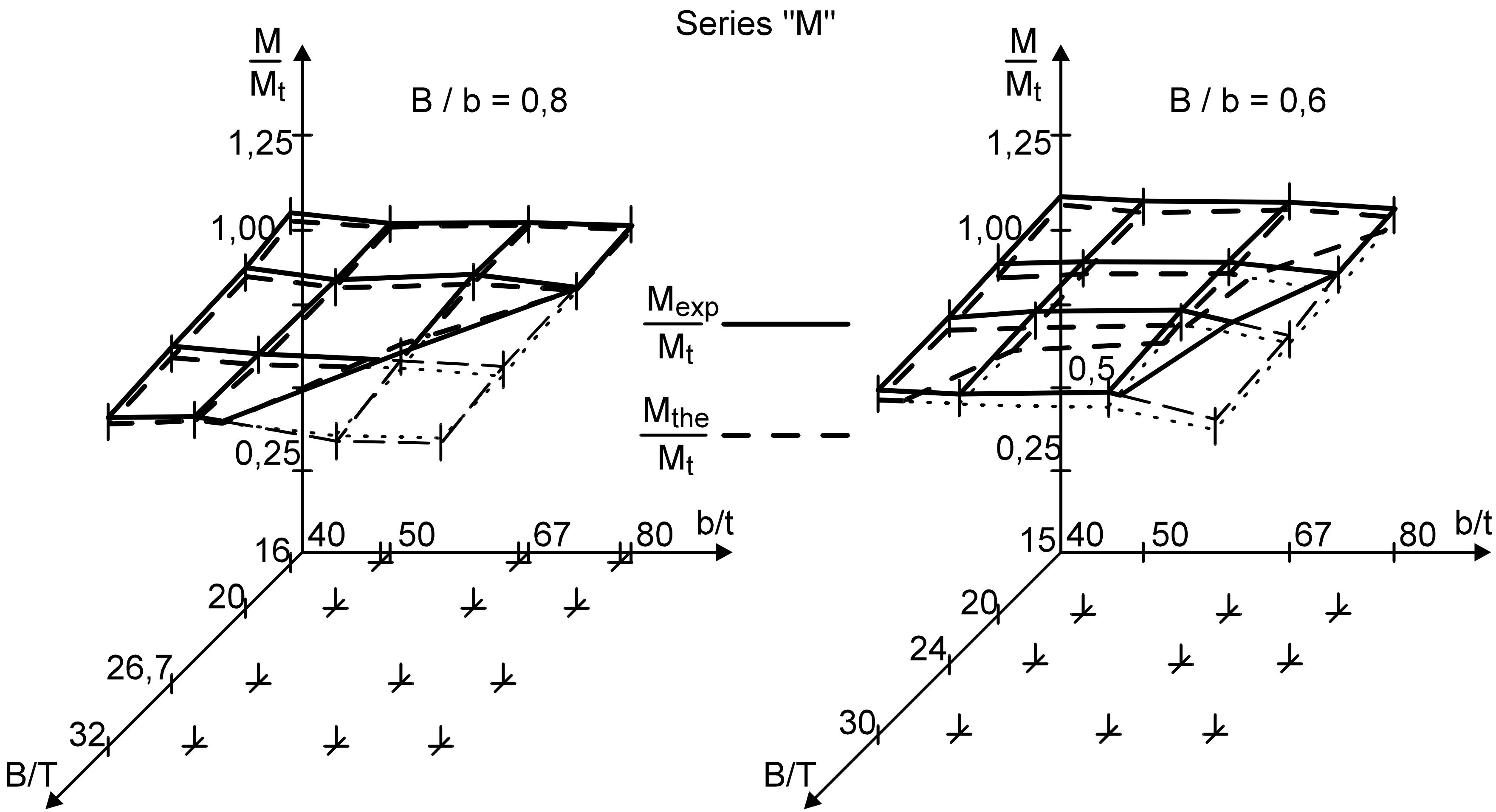
$$0 \le \frac{N}{N_t} \le \alpha_g \qquad M_{tN} = \left[\frac{1}{2} - \frac{1}{4}\alpha_g - \frac{1}{4\alpha_g}\left(\frac{N}{N_t}\right)^2\right] \cdot b \cdot N_t$$



Comparison of experimental and theoretical load-carrying capacities for specimens
(a) in pure compression and (b) in pure bending

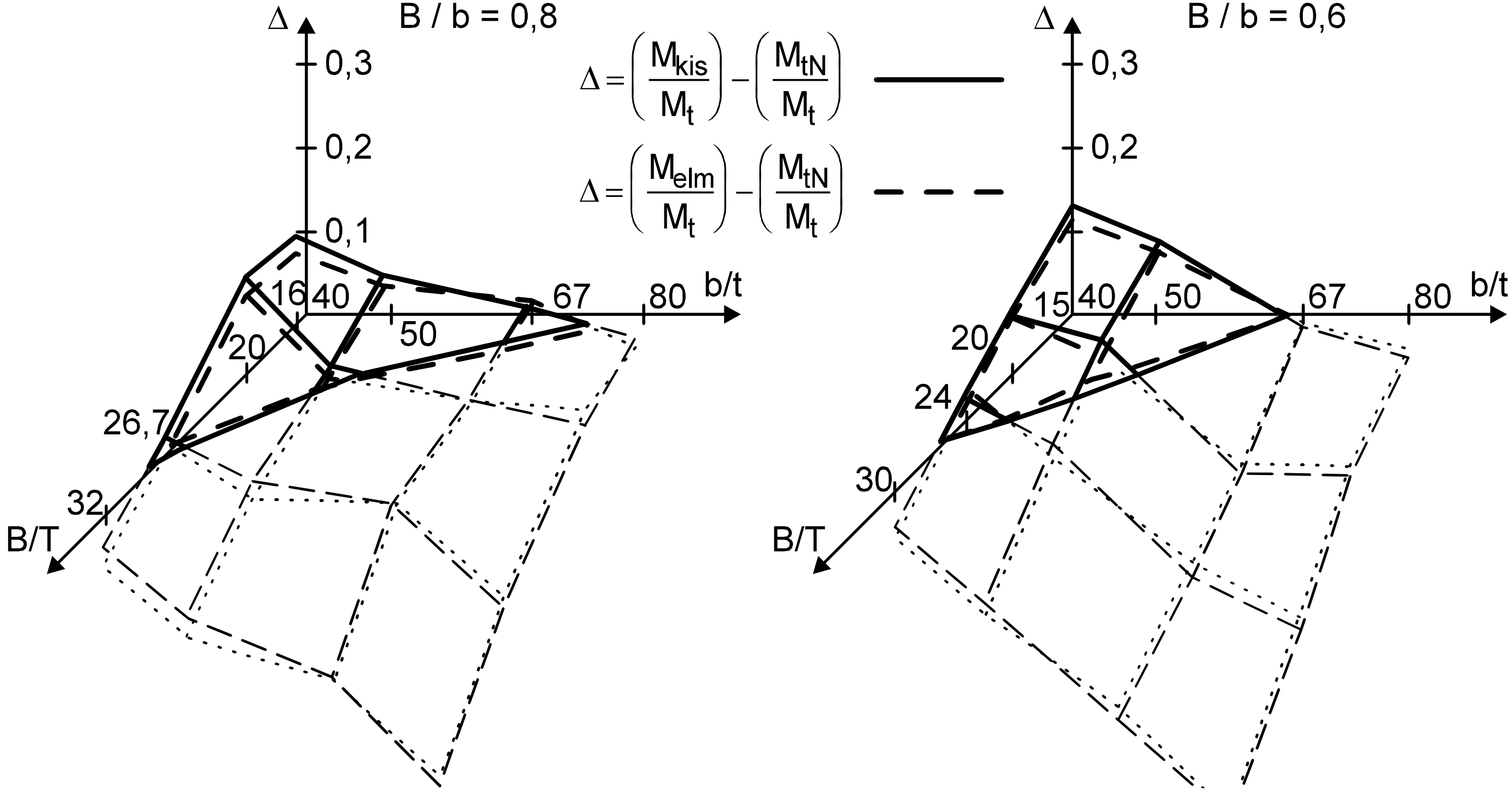


Influence of geometrical parameters on load-carrying capacities
of compressed members

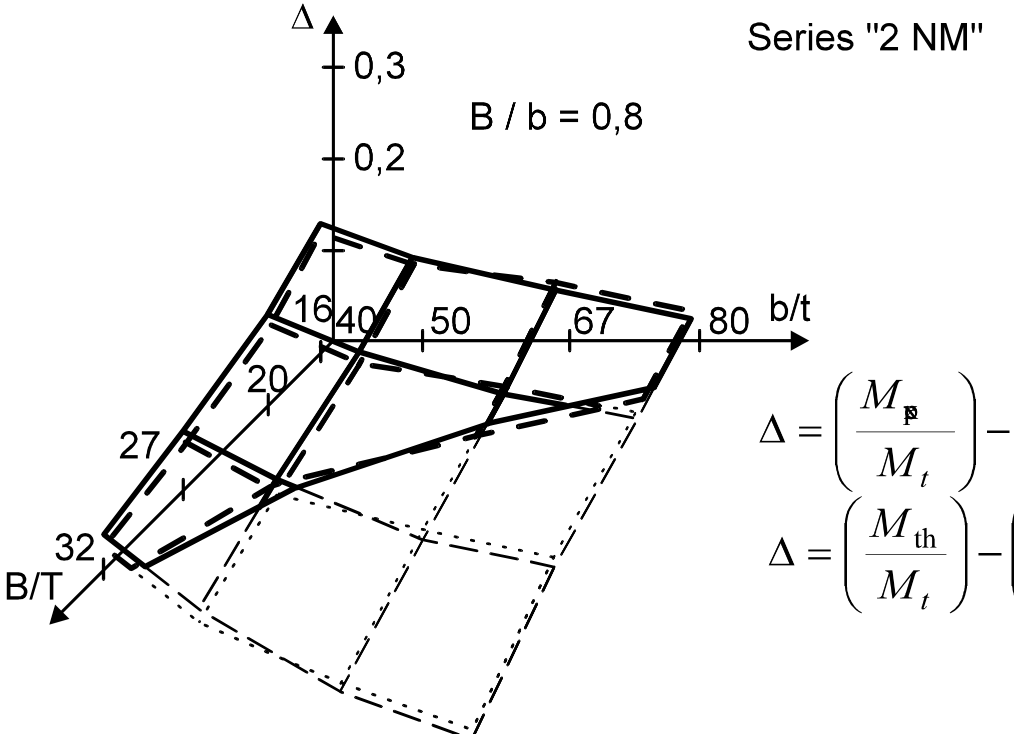


Influence of geometrical parameters on load-carrying capacities
of bent members

"1 NM" sorozat



Difference of experimental and calculated moment capacities compared to simple bent case for combined loading ($N=N_t/3$)

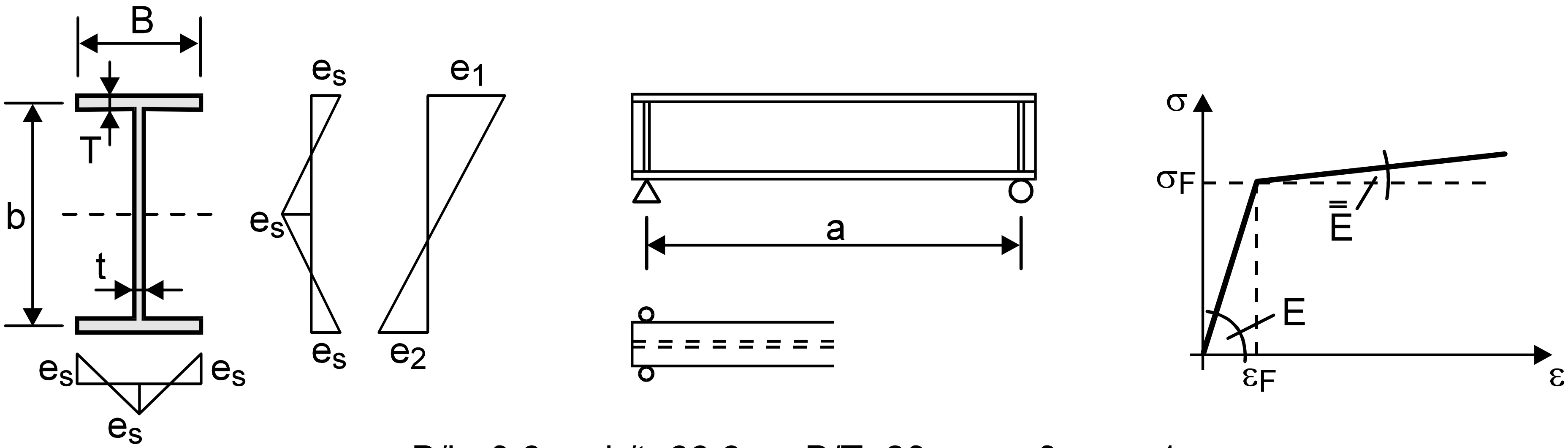


$$\Delta = \left(\frac{M_{p}}{M_t}\right) - \left(\frac{M_{tN}}{M_t}\right)$$ (solid line)

$$\Delta = \left(\frac{M_{\mathrm{th}}}{M_t}\right) - \left(\frac{M_{tN}}{M_t}\right)$$ (dashed line)

Difference of experimental and calculated moment capacities compared to simple bent case for combined loading ($N=2N_t/3$)

7.3.6 Parametric Study of Interaction [Iványi, 1986]

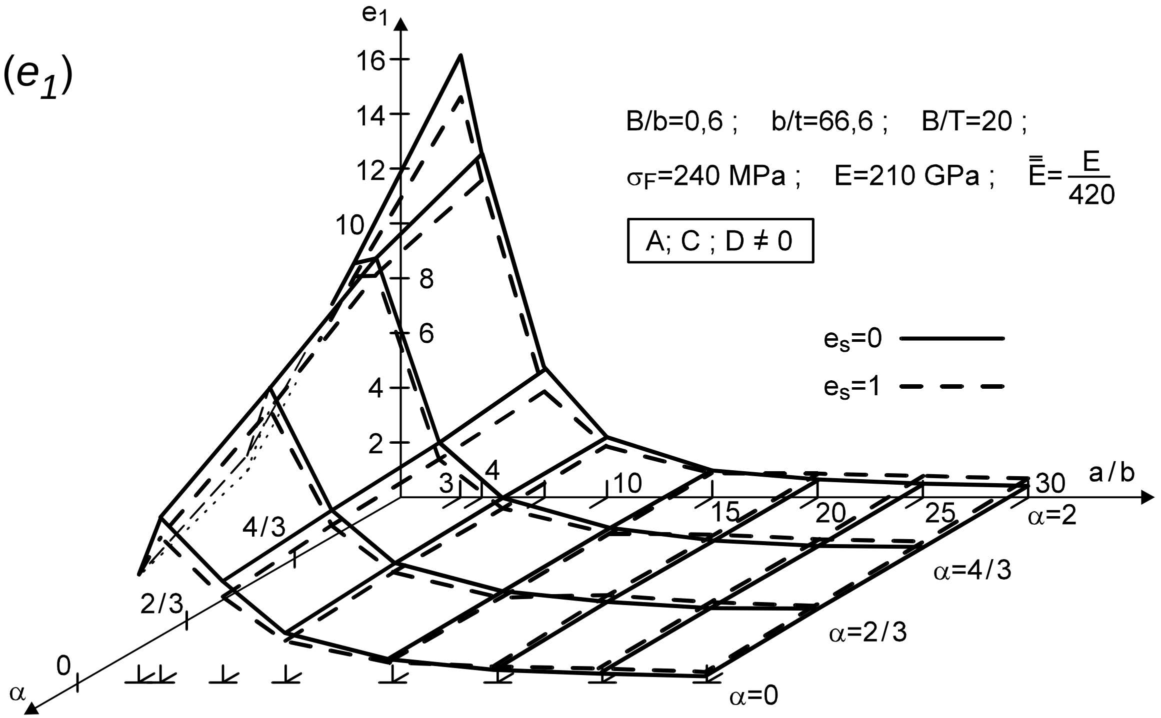


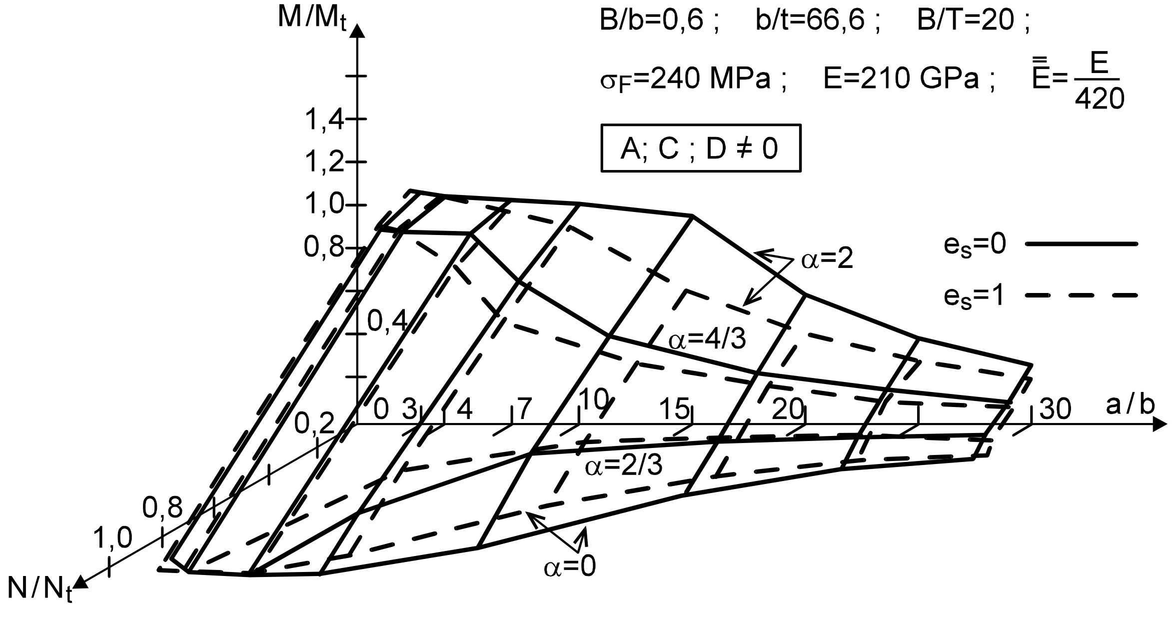
B/b=0,6 ; b/t=66,6 ; B/T=20 ; e_s=0 vagy 1 ;

σ_F=240 MPa ; E=210 GPa ; $\bar{\bar{E}} = \frac{E}{420}$ vagy $\frac{E}{42}$

Models for the parametric study

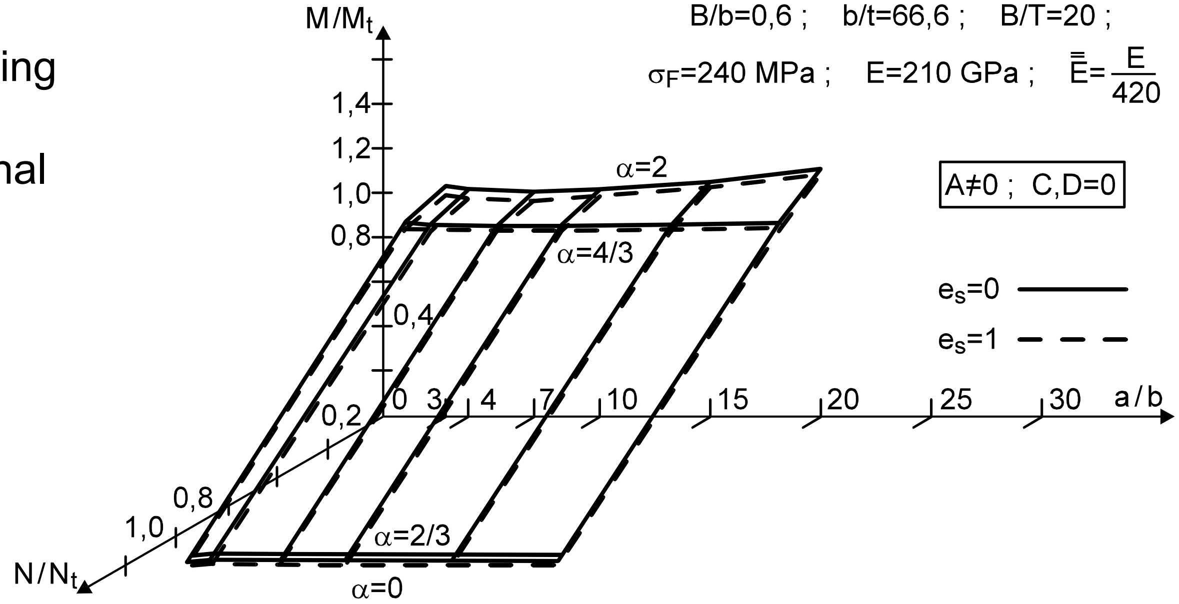
Compression fibre strains (e_1)

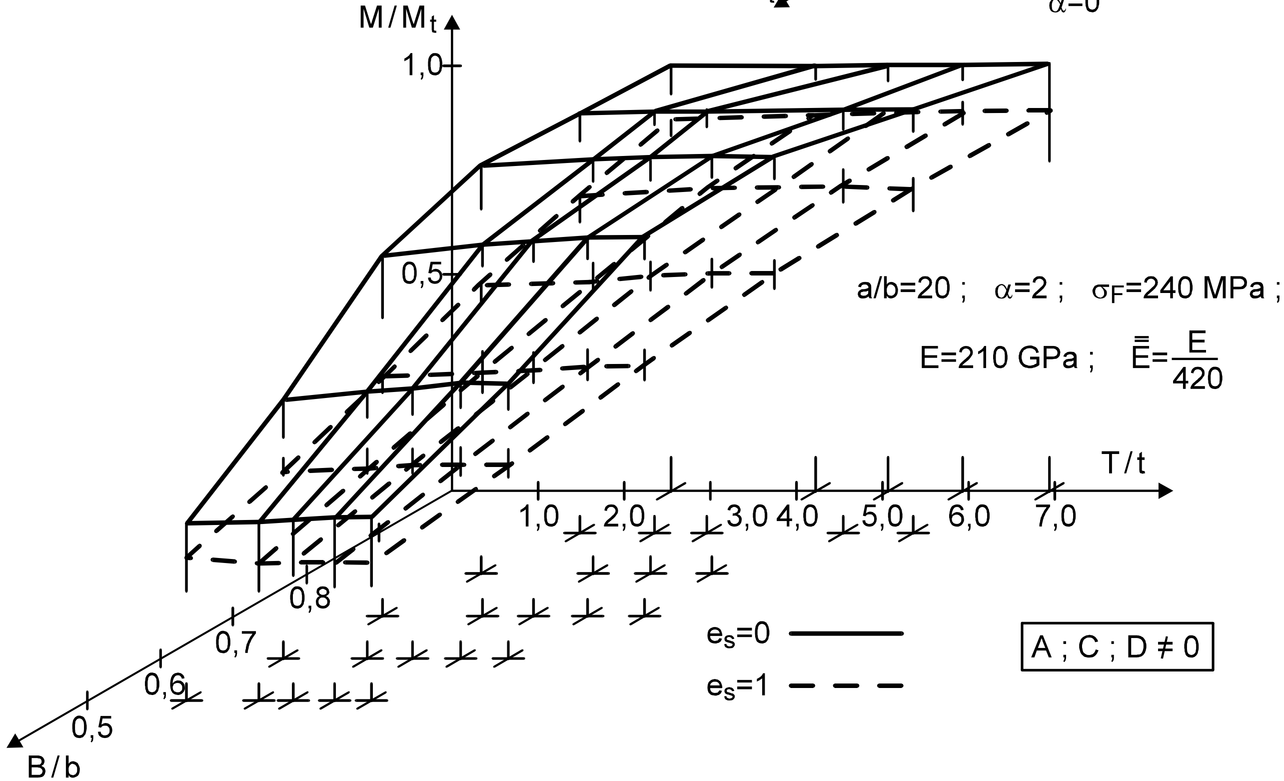




Combination of normal forces and bending moments

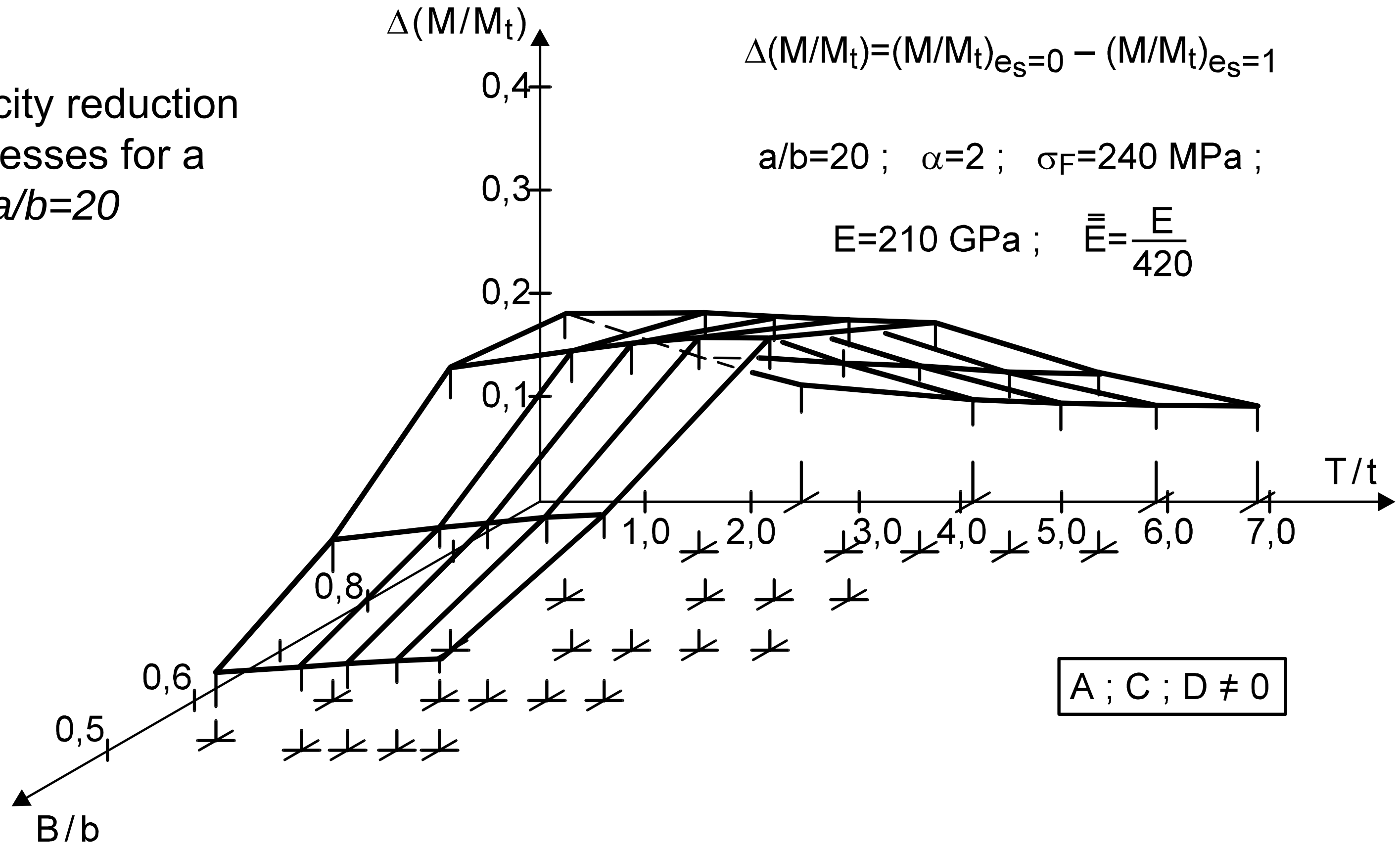
Characteristic normal force - bending moment combination causing web plate buckling, when lateral-torsional buckling is prevented





Influence of flange width to web depth ratio for load-carrying capacity of a bent member with *a/b=20*

Load-carrying capacity reduction effect of residual stresses for a bent member with *a/b=20*



7.3.7 Conclusion

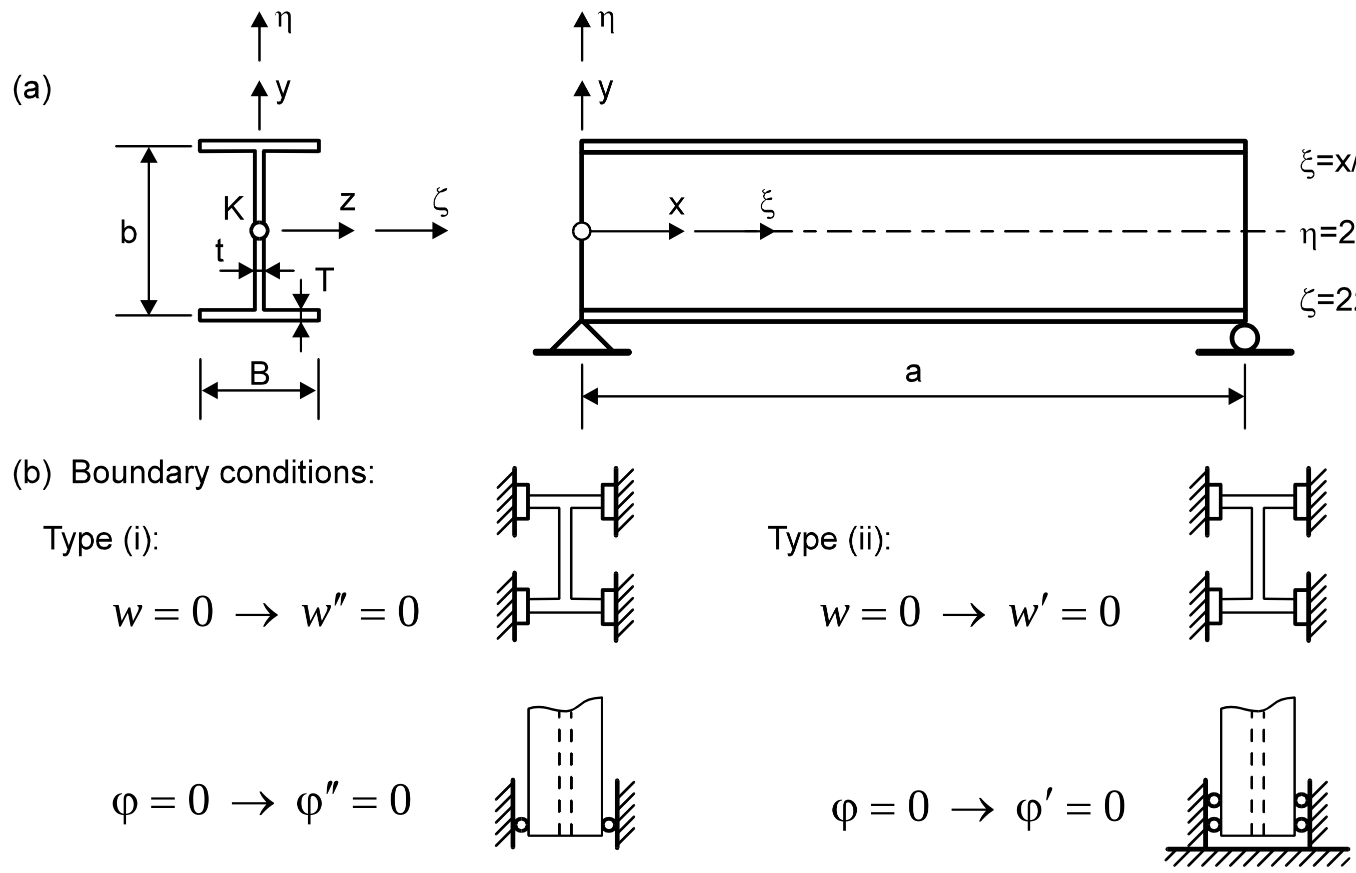
Investigation has been carried out theoretically and experimentally to analyse the effect of strain-hardening of structural steel and of residual stresses affecting the stability conditions of beam-columns.

Important results have been determined and utilised in the Hungarian Code for steel construction.

7.4. Interaction Between Local and Lateral-Torsional Buckling

7.4.1 Plastic Rotational Capacity

[Rzanitsin, 1964] [Belski, 1973] [Iványi, 1983]



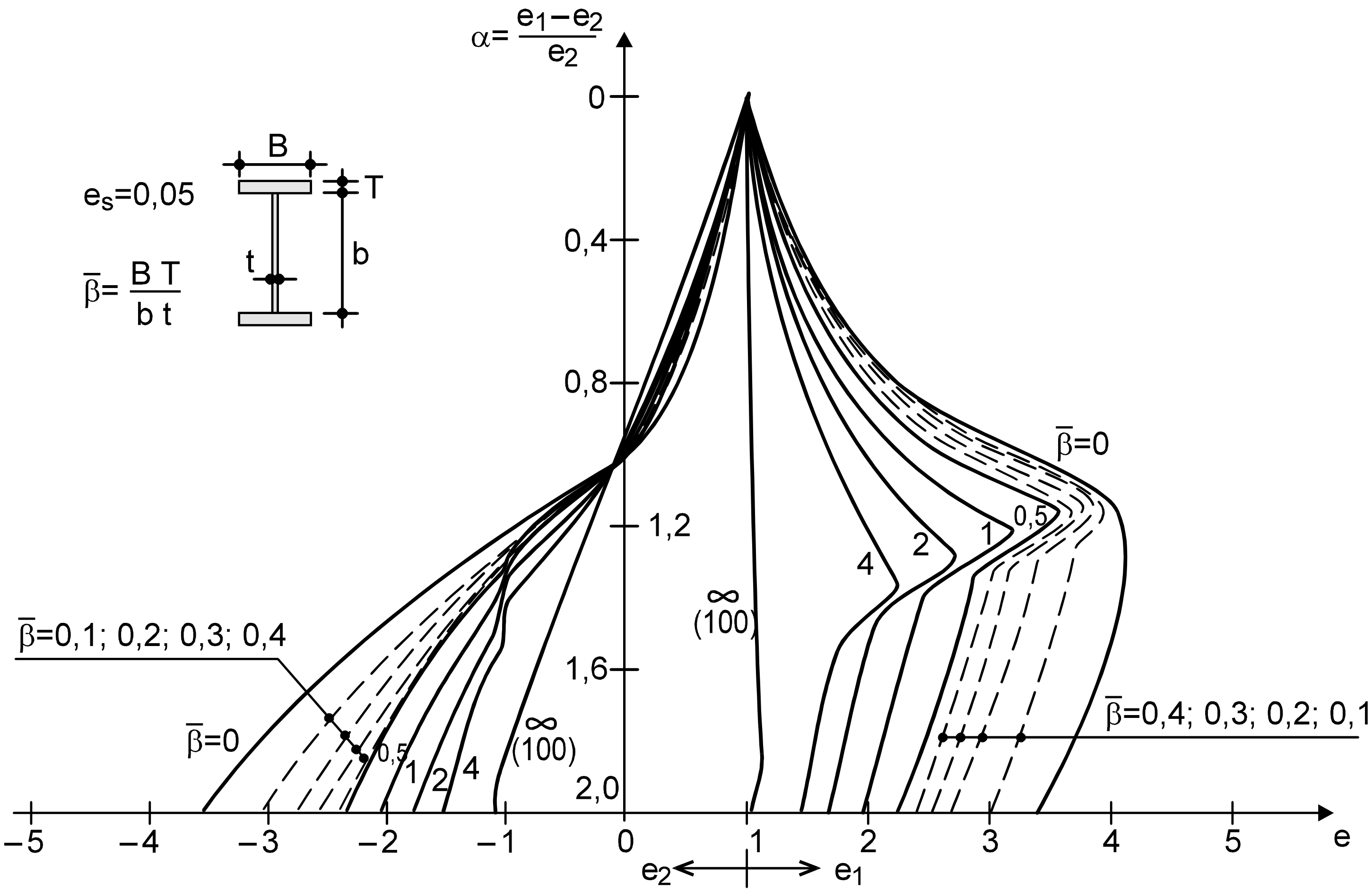
Geometry and boundary conditions

$$I_2 + I_1^2 = 1 + 4\bar{\beta};\qquad \bar{\beta} = \frac{BT}{bt}$$

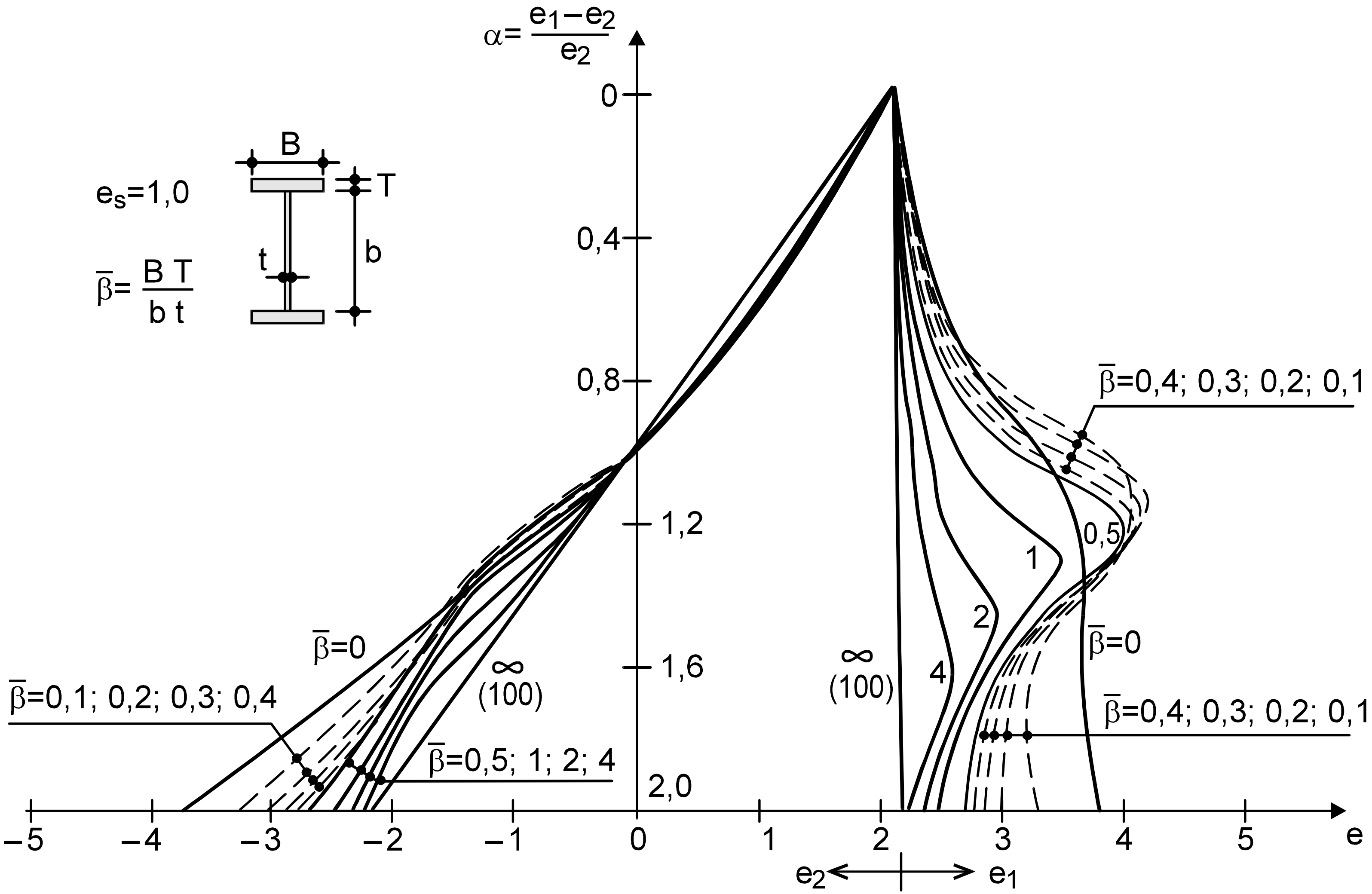
$$I_2 + 2I_1 = 2(1+\bar{\beta}),$$

$$I_1 = \frac{1}{b}\int_{-b/2}^{+b/2} s_g \cdot \mathrm{d}\,y + \bar{\beta}T\,\frac{B}{4}\int_{-B/2}^{+B/2}(s_{\check{s}v1} + s_{\check{s}v2})\cdot \mathrm{d}\,z$$

$$I_2 = \left(\frac{2}{b}\right)^2\int_{-b/2}^{+b/2} s_g \cdot y \cdot \mathrm{d}\,y + \bar{\beta}T\,\frac{B}{4}\int_{-B/2}^{+B/2}(s_{\check{s}v1} - s_{\check{s}v2})\cdot \mathrm{d}\,z$$



Strain distribution with small residual strain



Strain distribution with large residual strain

7.4.2 Plastic Buckling [Iványi, 1983]

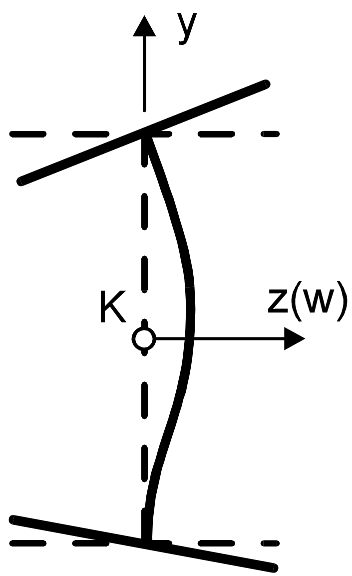
(a) Web buckling of "I"-shape cross-section

$$\overline{\beta} = BT / bt = 1{,}0$$

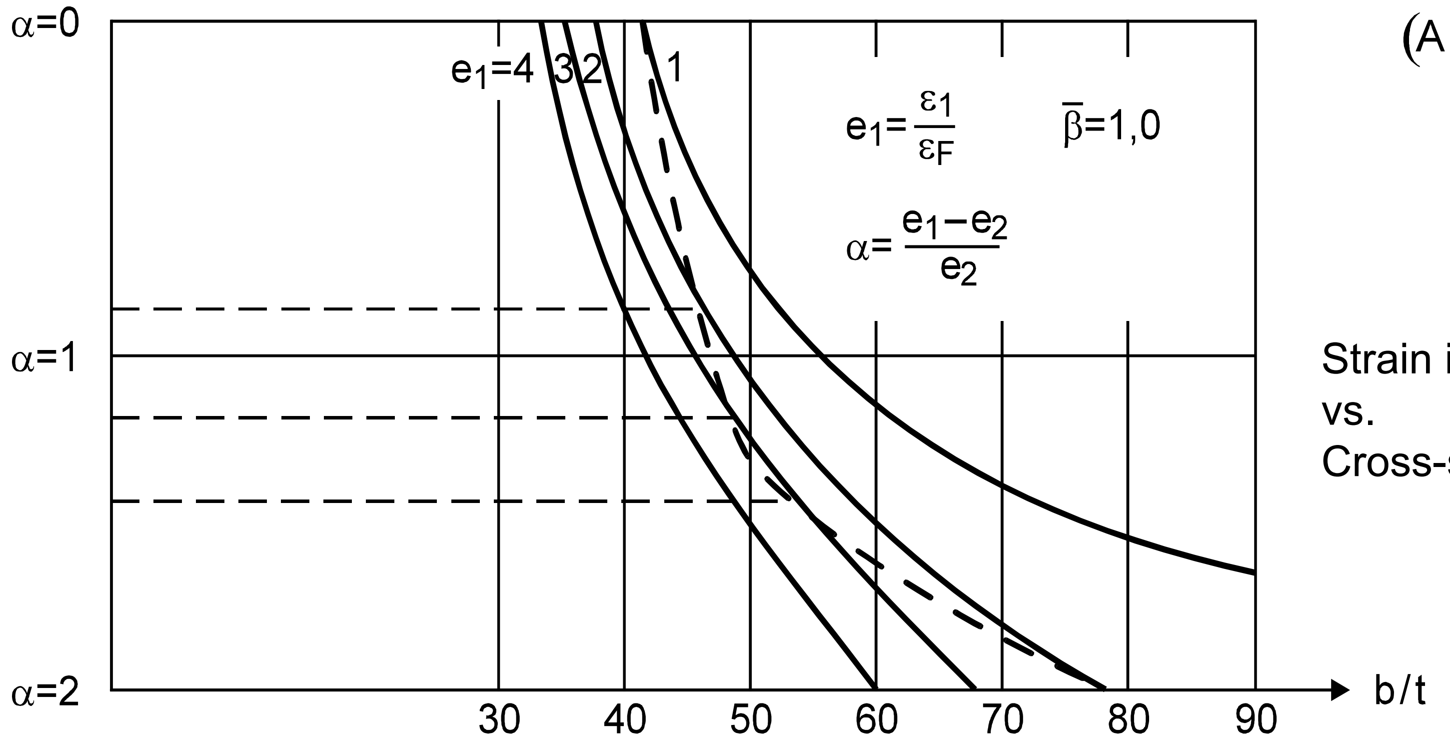
$$A \neq 0 \qquad C = D = 0$$

[Haaijer, Thurlimann, 1958]

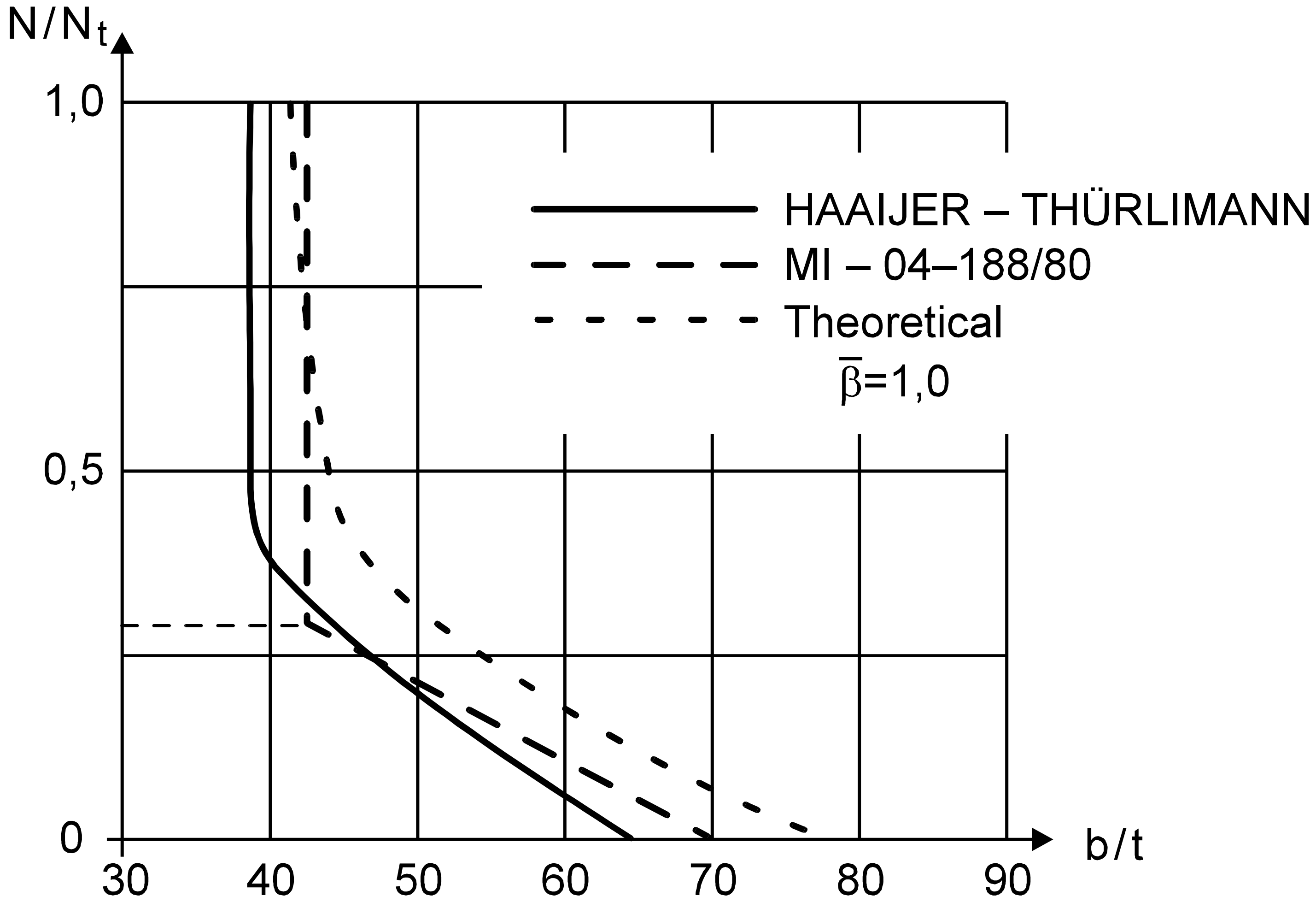
$$(e = \varepsilon / \varepsilon_F = 4)$$



$(A \neq 0\,;\, C = D = 0)$



Strain in extreme fibre
vs.
Cross-section geometry

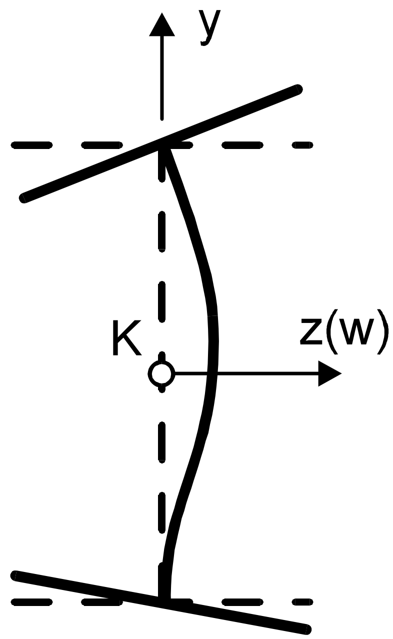


Change of normal force by web geometry

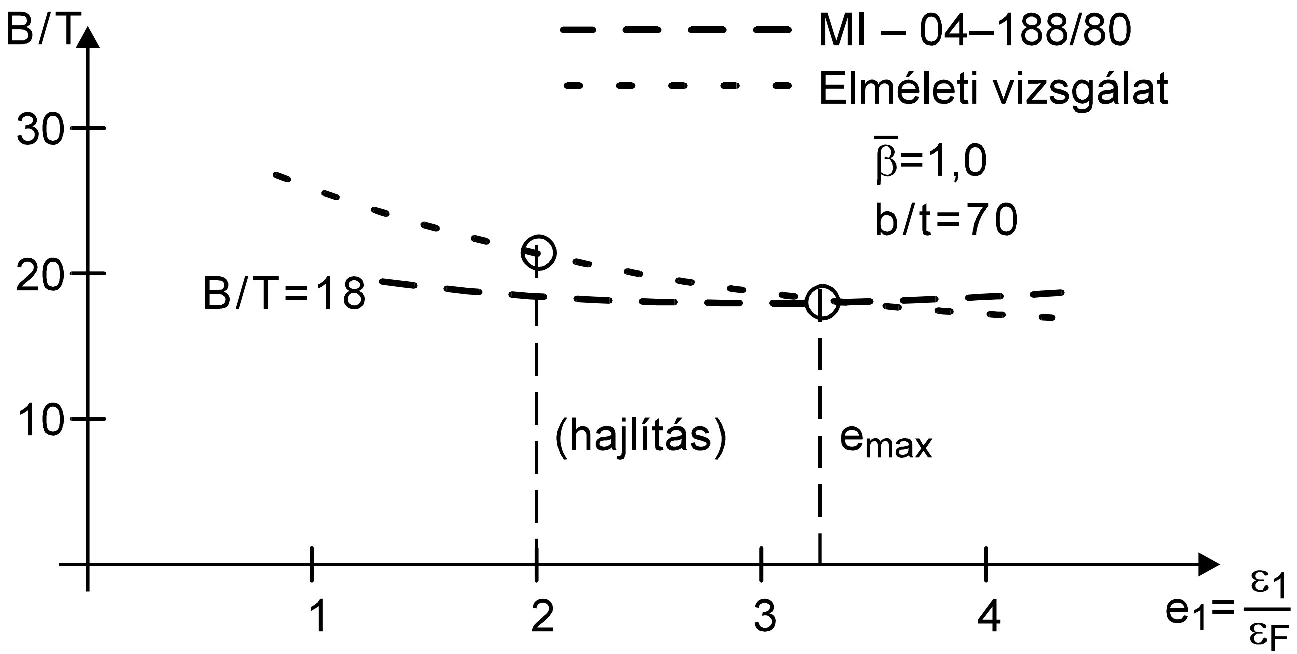
(b) Flange buckling of "I"-shape cross-section

$A \neq 0 \qquad C = D = 0$

[Moyseyev, 1975]



$(A \neq 0\,;\, C = D = 0)$



(c) Plastic lateral-torsional buckling of “I”-beam