Készült az ERFP – DD2002 – HU – B – 01 szerzősésszámú projekt támogatásával

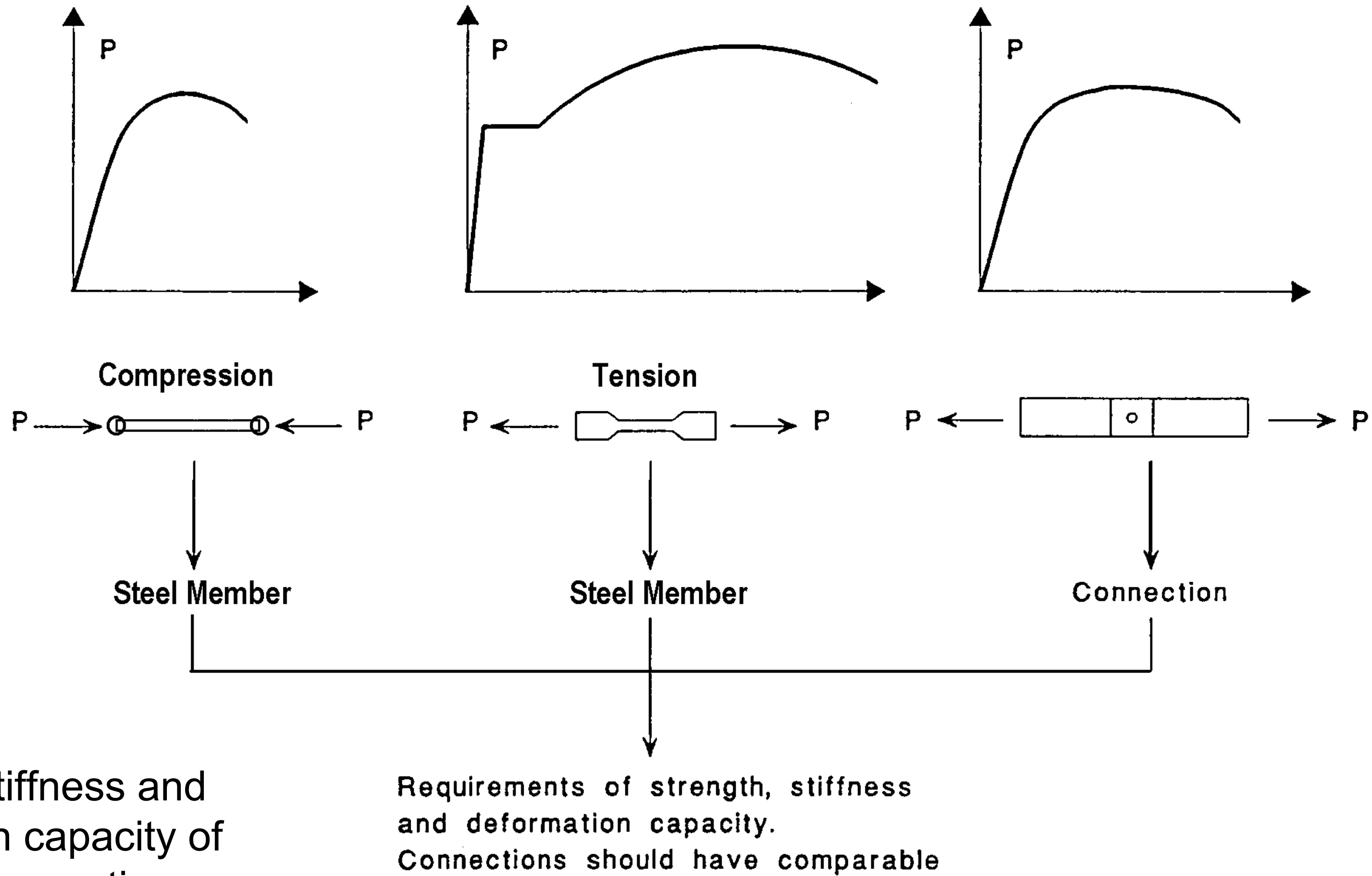
C h a p t e r 9

Semi-Rigid Connections in Steel Construction

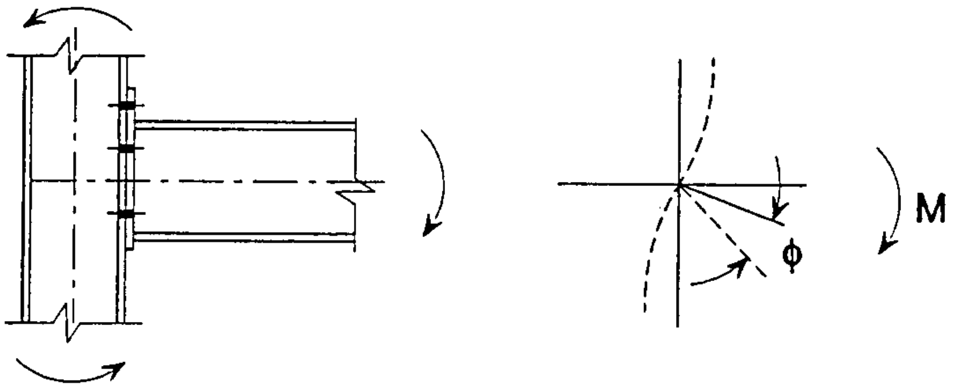
9.1. Introduction: A Reasonable Principle for Connection Design

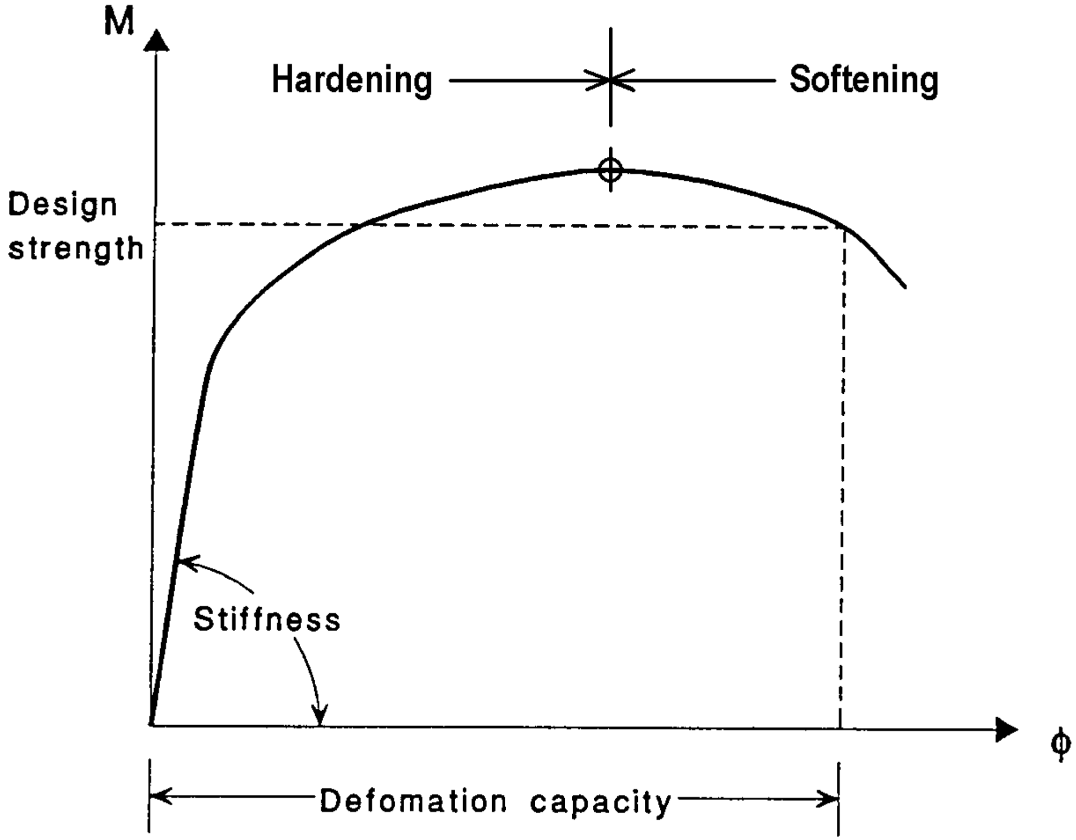
9.1.1 Introduction to Connection

9.1.2 Basic Criteria for Structural Behaviour



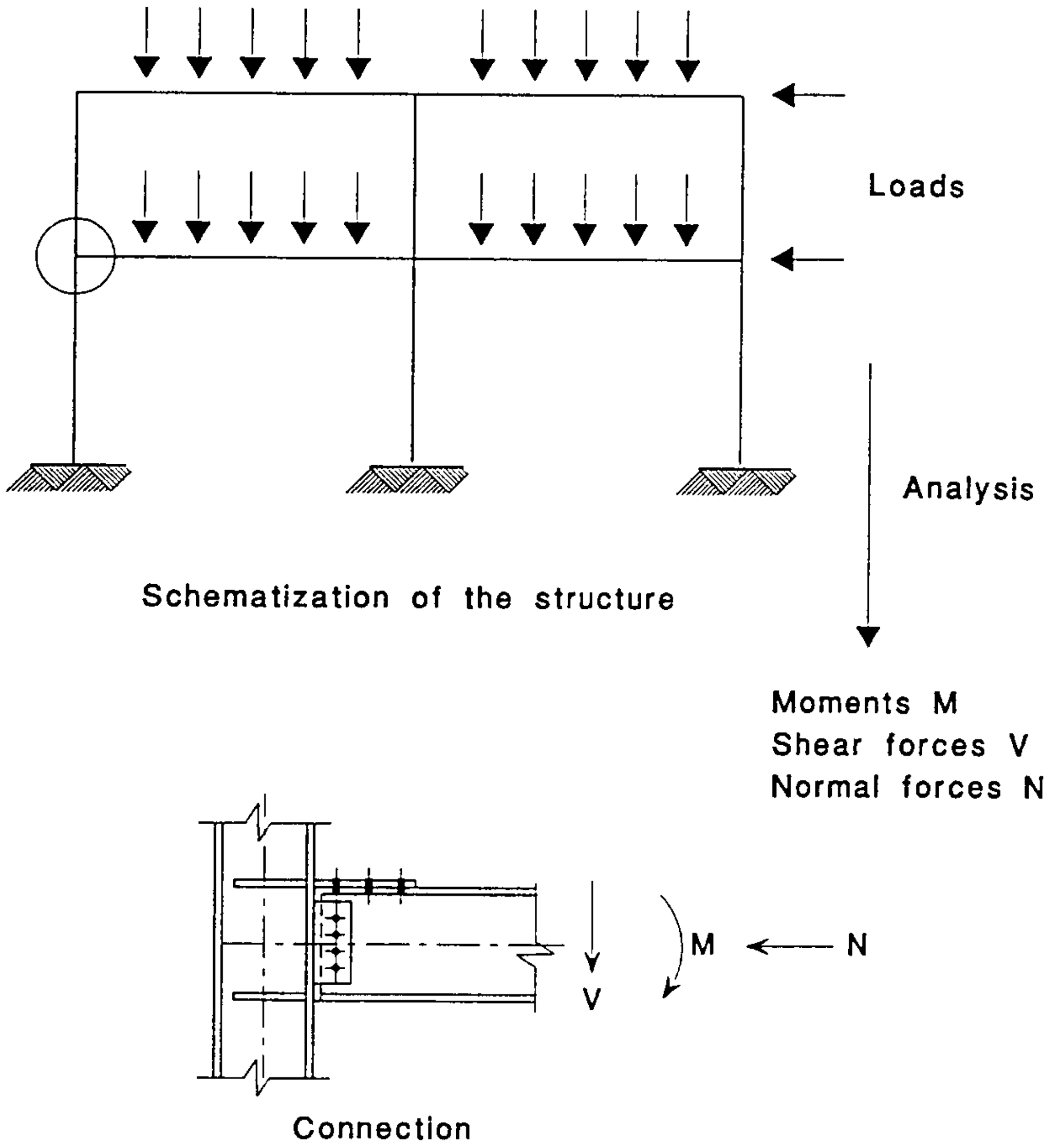
Strength, stiffness and deformation capacity of steel and connections





Moment–rotation diagram of a beam-to-column connection (M–ϕ curve)

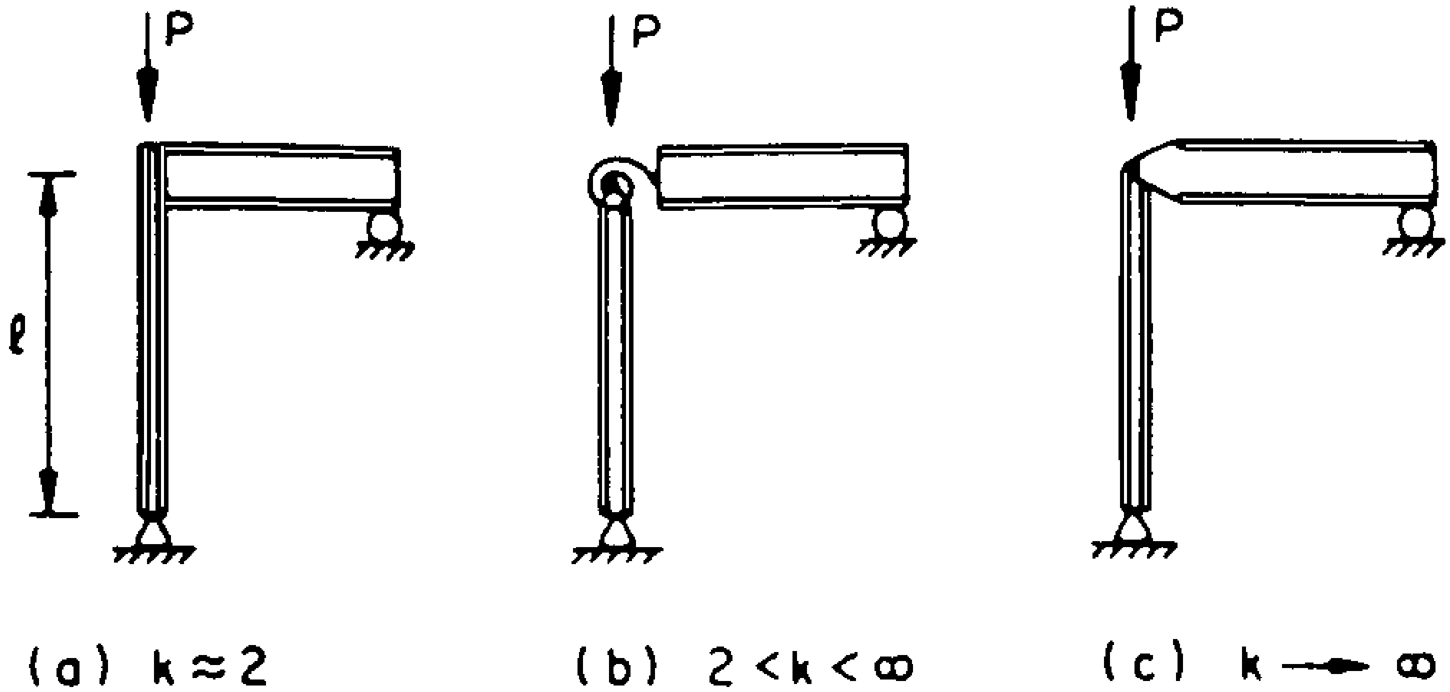
(a) Strength



Analysis of the forces on the connection

(b) Stiffness

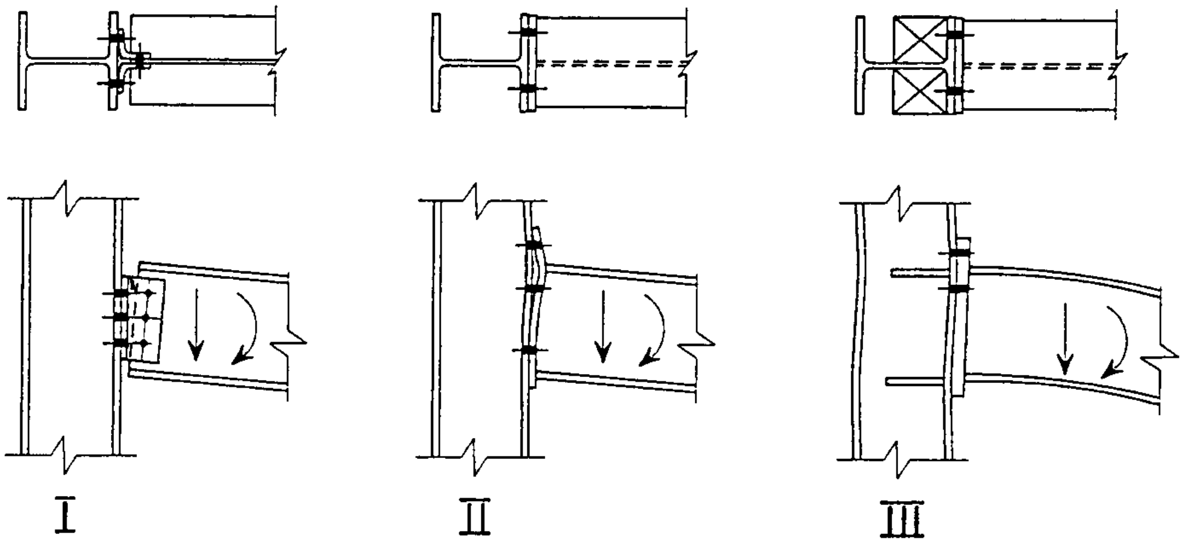
$$P_E = \frac{\pi^2 EI}{(2l)^2}$$

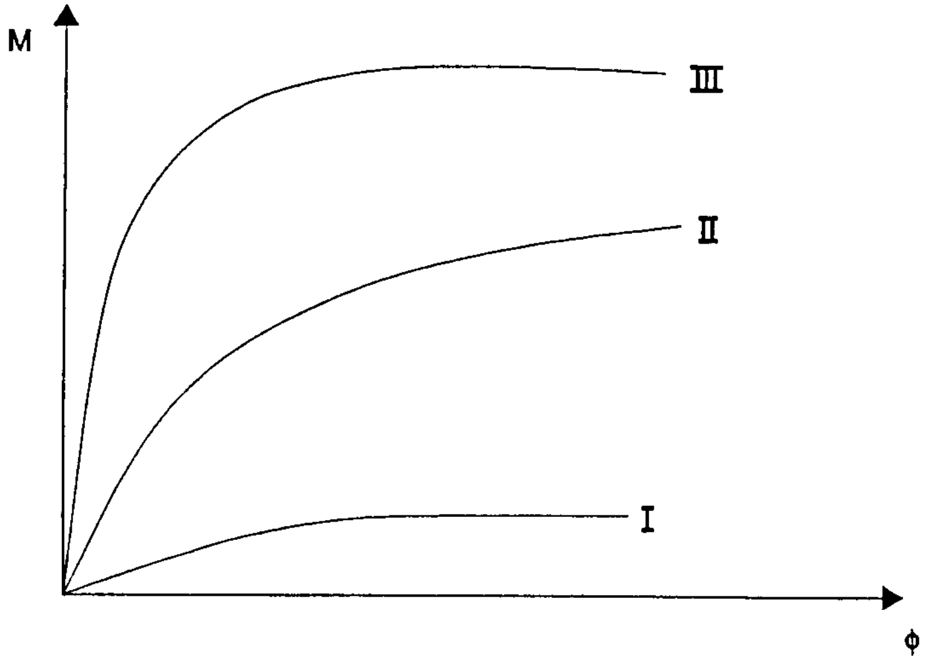


Frame stability

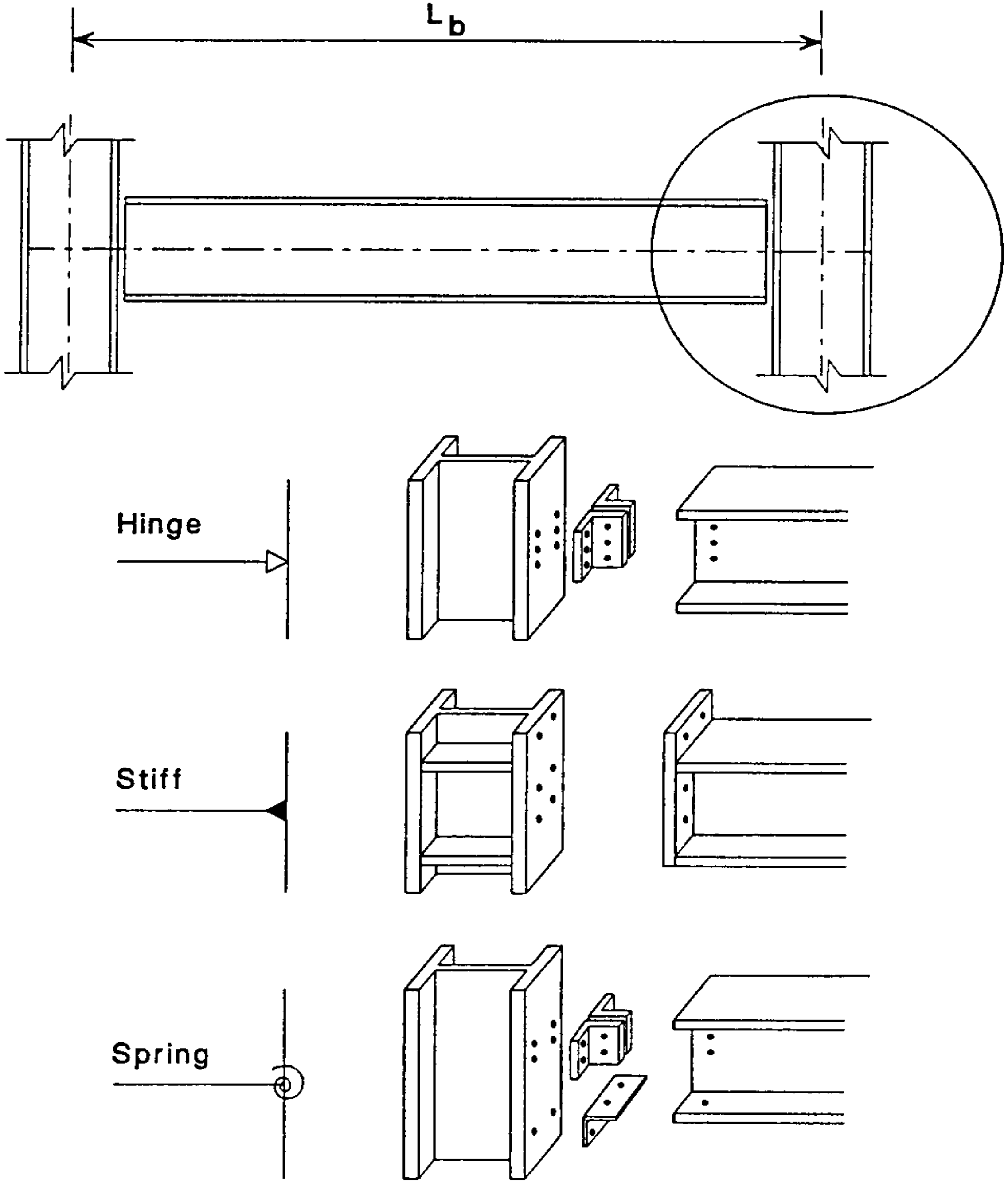
(c) Deformation Capacity

9.1.3 Classification as a Basis for Design

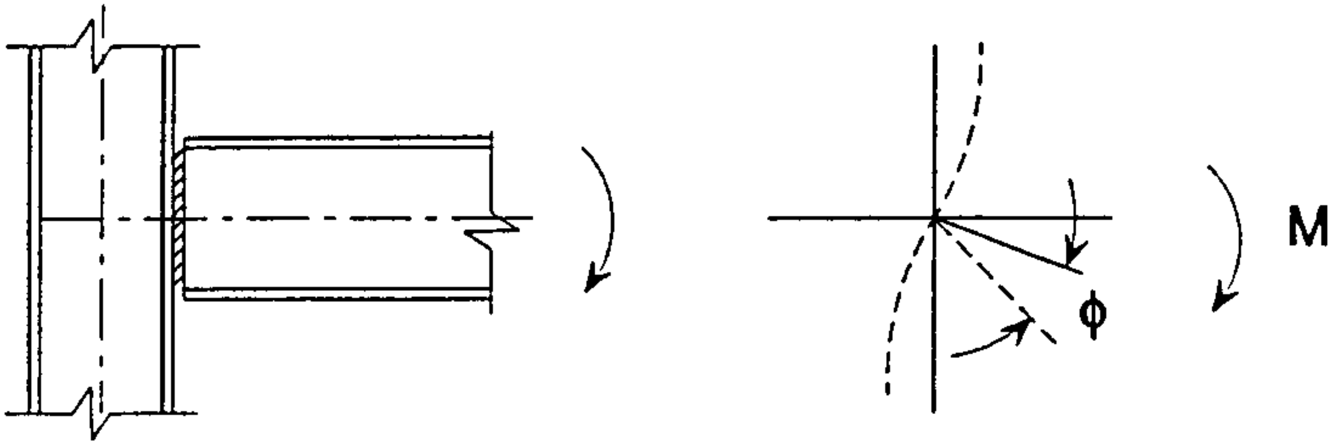


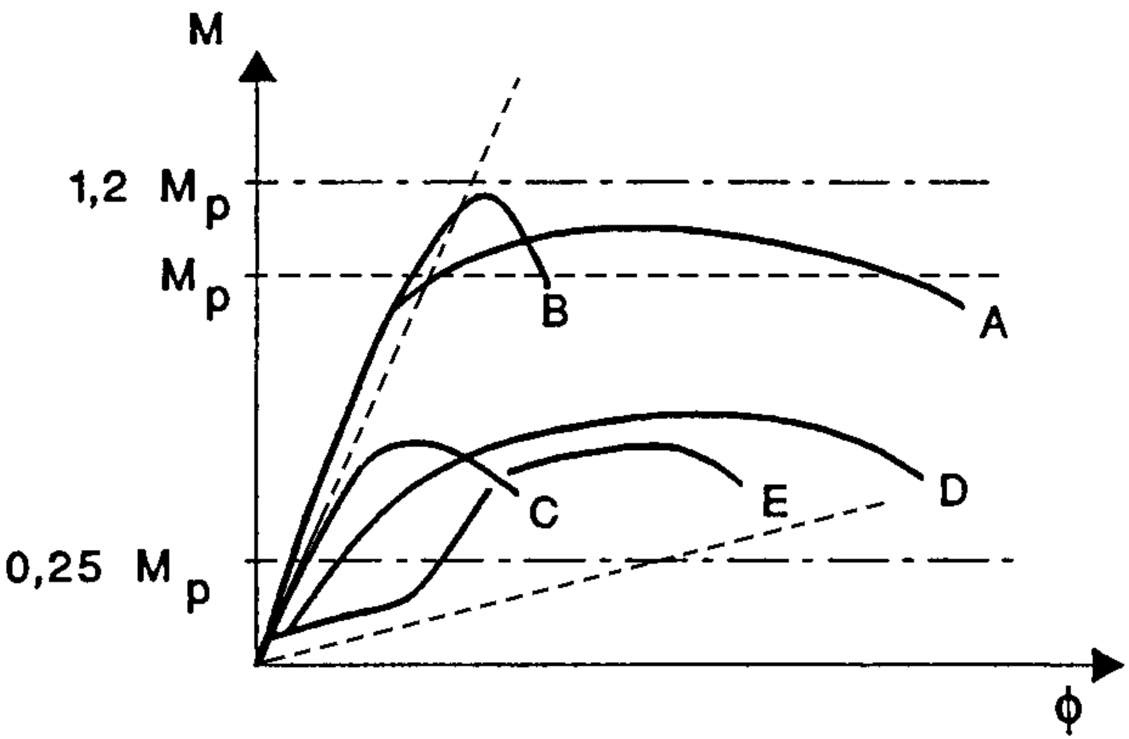


Moment–rotation diagrams (M–ϕ curves)

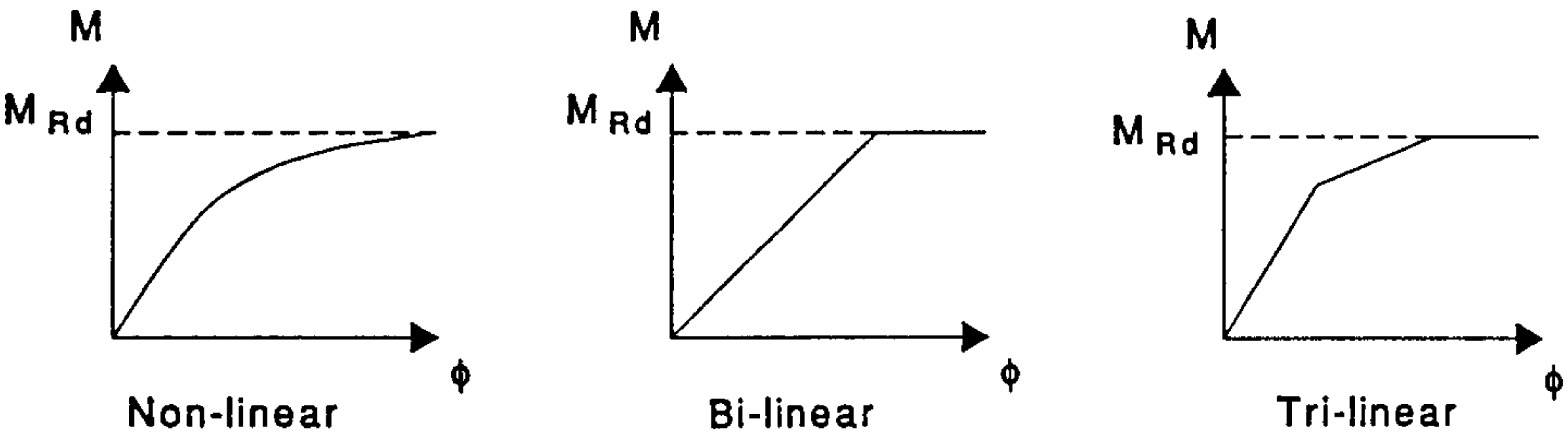


Schematisation of rotational stiffness





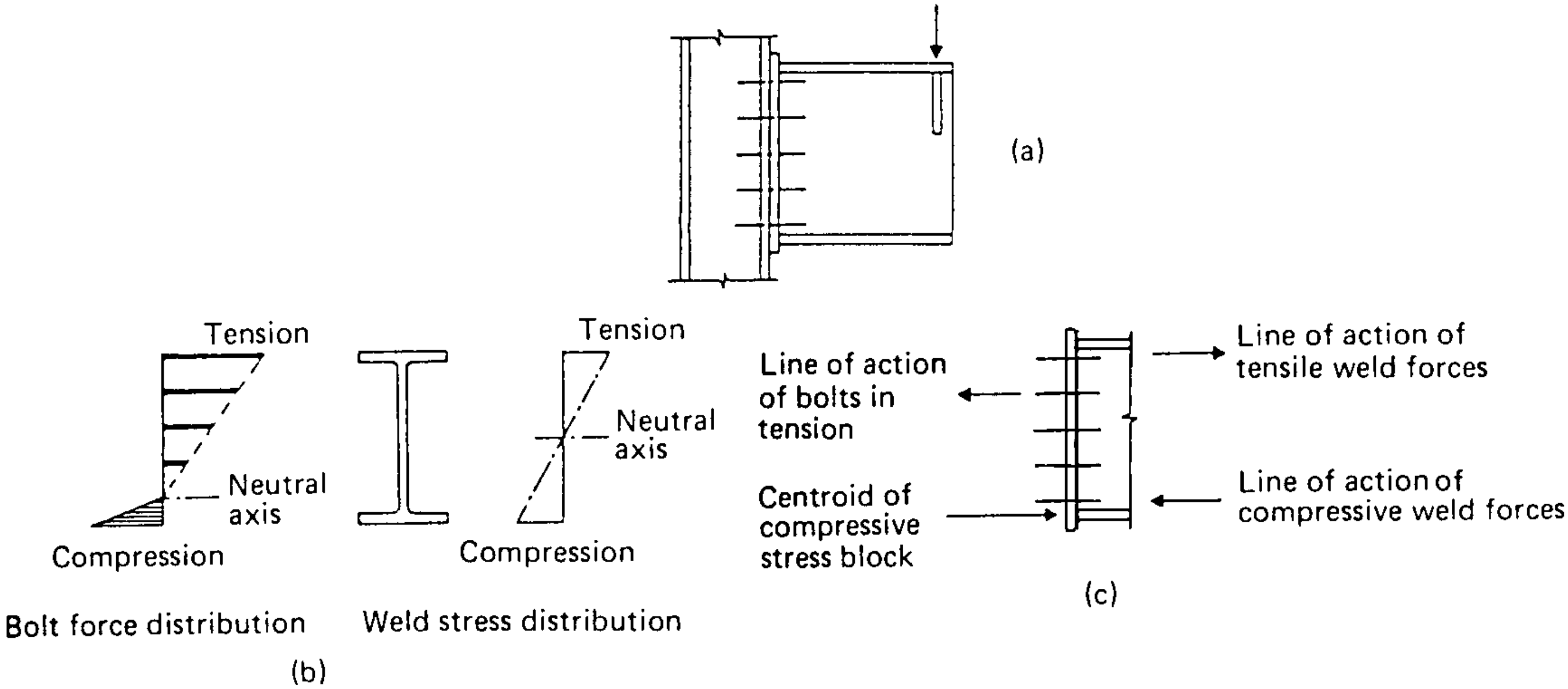
Various forms of M–ϕ curves



Possible idealisations for *M*–ϕ curves

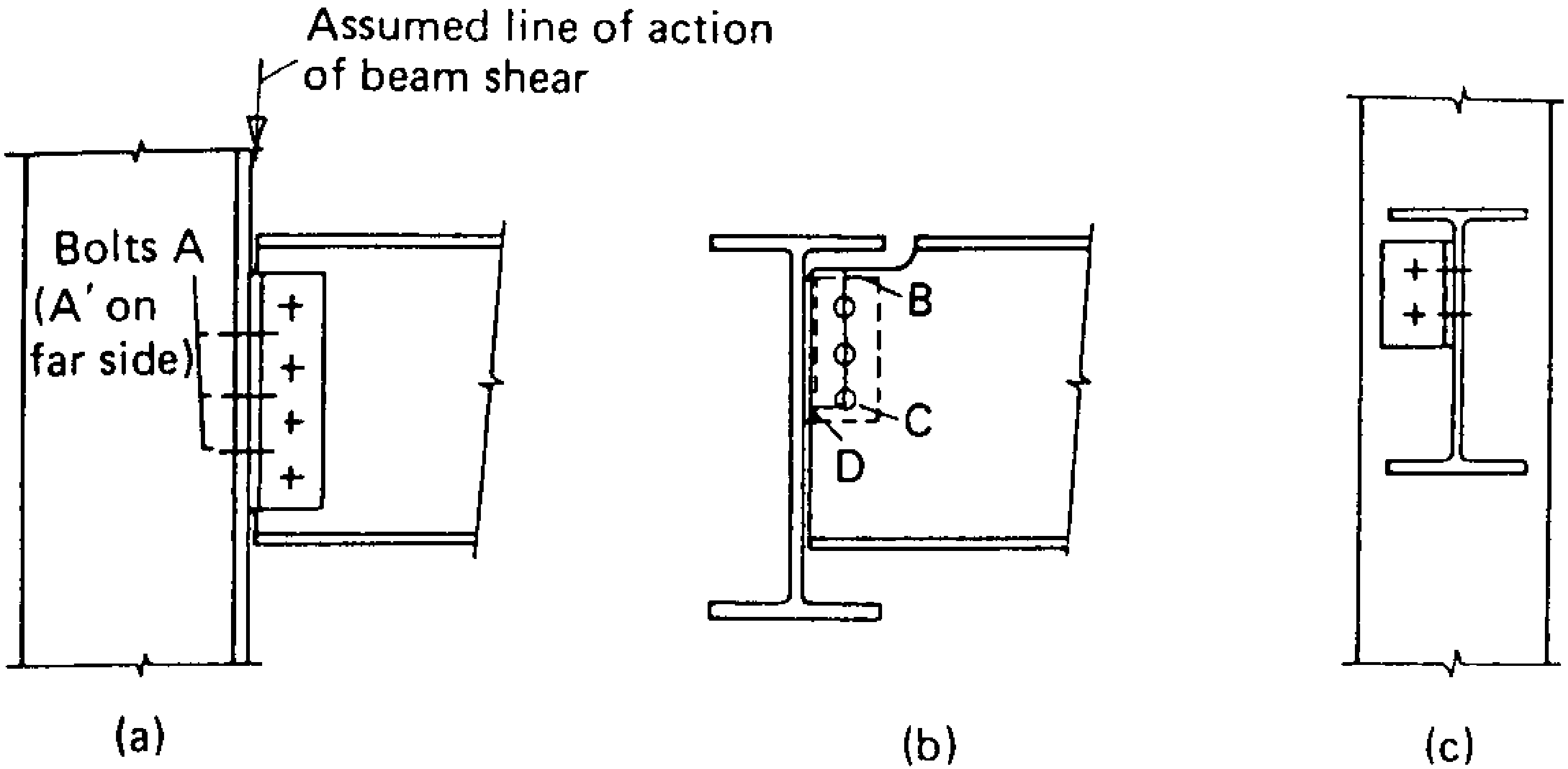
9.1.4 An Adapted Design Philosophy for Connections

1. *Taking account of overall connection behaviour, carry out an appropriate simple analysis to determine a realistic distribution of forces within the connection.*
2. *Ensure that each component of each force path has sufficient strength to transmit the required force*
3. *Recognising that the above procedure can only give a connection where equilibrium is capable of being achieved but where compatibility is unlikely to be satisfied, ensure that the components are capable of ductile behaviour.*
4. *Recognising that the preceding steps only relate to static ultimate capacity, ensure that the connection will achieve satisfactory serviceability, fatigue resistance, etc.*

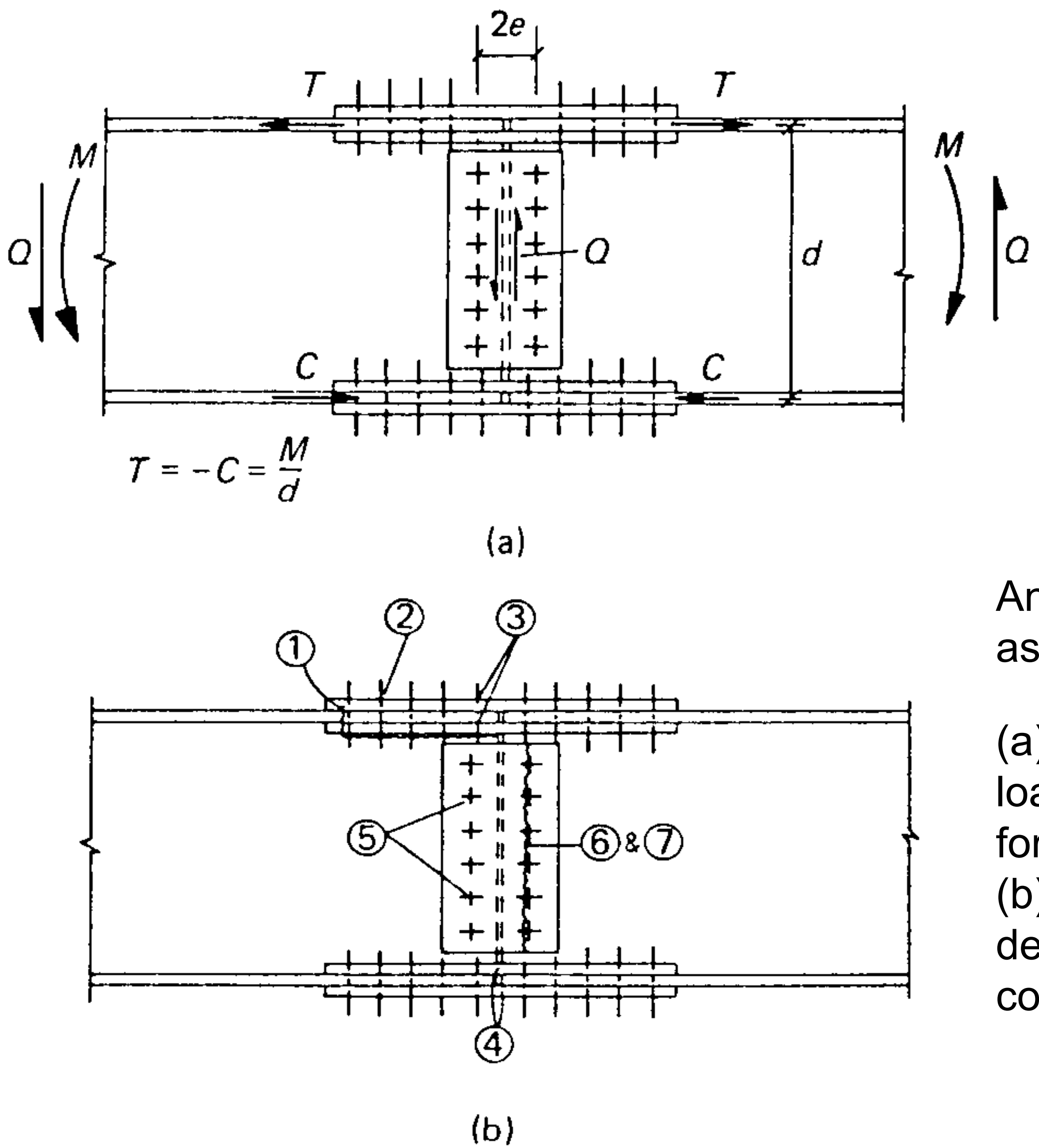


Inconsistency in connection analysis.
(a) Bracket connection; (b) conventional elastic analyses;
(c) stress resultants

9.1.5 Application of the Design Philosophy

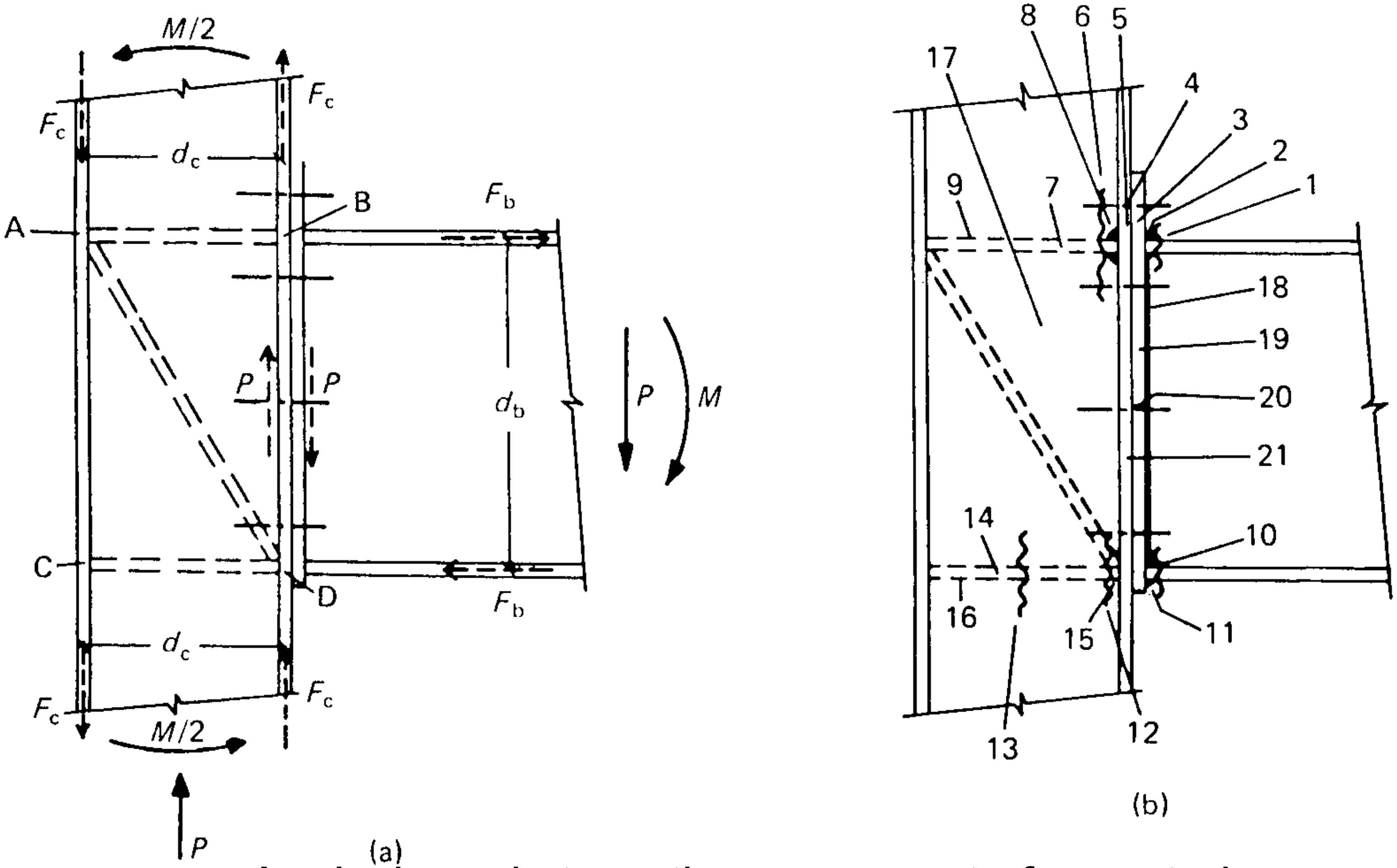


"Simple" beam-to-column and beam-to-beam connections.
(a) Conventional beam-to-column connection with double-web cleat;
(b) beam-to-beam grillage connection with double-web cleats;
(c) single-web cleat connection



Analysis and strength assessment of beam splice.

(a) Conversion of applied loading to equivalent system of forces;
(b) strength checks required to demonstrate adequacy of connection



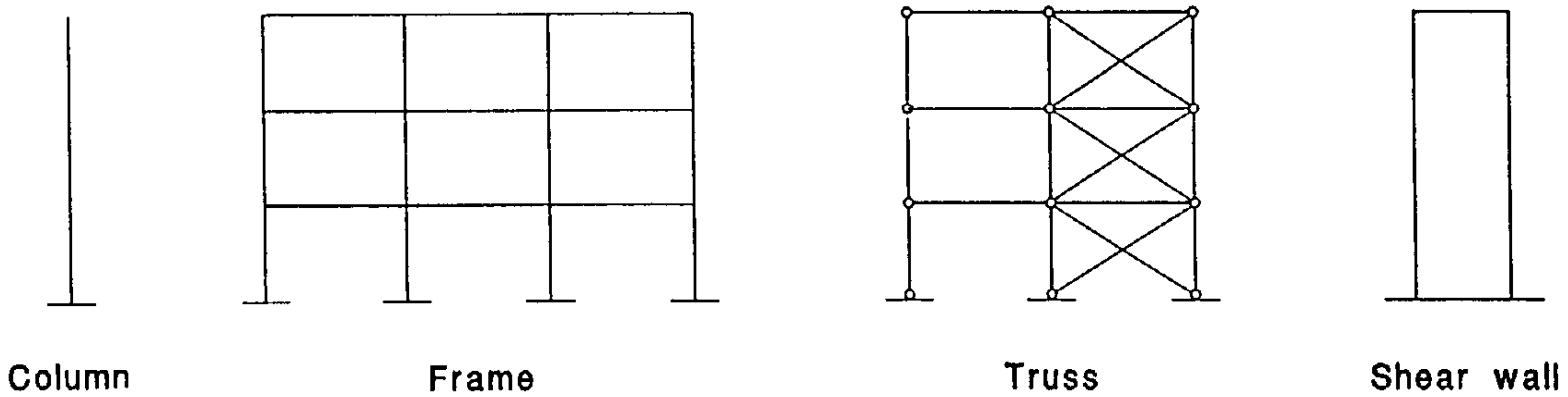
Analysis and strength assessment of an exterior beam-to-column connection

$$F_b = \frac{M}{d_b} \qquad F_c = \frac{M}{2d_c} \qquad \frac{F_b}{d_c} = \frac{M}{d_b \cdot d_c} \qquad \frac{2F_c}{d_b} = \frac{M}{d_c \cdot d_b}$$

9.2. Classification of Connections by Structural Eurocode

9.2.1 Classification of Framed Systems

(a) Introduction

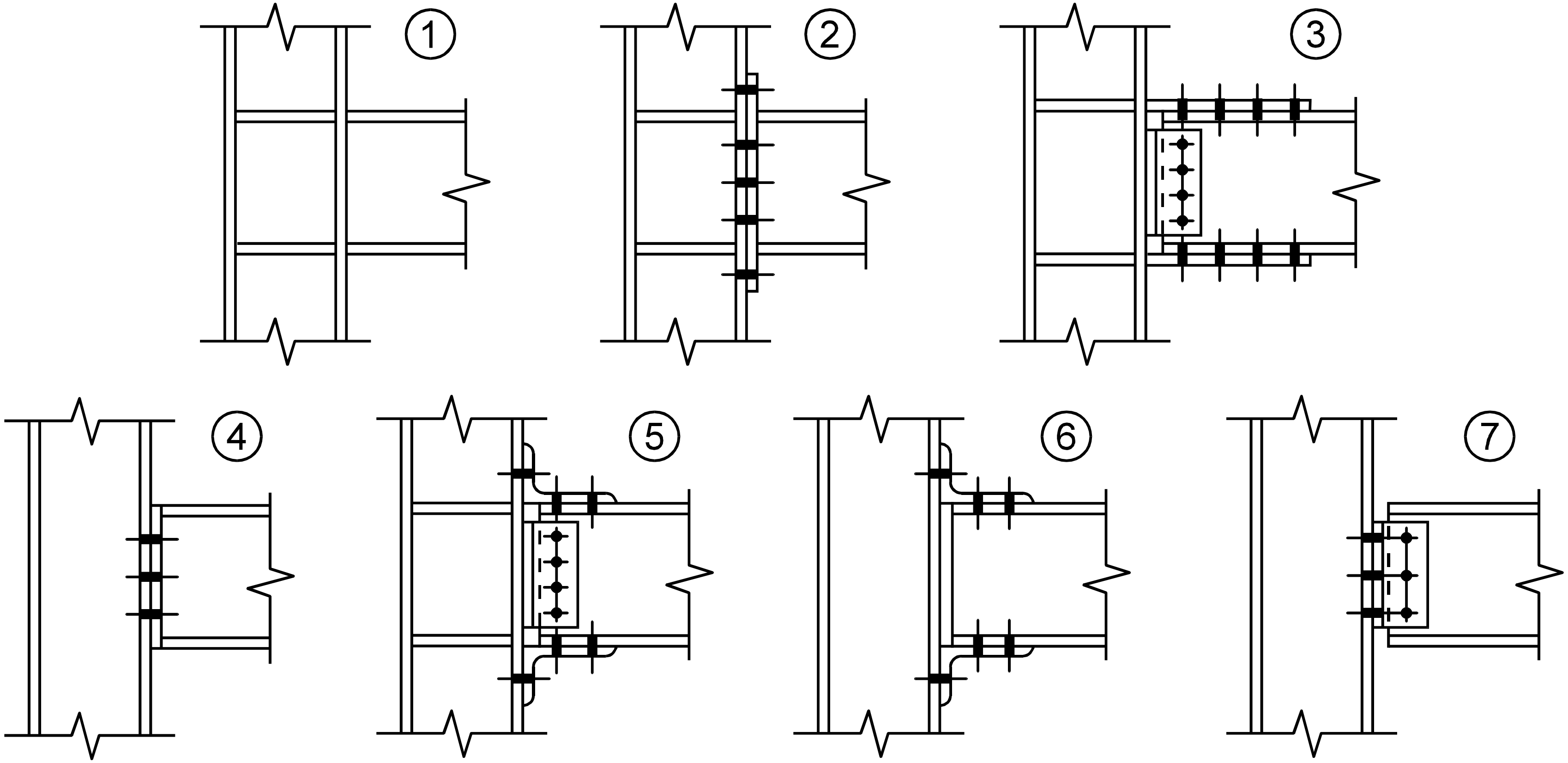


Continuous and hinged systems

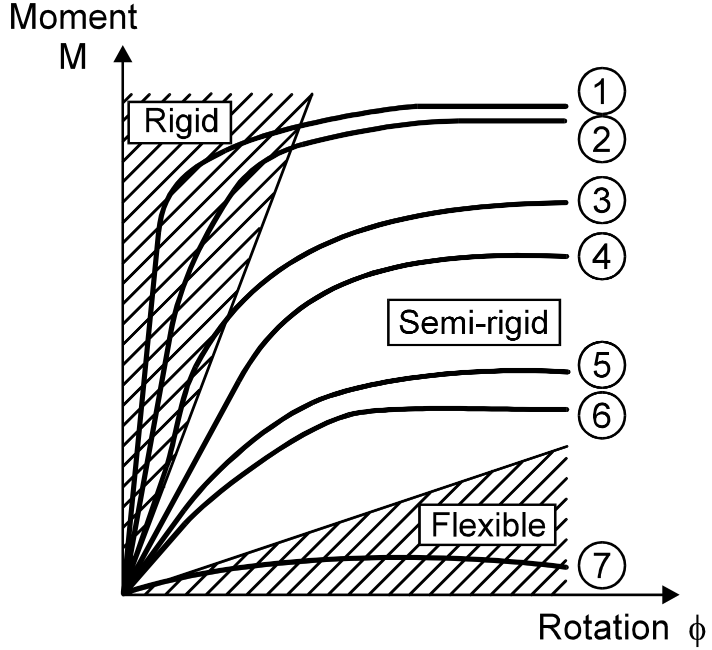
– *Engineering Definition* (ESDEP, 1994)
– Eurocode Definition (EC3, 1993)

– *Engineering Definition* (ESDEP, 1994)

Experimental M–ϕ relations of connections /a

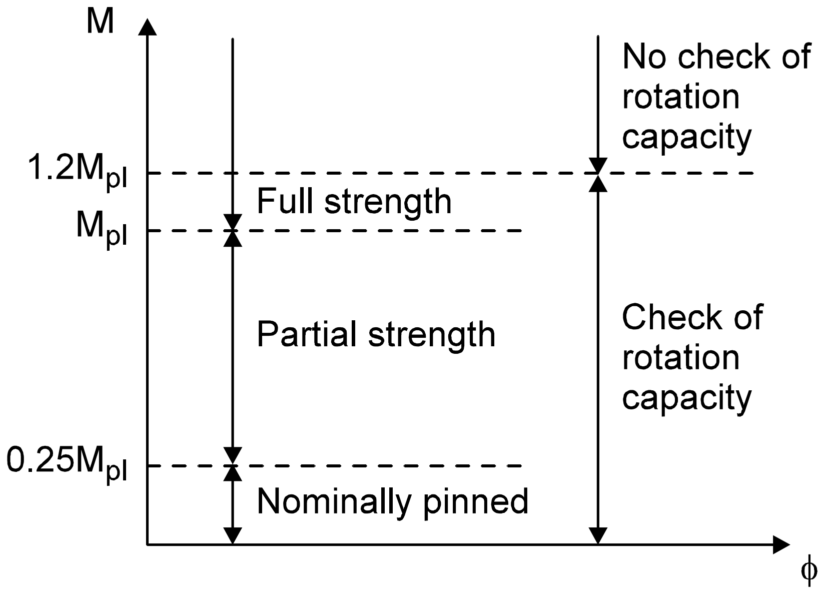


Experimental M–ϕ relations of connections /b

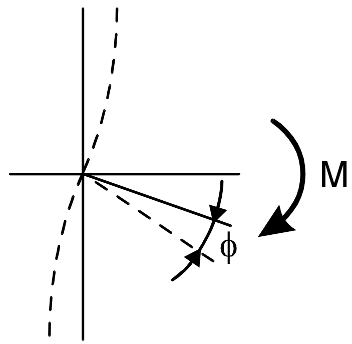


Classification of Stiffness

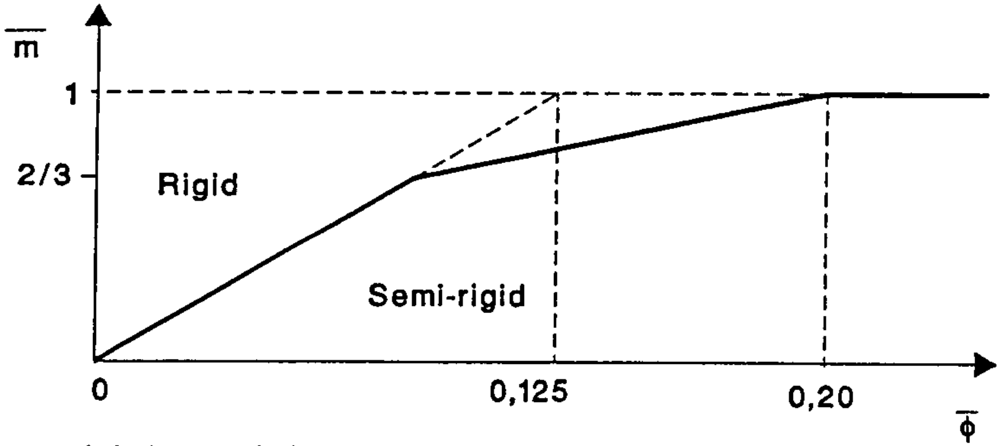
1. Fully welded
2. Extended end plate
3. Top and bottom flange splices
4. Flush end plate
5. Flange cleats and web angles
6. Flange cleats
7. Double web angle



Classification of Strength and Ductility



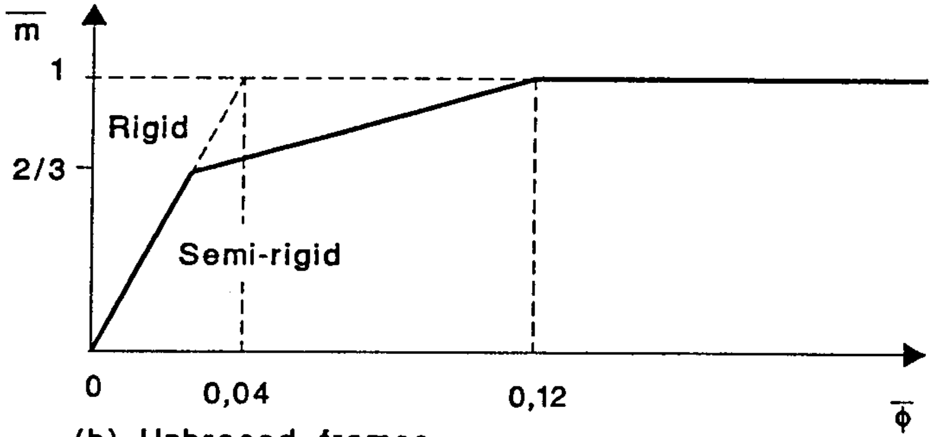
– Eurocode Definition (EC3, 1993)



(a) Braced frames

when $\overline{m} \leq 2/3$: $\overline{m} = 8\,\overline{\phi}$

when $2/3 < \overline{m} \leq 1{,}0$: $\overline{m} = (20\,\overline{\phi}+3)/7$



(b) Unbraced frames

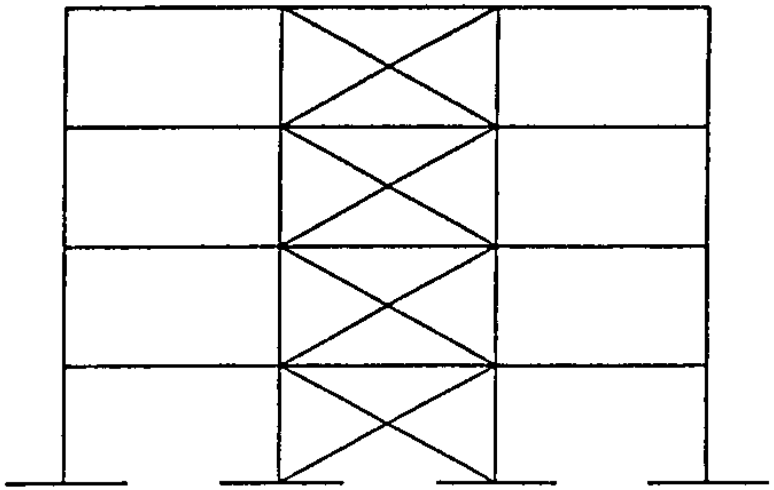
when $\overline{m} \leq 2/3$: $\overline{m} = 25\,\overline{\phi}$

when $2/3 < \overline{m} \leq 1{,}0$: $\overline{m} = (25\,\overline{\phi}+4)/7$

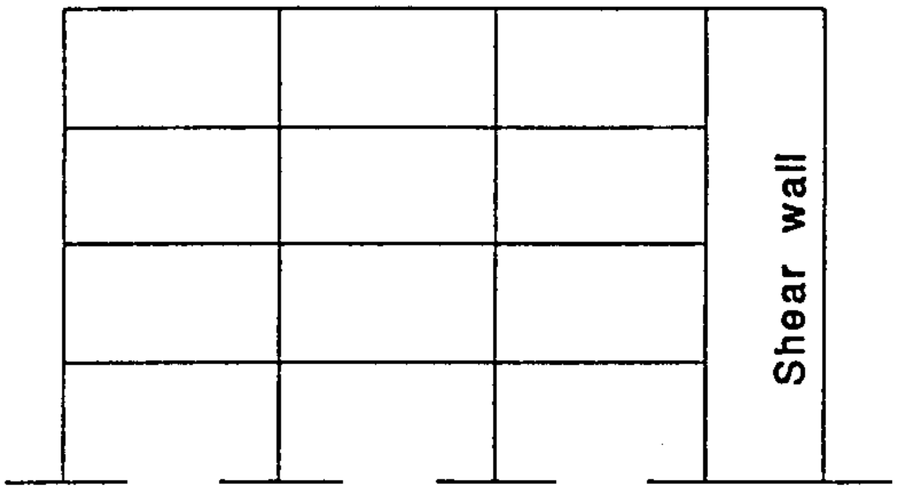
$$\overline{m} = \frac{M}{M_{p\ell.Rd}} \qquad \overline{\phi} = \frac{EI_b}{L_b M_{p\ell.Rd}}$$

Eurocode classification boundaries for rigid beam-to-column connections in unbraced and braced frames

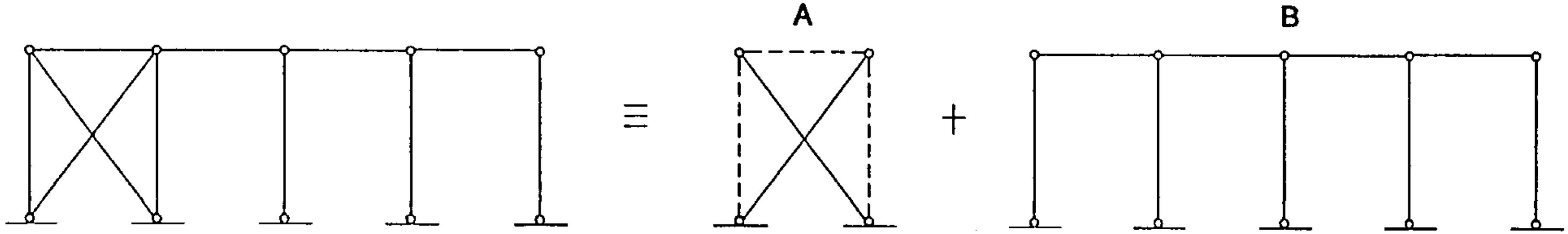
(a) Braced and unbraced Frames



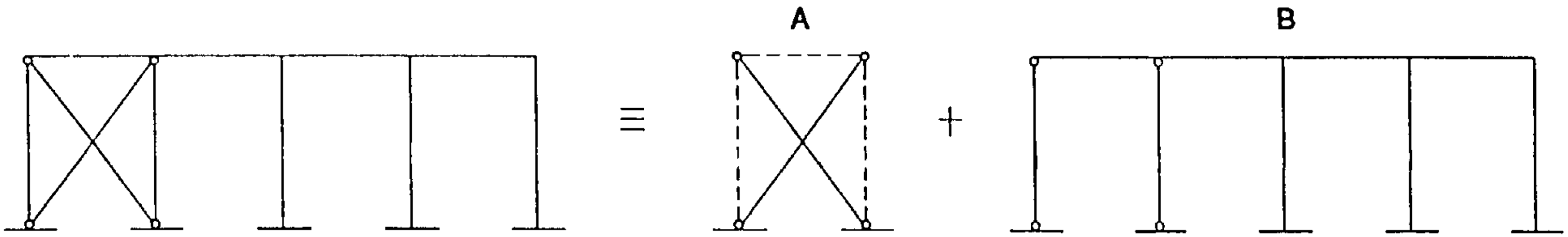
Common bracing systems
[ESDEP, 1994]



– *Engineering Definition* (ESDEP, 1994)



Pinned connection structure split into two sub-assemblies



Partly framed structure split into two sub-assemblies

– *Eurocode Definition* (EC3, 1993)

$$K_a > 0.8(K_a + K_b) \qquad K_a > 4K_b$$

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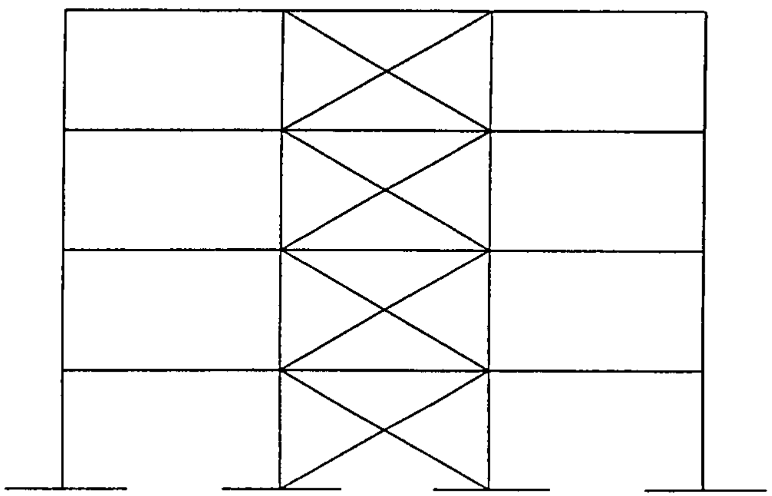
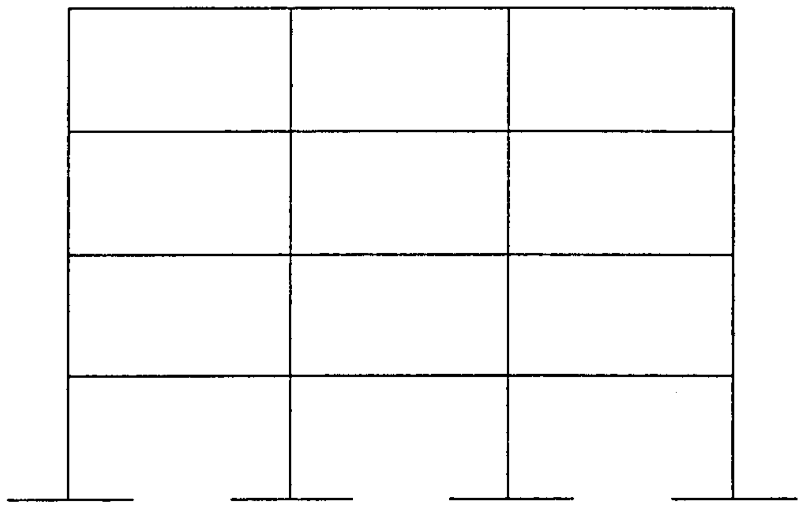
(b) Sway and Non-sway Frames



Braced frame (but may be a sway frame if bracing is very flexible)



Unbraced frame (but may be a non-sway frame if it is sufficiently rigid i.e. insensitive to horizontal loading)

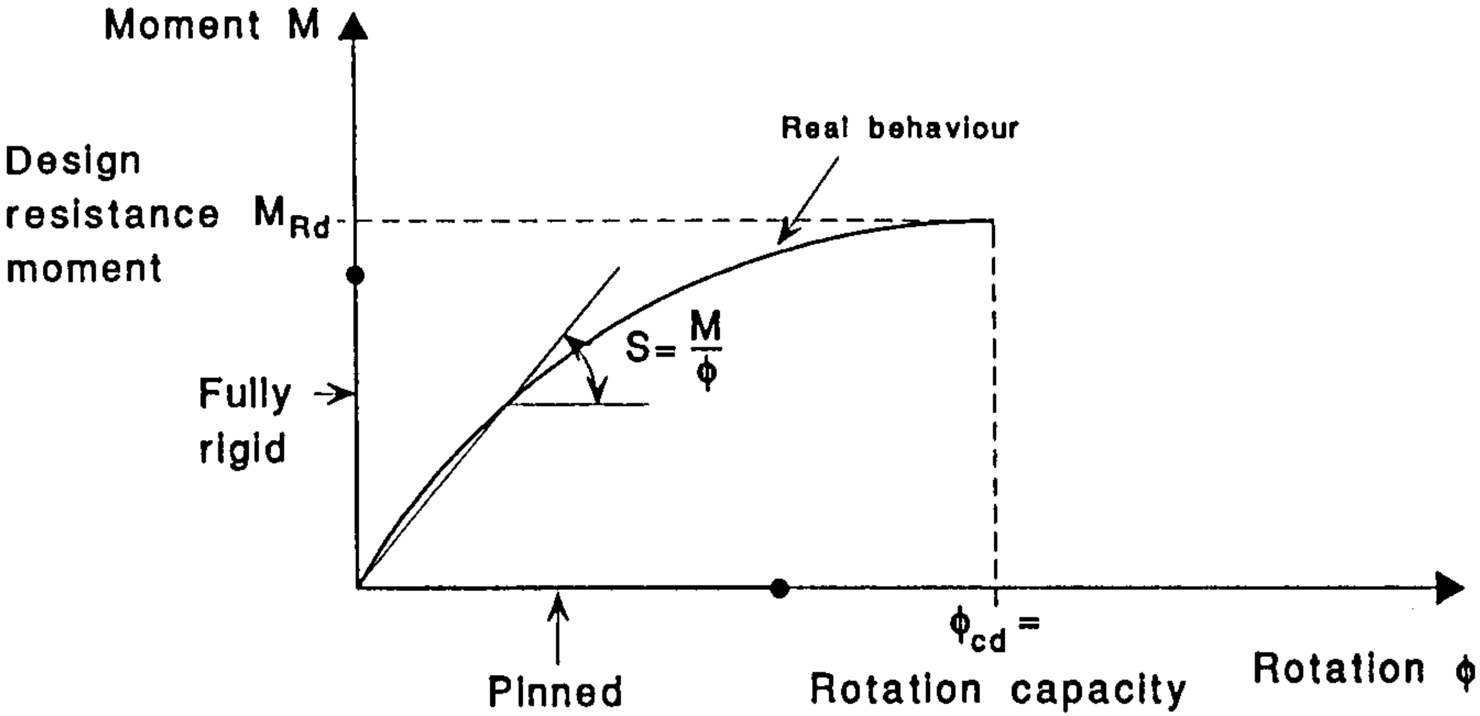
– *Engineering Definition* (ESDEP, 1994)

– *Eurocode Definition* (EC3, 1993)

$$\frac{V_{Sd}}{V_{cr}} \leq 0.1$$

9.2.2 Influence of Connections on the Behaviour of Frames

(a) Introduction

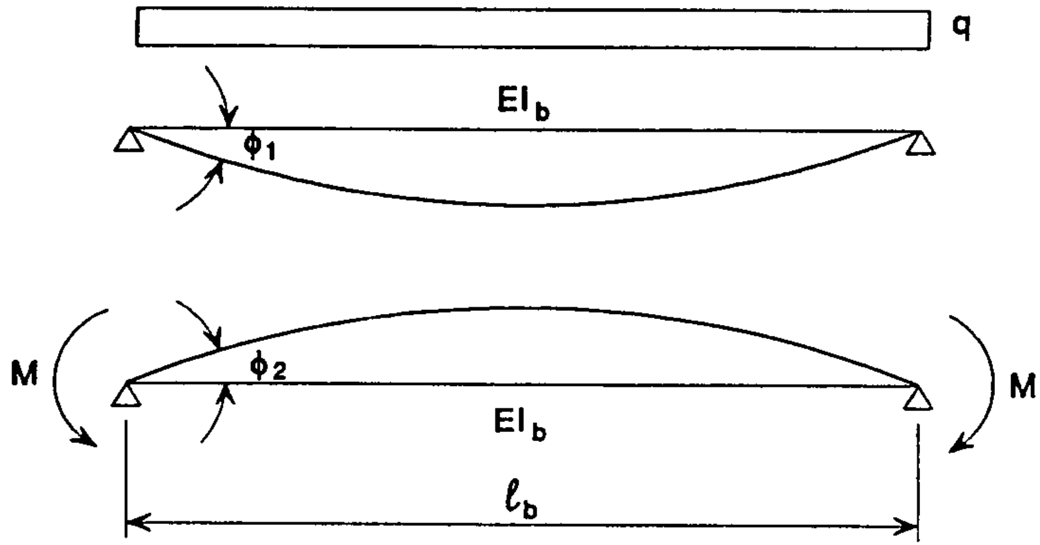


Characteristics of beam-to-column connections

(b) Classification of Connections

– Influence of Connection Flexibility on Elastic Frame Stability

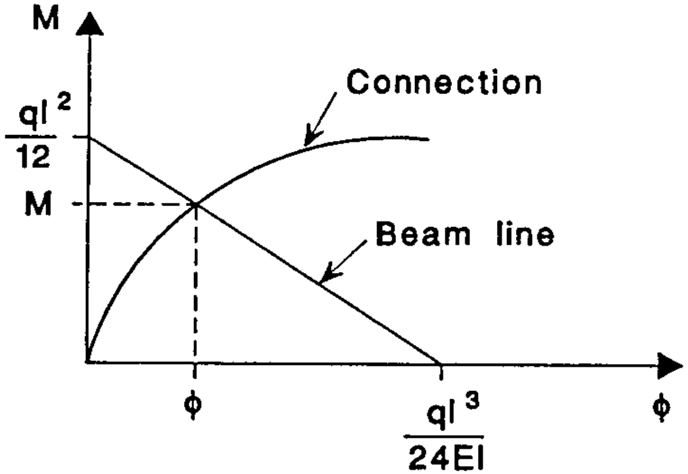
Beam-line and connection behaviour



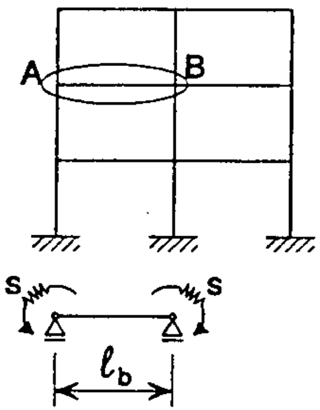
$$\phi_1 = \frac{q\ell_b^3}{24EI}$$

$$\phi_2 = \frac{M\ell_b}{2EI}$$

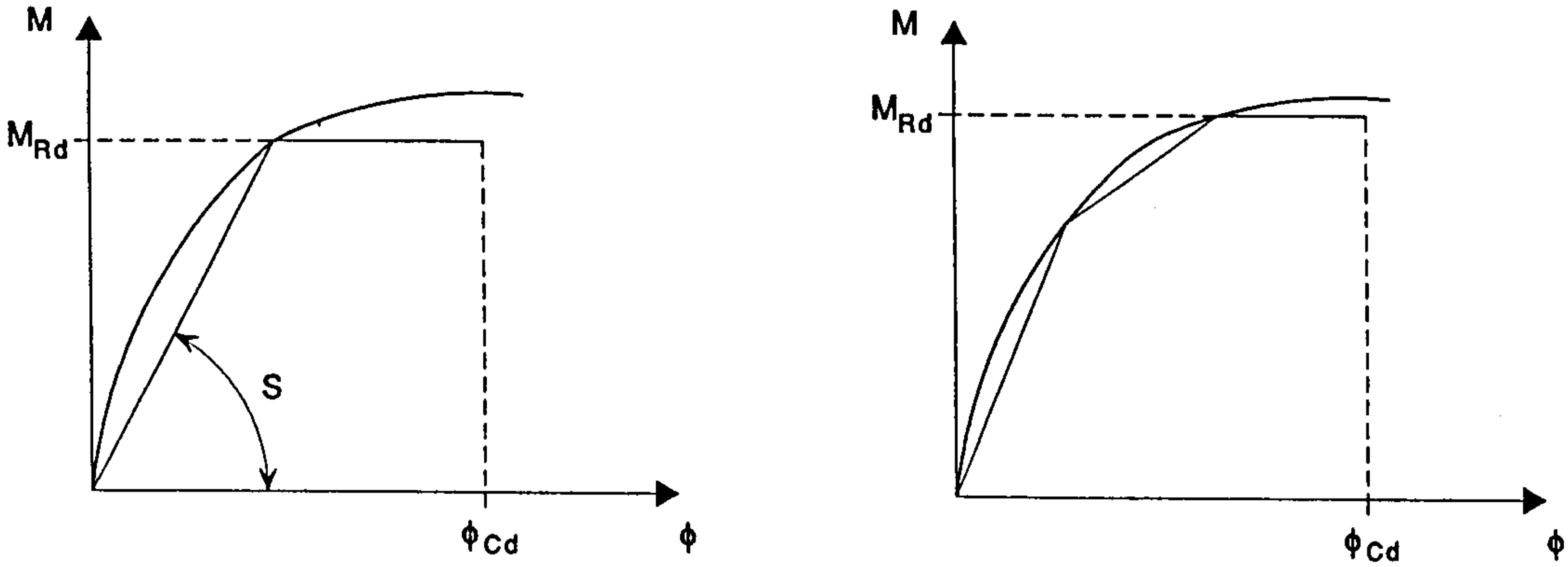
(a)



(b)



(c)



M_{Rd} = Connection resistance

ϕ_{Cd} = Rotational capacity of the connection

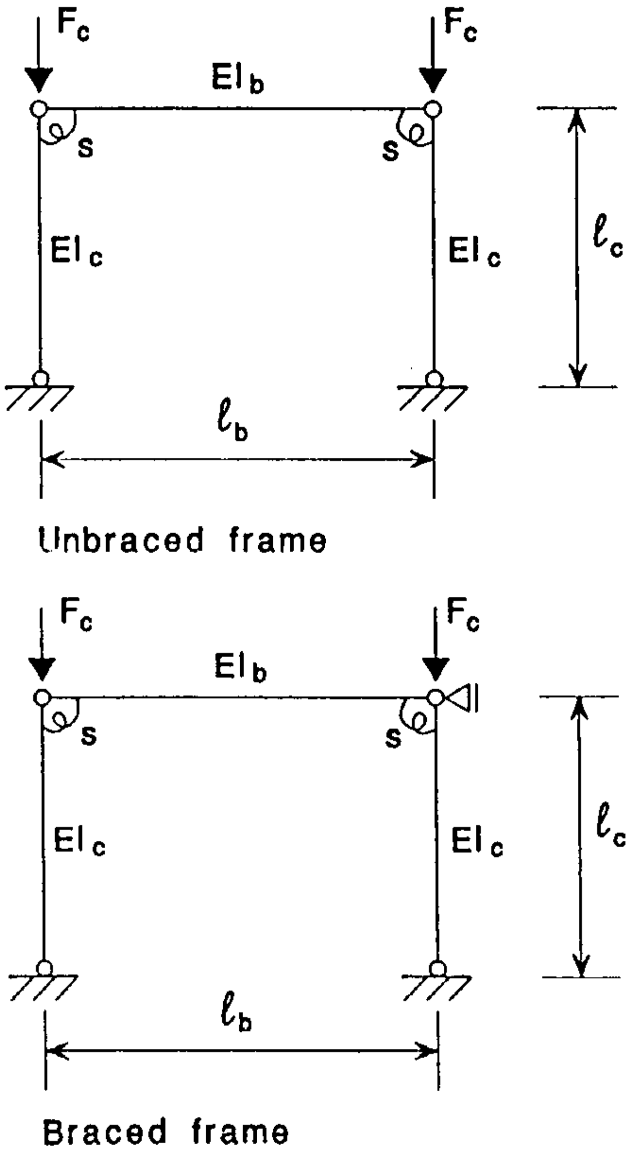
S = Connection rigidity

(a) Bi-linear response

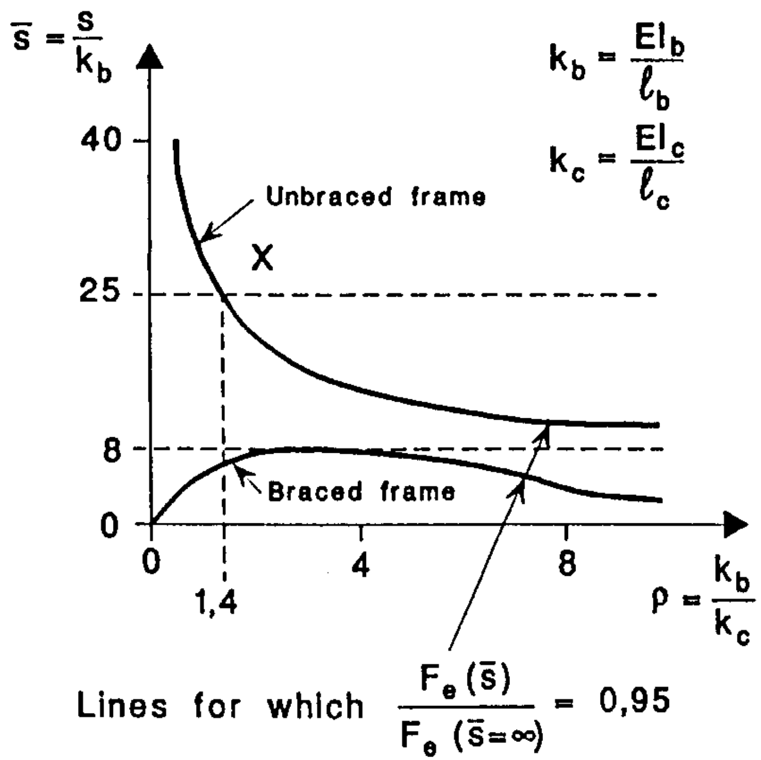
(b) Tri-linear response

Derivation of approximate moment–rotation characteristics

[ESDEP, 1994] [Bjorhovde, Colson, 1989]



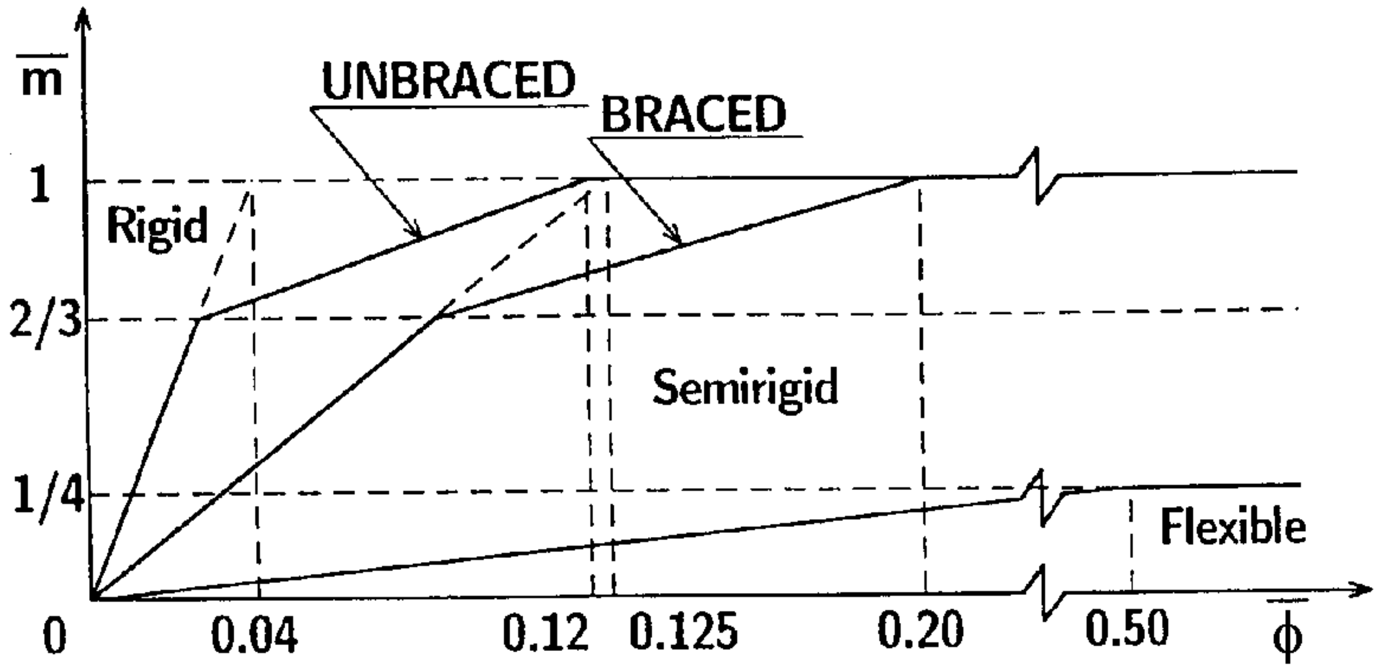
(a)



(b)

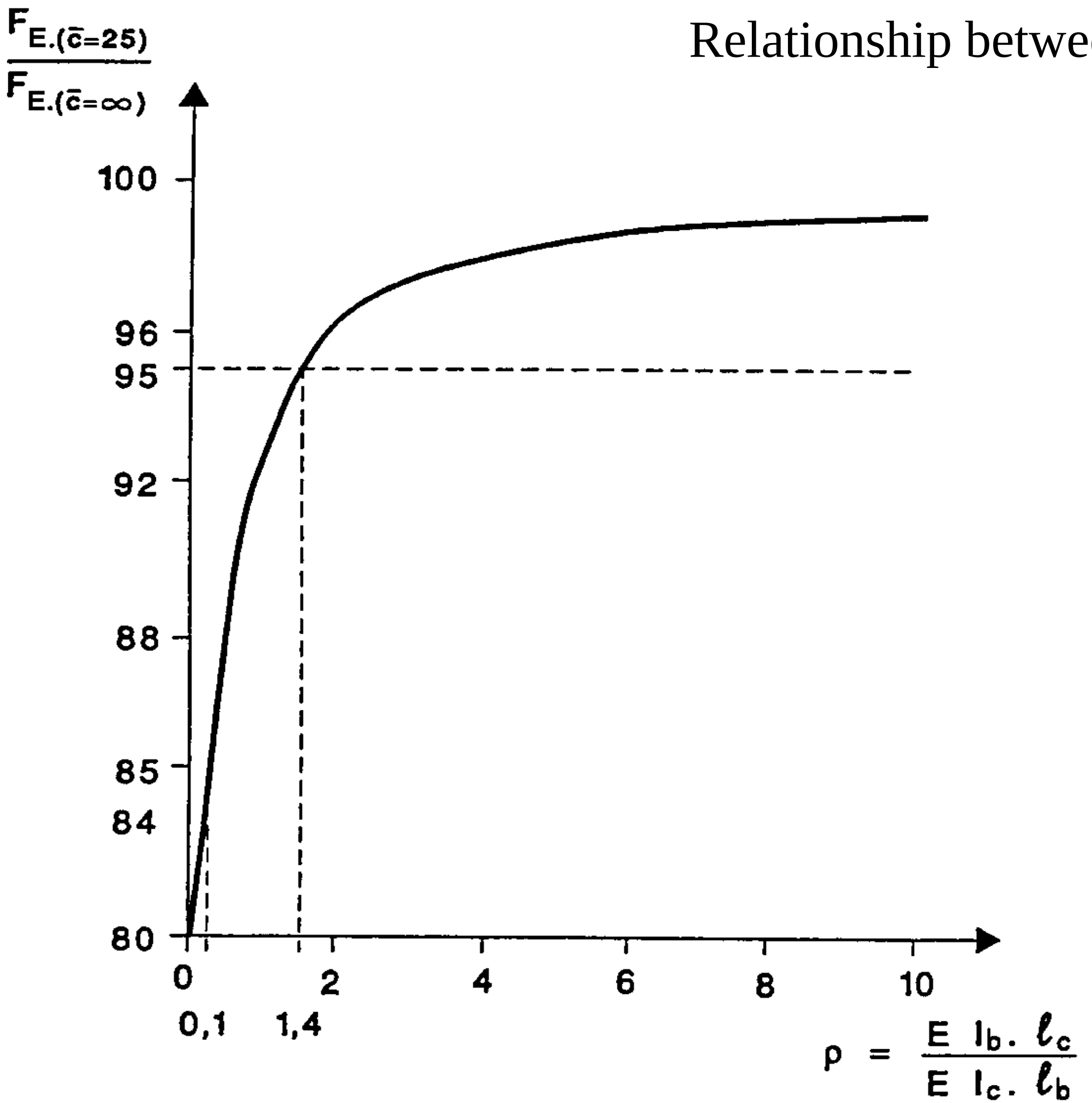
Influence of connection rigidity on frame behaviour

– Influence of Connection Flexibility on Frame Strength



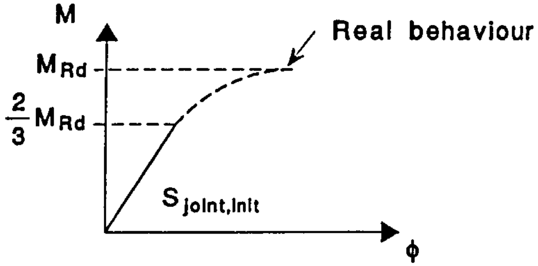
Classification boundaries for connections in respect to their rigidity

Relationship between ρ and $\frac{F_{E(\bar{s}=25)}}{F_{E(\bar{s}=\infty)}}$



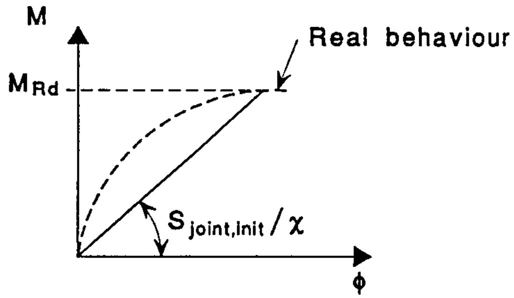
– Influence of Connection Strength on Frame Behaviour

(c) Modelling of Connections



(a) Elastic global analysis, elastic verification of the joint

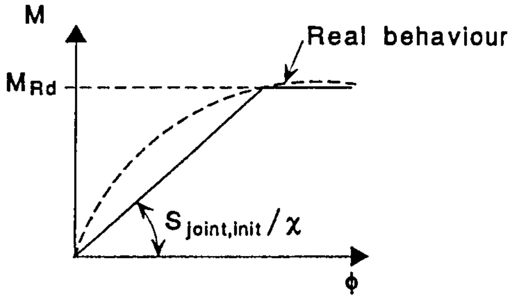
$$\frac{2}{3}M_{Rd} \geq 1.2M_{pl,b}$$



χ - value		
Type of connection	Braced	Unbraced
Welded	3	2
Endplate	3	2

(b) Elastic global analysis, plastic verification of the joint

$$M_{Rd} \geq 1.2M_{pl,b}$$

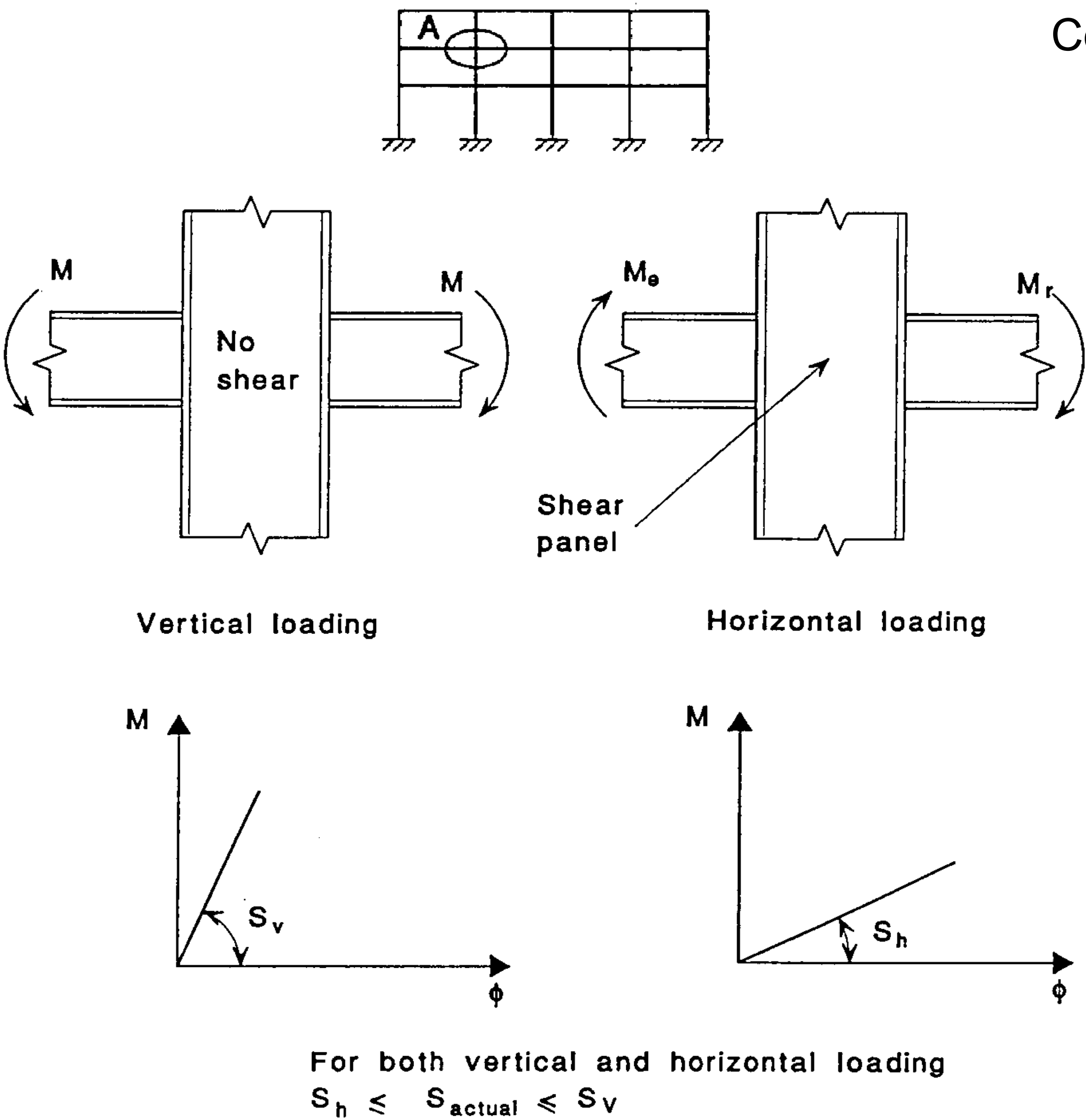


k - value		
Type of connection	Braced	Unbraced
Welded	3	2
Endplate	3	2

(c) plastic global analysis, elastic or plastic verification of the joint

Note : For plastic global analysis, models (a) and (c) may be used provided that the joints are full strength with $M_{Rd} > M_{pl,b}$

Connection rigidity for unbraced frames



(d) Relation between Frame and Connection Behaviour

[ESDEP, 1994] [Bijlaard, Zoetemeijer, 1986]

– Plastically Designed Connections in Elastically Designed Frames

– Elastically Designed Connections in Plastically Designed Frames

9.3. Failure Tests of Two-Storey Steel Frames

9.3.1 Introduction

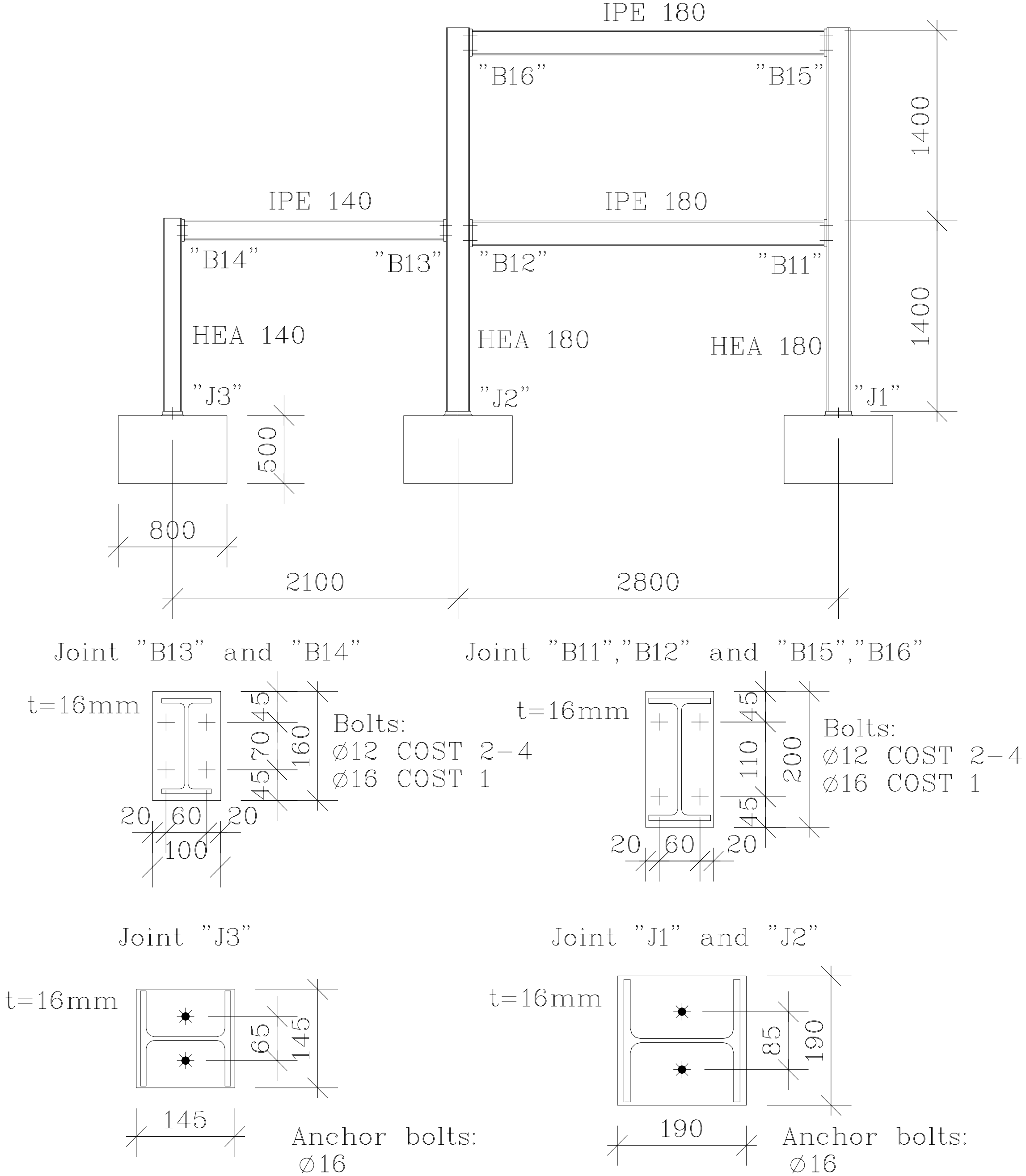
Full-scale frame tests:

- It enables the effect of column continuity through a loading level to be investigated
- it confirms whether the experimentally observed performance of isolated joints and sub-frames is indeed representative of their behaviour when they form part of an extensive frame

9.3.2 General arrangement of experiments

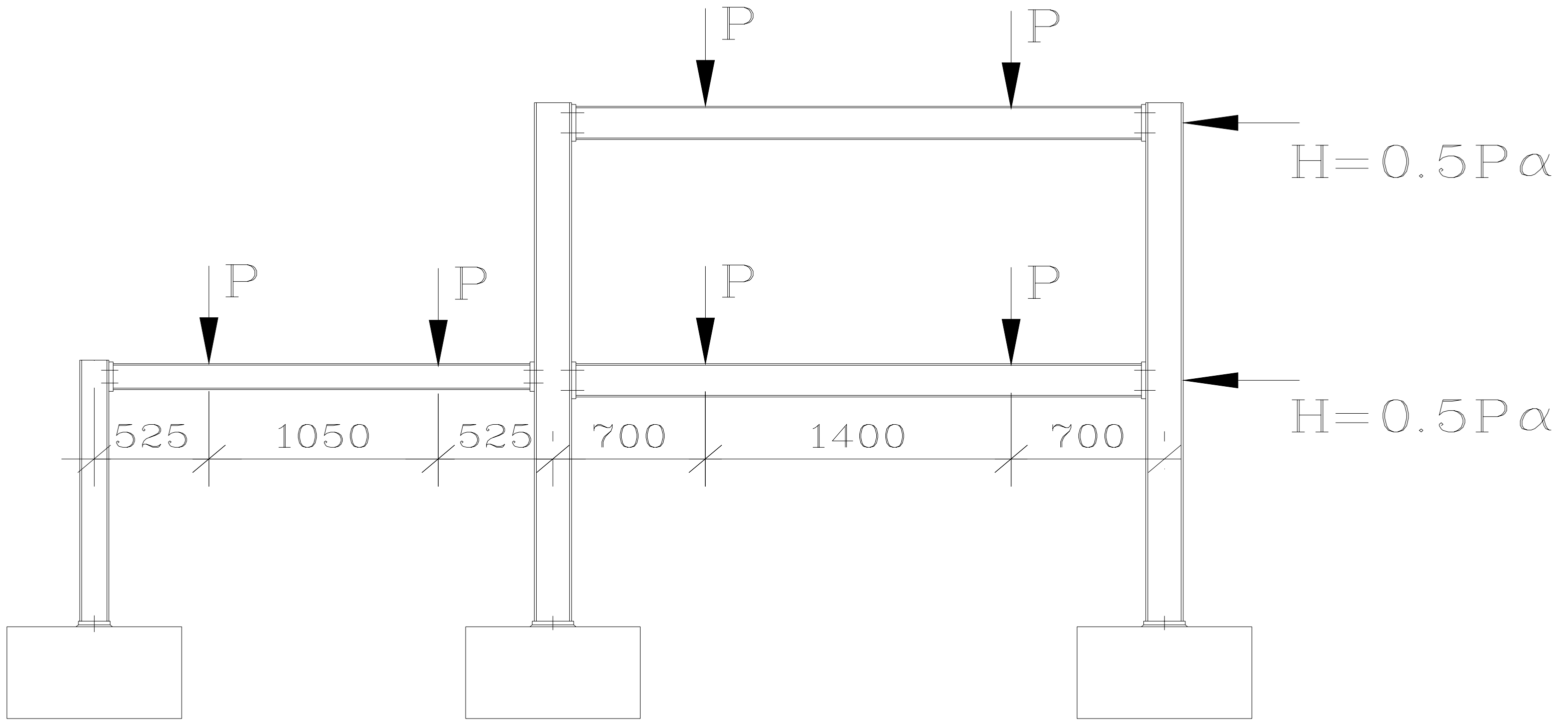
(a) Test Program and Description of Test Frames

- Frame COST 2: proportional loading process, horizontal load ratio: $R=H_{min}/H_{max}=1$
- Frame COST 3: pulsating loading process, horizontal load ratio: $R=H_{min}/H_{max}=0$
- Frame COST 4: alternating loading process, horizontal load ratio: $R=H_{min}/H_{max}=-1$



Overall view of test frames

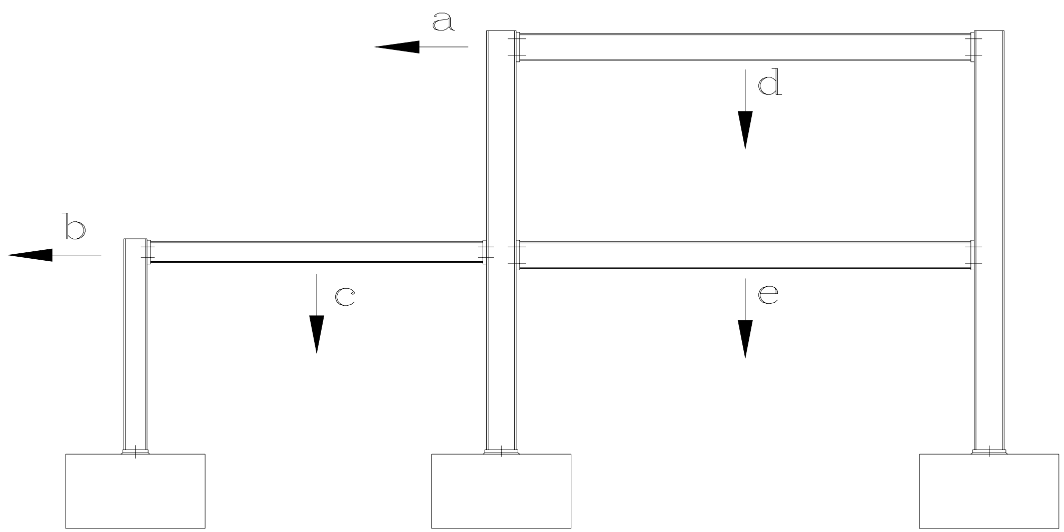
(b) Loading System



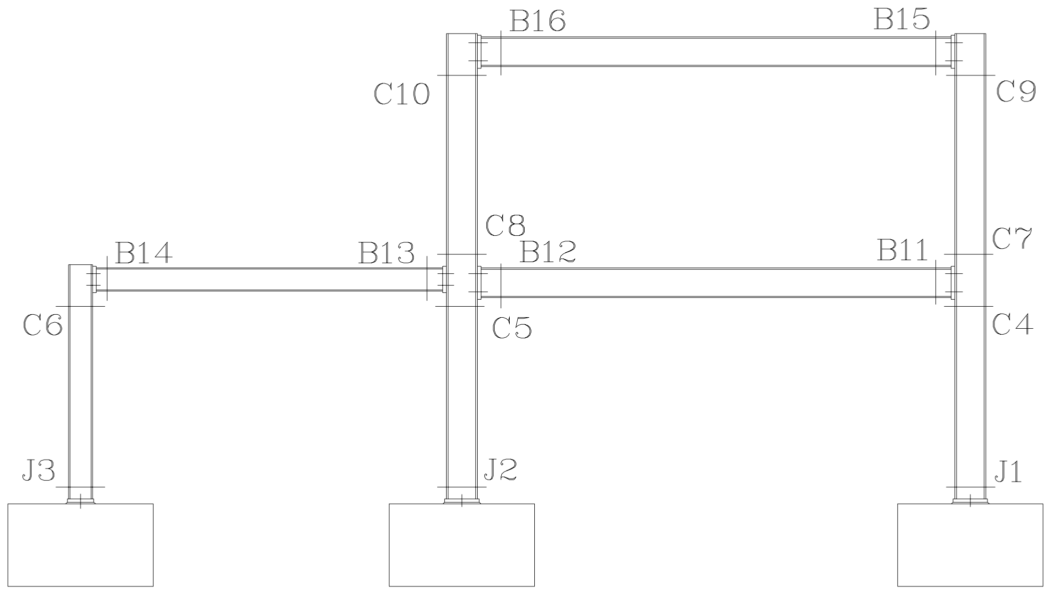
Loading arrangement

(c) Measuring Techniques

Measured displacements



Cross-sections with relative rotation measuring scales



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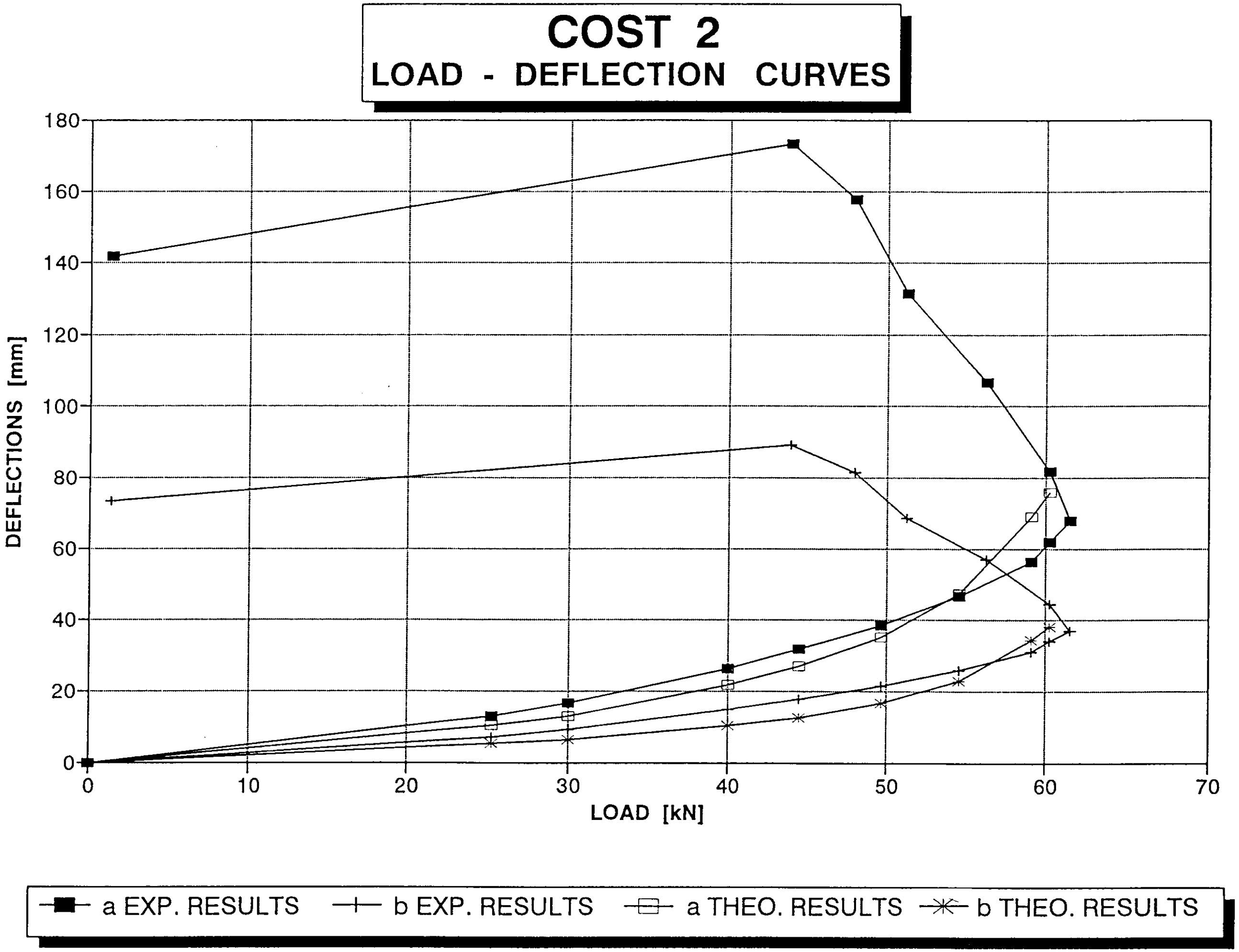
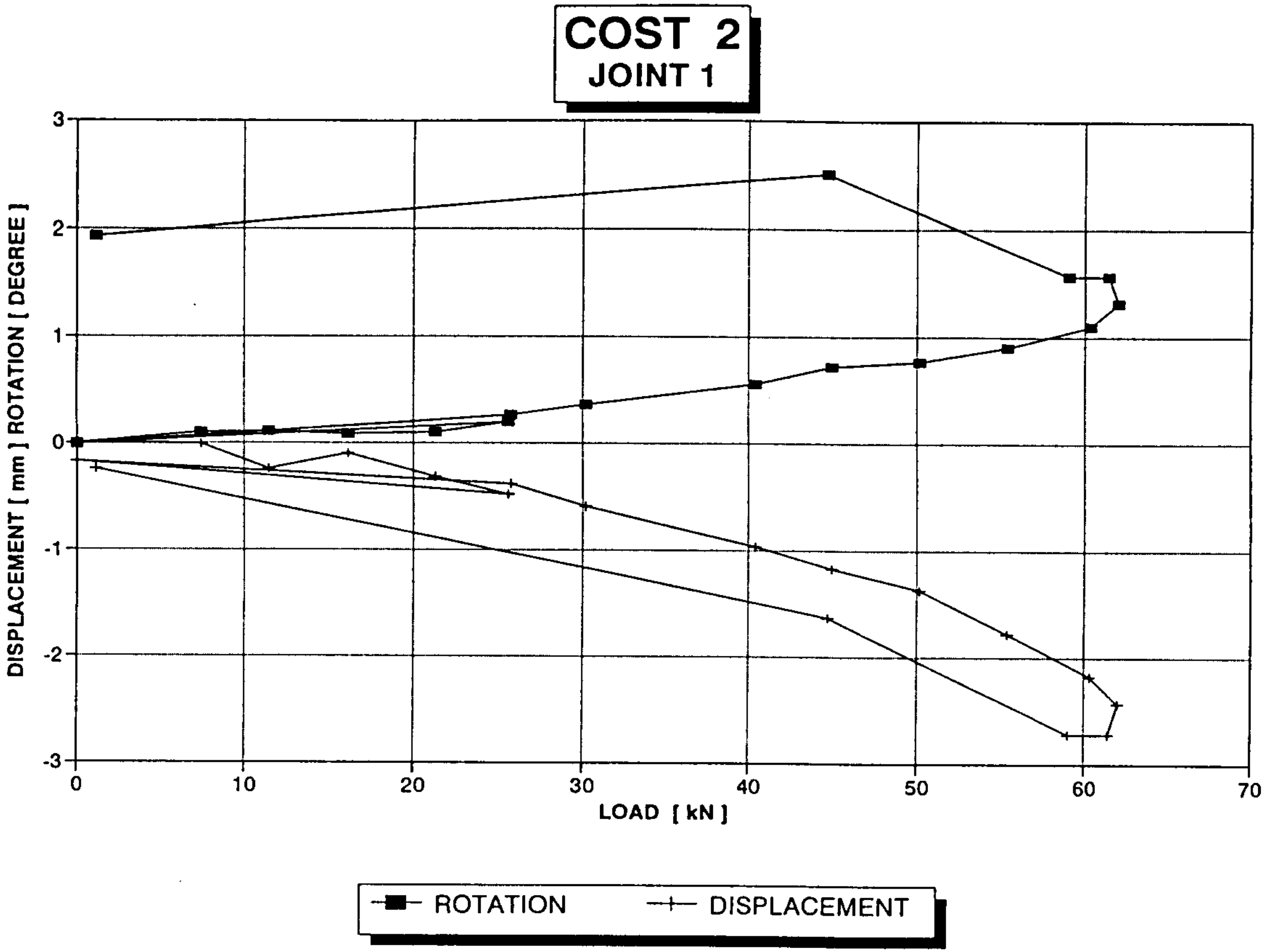
9.3.3 Some results of the experiments

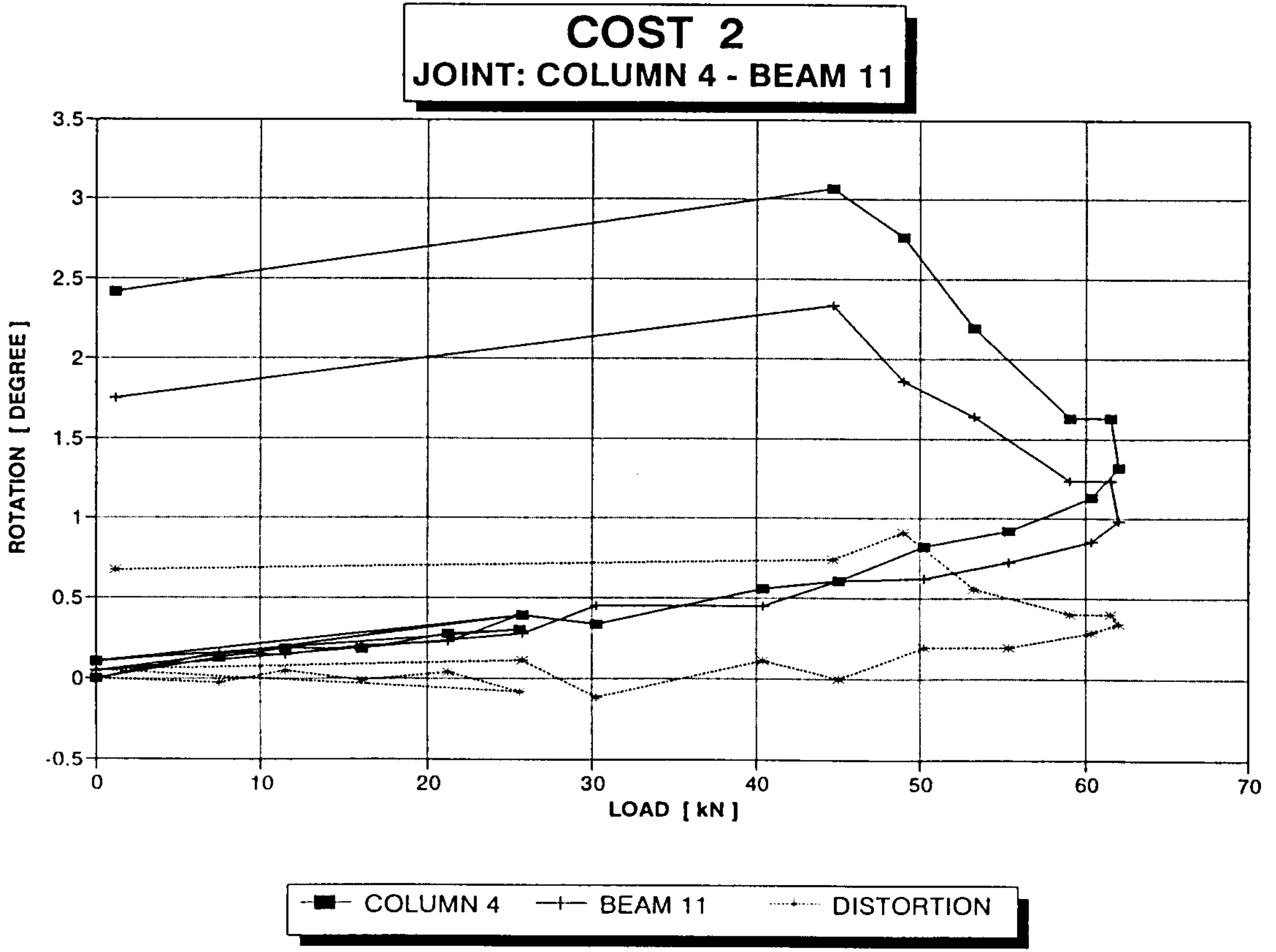
(a) Test Frame COST 2: Proportional Loading Process



Load – displacement curves

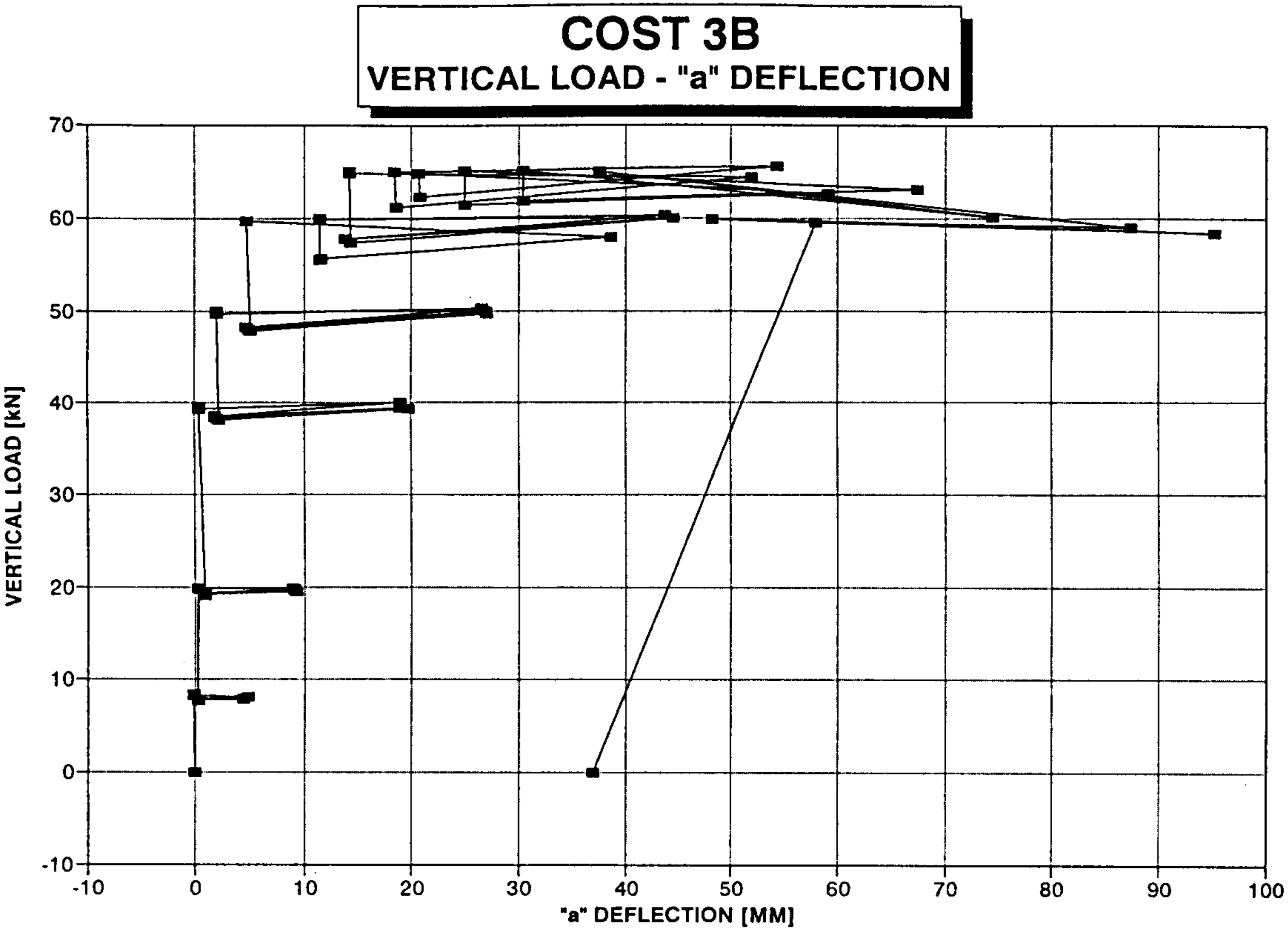


Load – relative displacement curves at the column bases

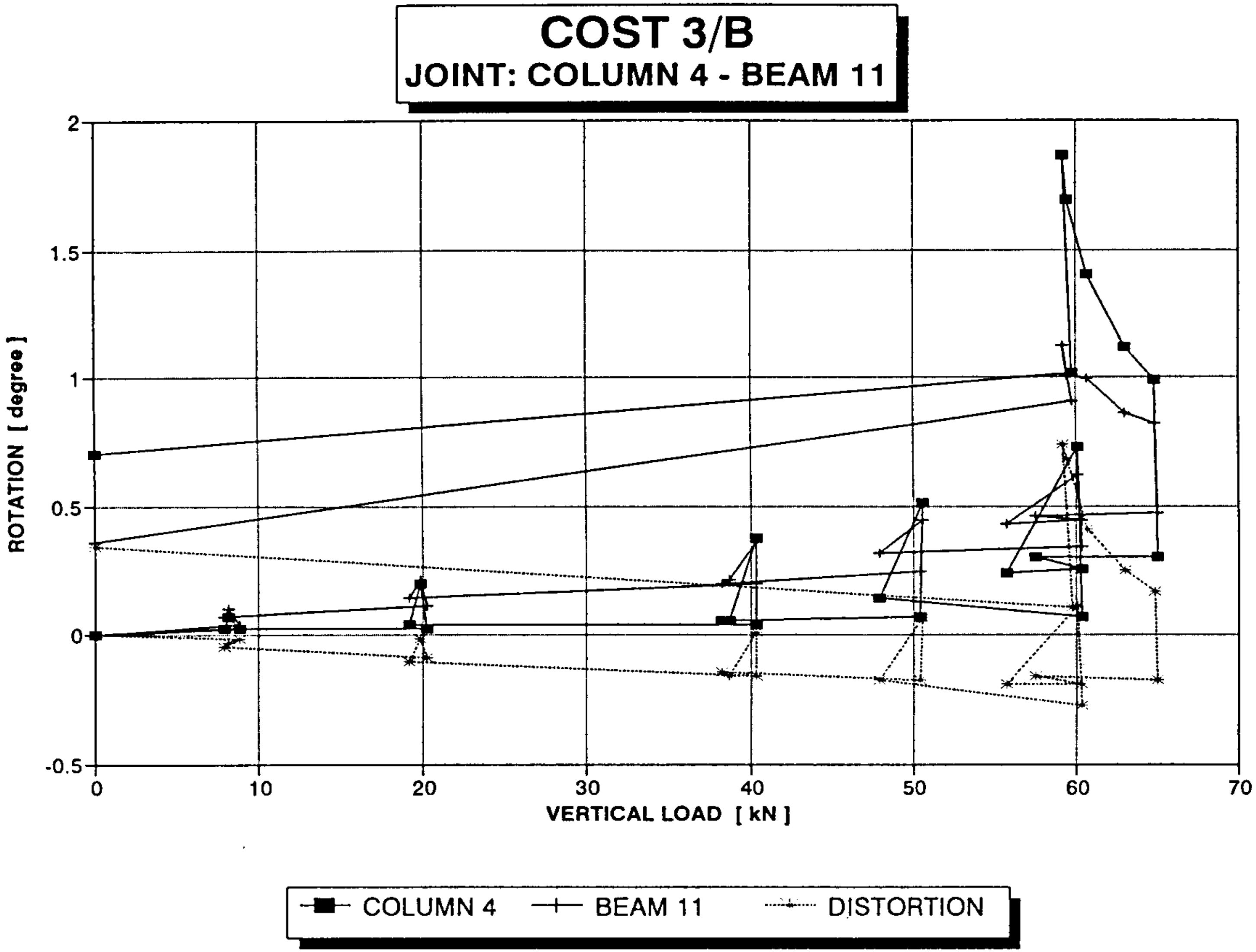


Load – joint displacement curve

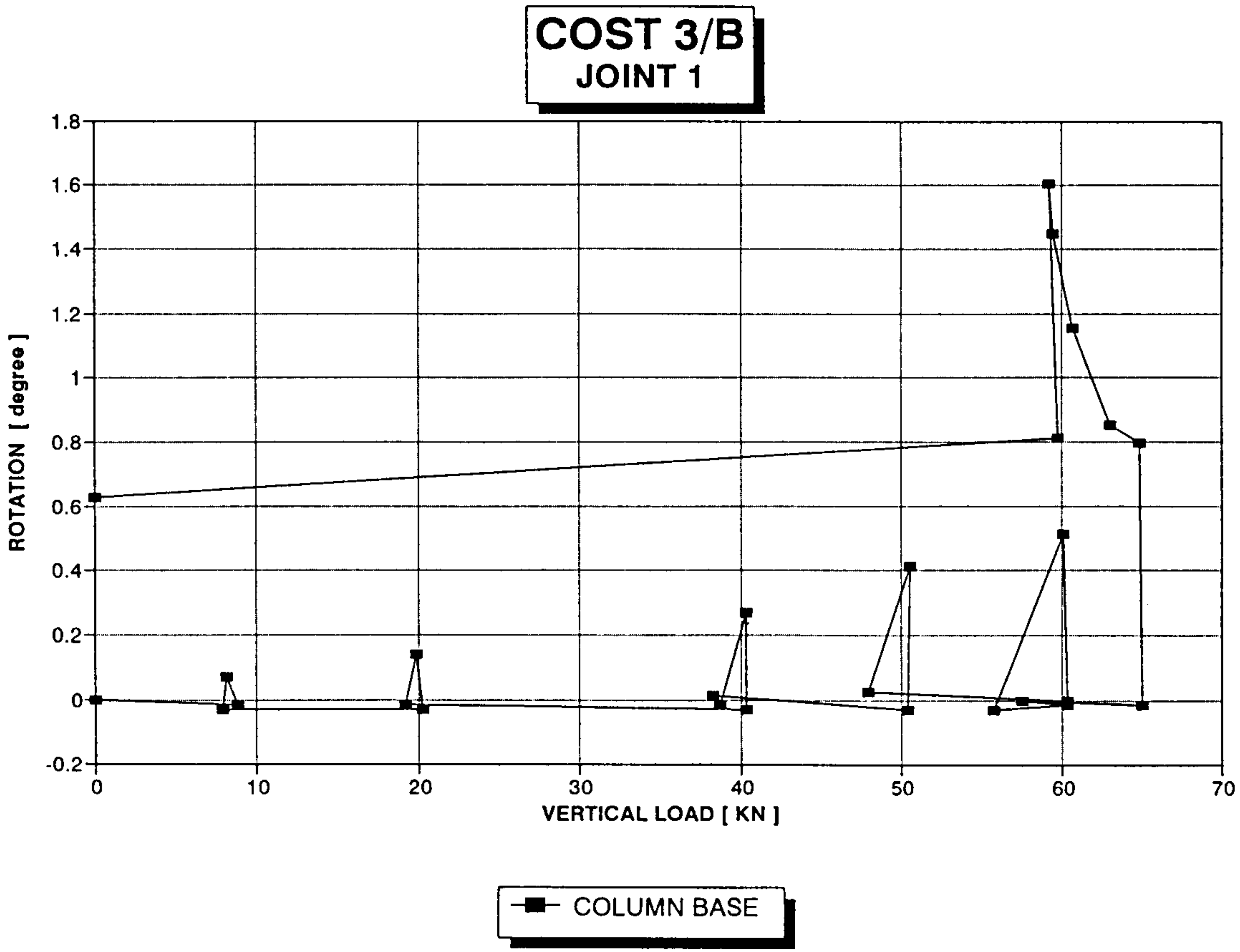
(b) Test Frame COST 3: Pulsating Loading Process



Vertical load – deflection curve

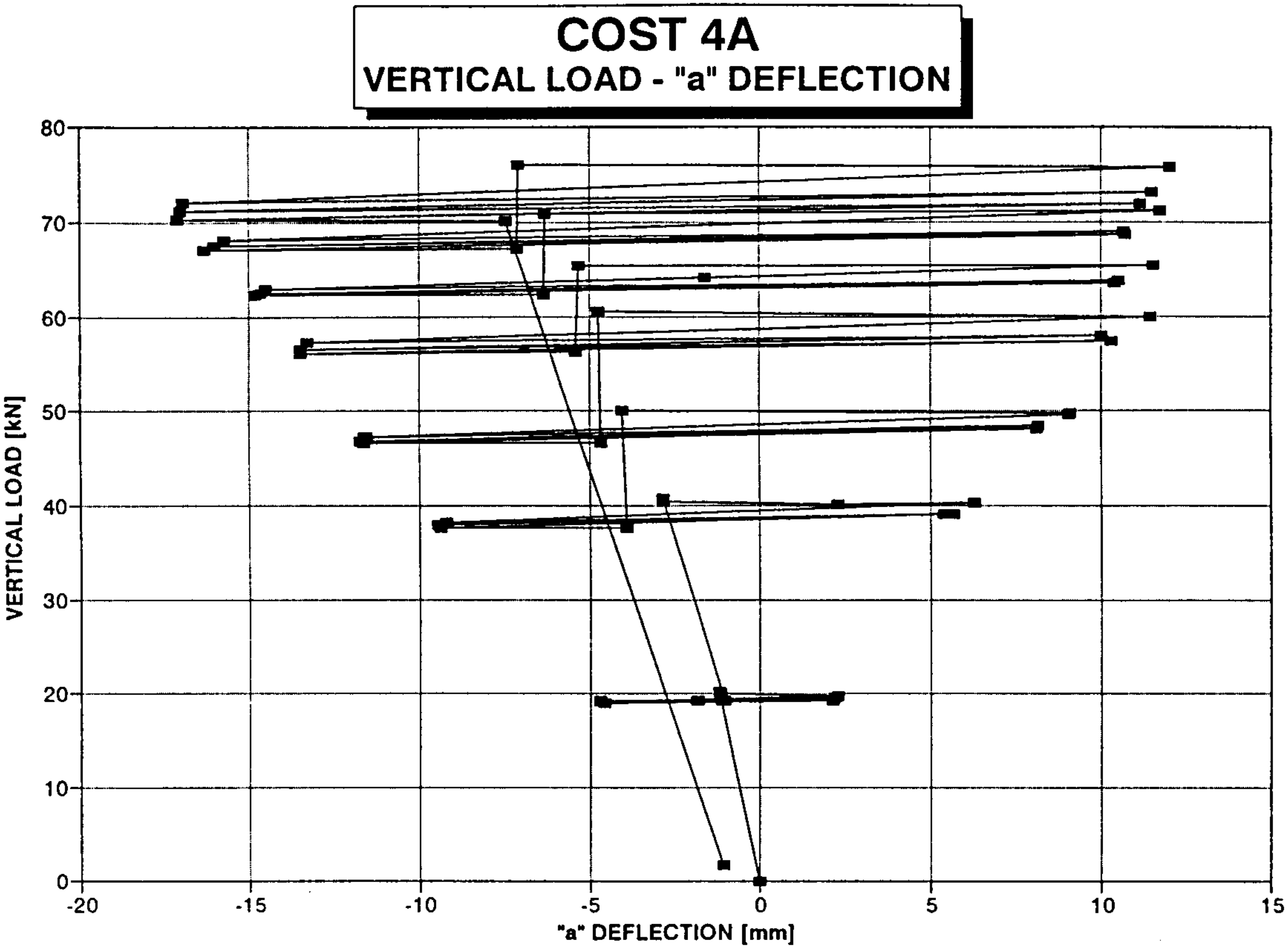


Vertical load – rotation curves

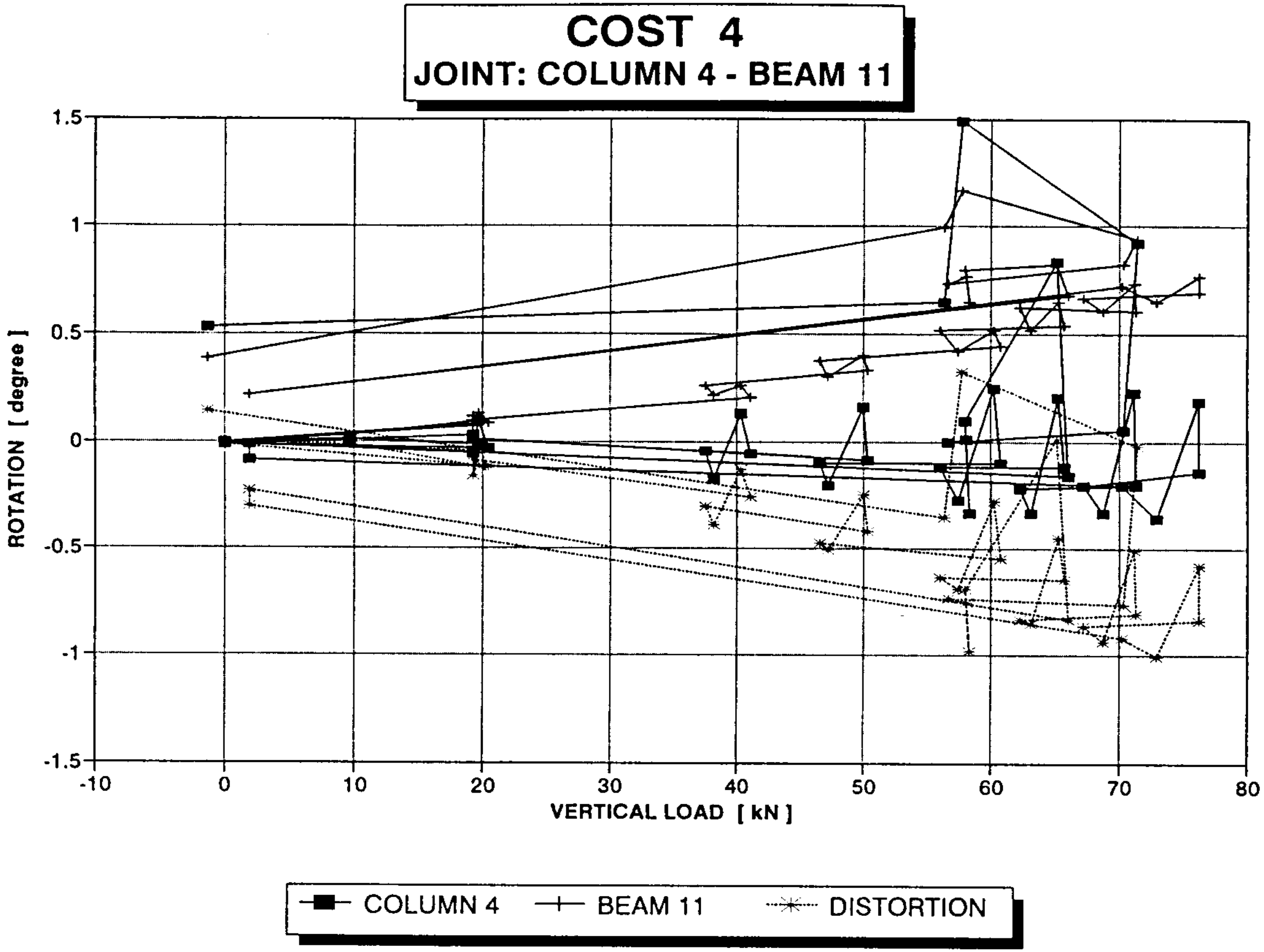


Vertical load – rotation curve

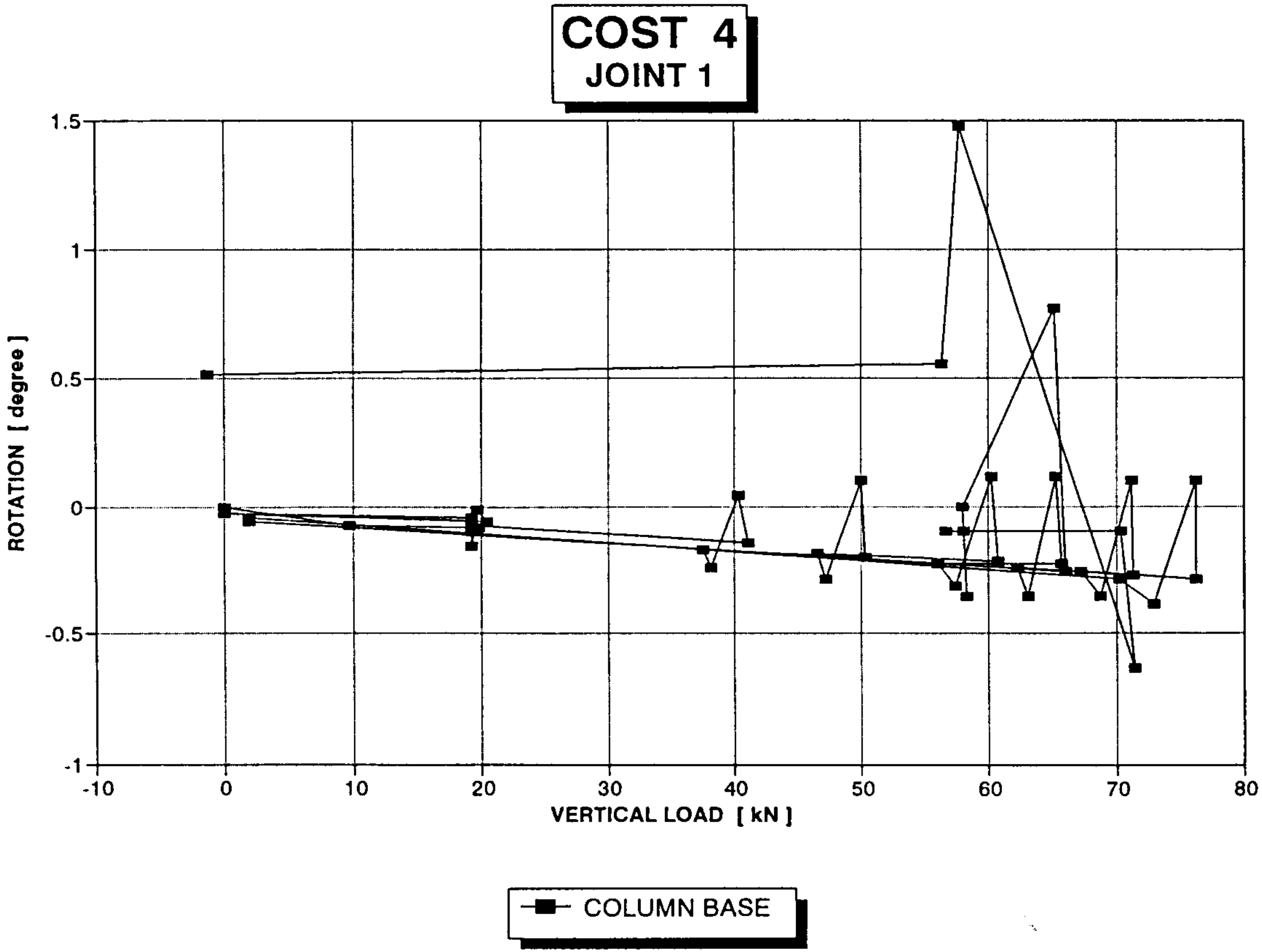
(c) Test Frame COST 4: Alternating Loading Process



Vertical load – deflection curve



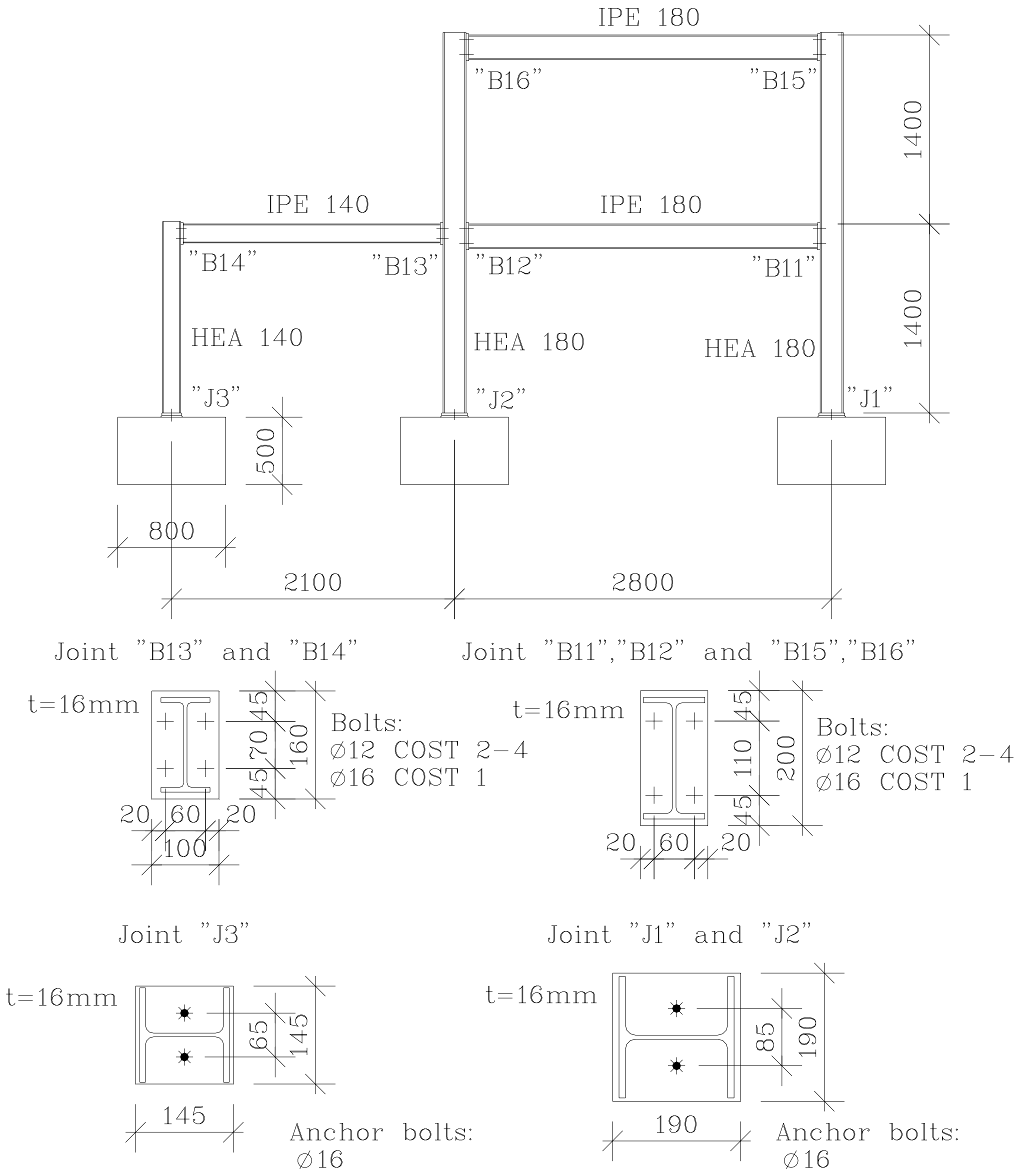
Vertical load – rotation curves



Vertical load – rotation curve

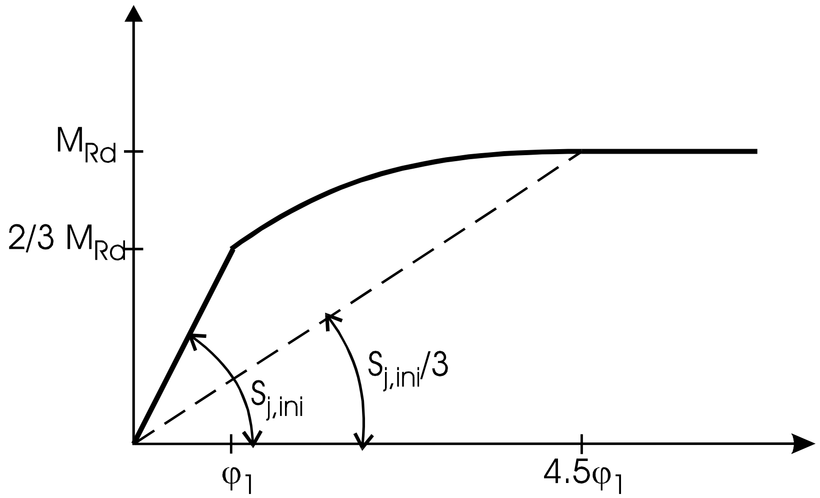
9.3.4 Comparison of test results to theoretical calculations

(a) Modelling of Beam-to-Column Connections and Column Bases

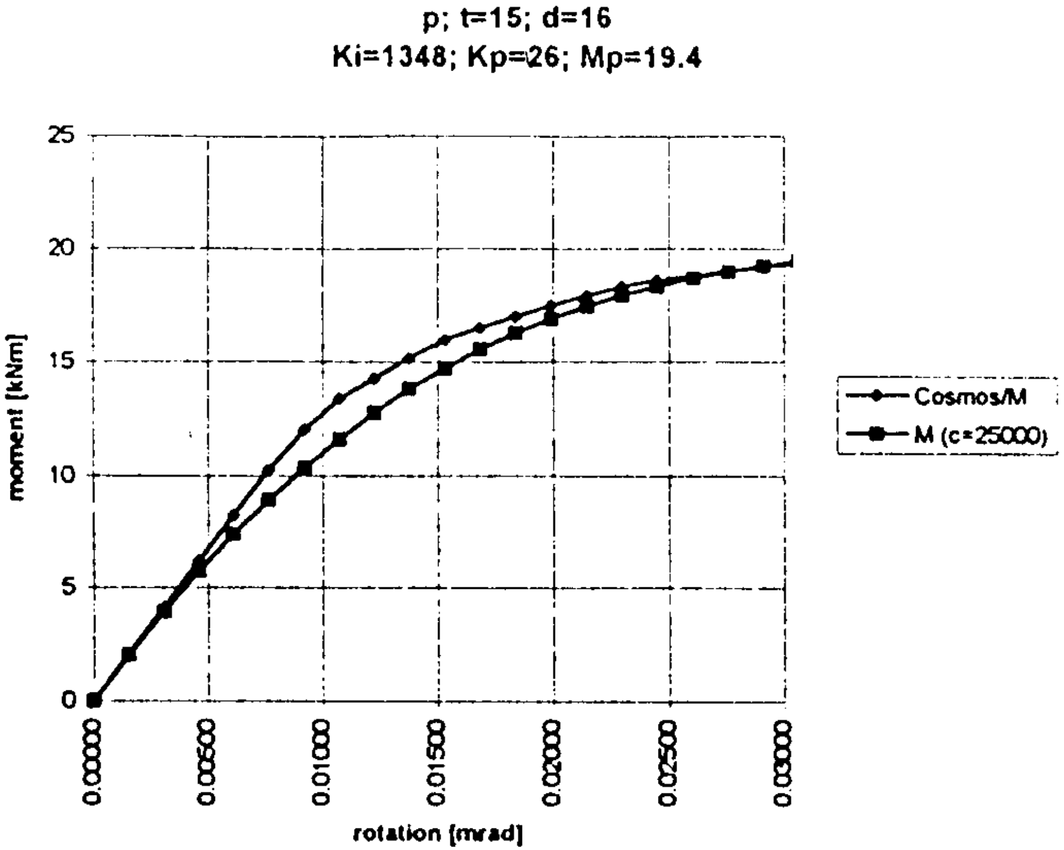


Layout of the connections

$$S_j = S_{j,ini} \cdot \left[\frac{M_{Rd}}{1.5 \cdot M} \right]^{2.7}$$

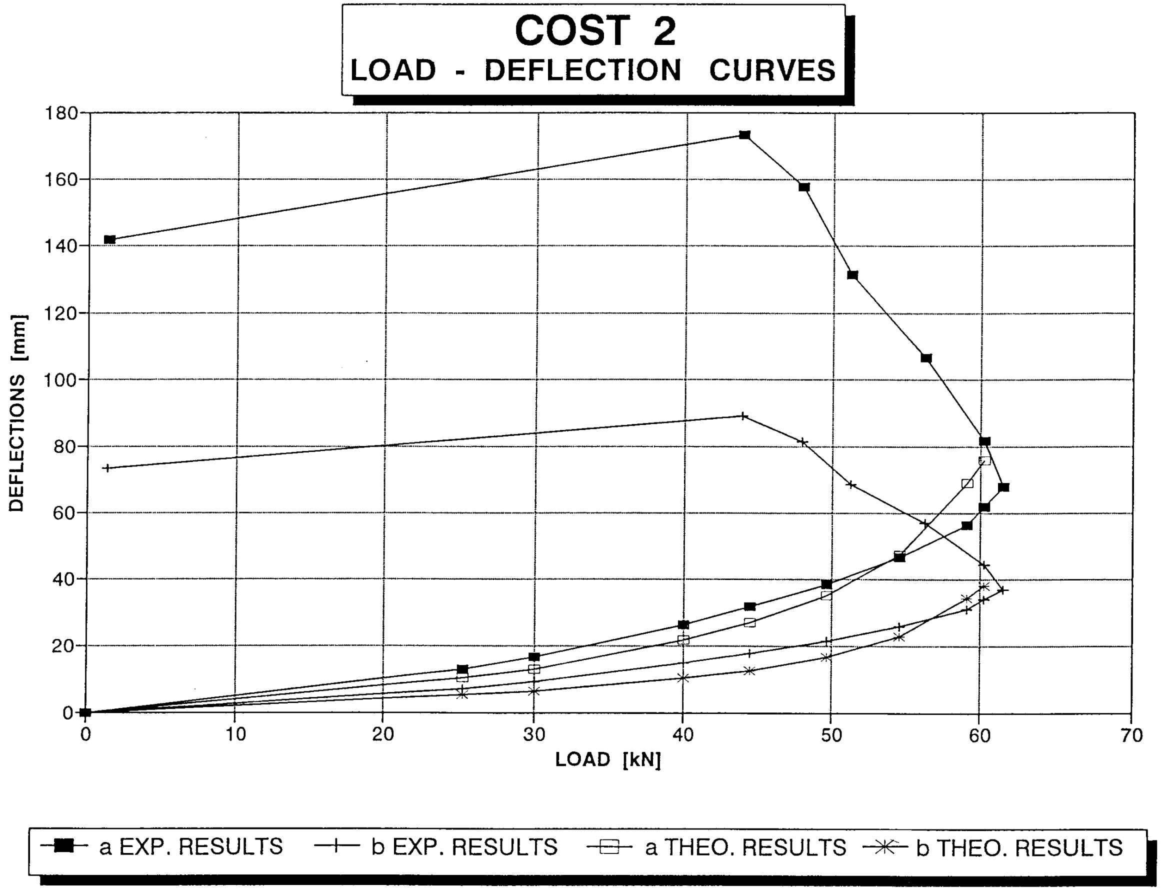


Moment – rotation curves for bolted end-plate connections according to Eurocode 3



Moment – rotation curves for column bases

(b) Effect of Proportional Loading



Load – displacement curve

For test frame COST 2: load-bearing capacity is P=60.2kN, test showed P=61.5kN

1st order theory - load-bearing capacity is P=69.2kN

(c) Effect of Variable Loading

Shakedown Analysis [Kaliszky, 1989]

$$m_b \cdot M_{res,i}^{\max} + M_i^s = M_{ti} \qquad m_b \cdot M_{res,i}^{\min} + M_i^s = -M_{ti}$$

m_b – Shakedown load parameter

$$M_{res,i}^{\max} \qquad M_{res,i}^{\min}$$

$$m_b \cdot \left[\Sigma \mathrm{M}_{res,i}^{\max} \cdot \left|\vartheta_i^+\right| - \Sigma \mathrm{M}_{res,i}^{\min} \cdot \left|\vartheta_i^-\right|\right] + \Sigma M_i^s \cdot \vartheta_i = \Sigma M_{ti} \cdot \left|\vartheta_i\right|$$

$$\Sigma M_i^s \cdot \vartheta_i = 0$$

$$m_b = \frac{\Sigma M_{ti} \cdot \left|\vartheta_i\right|}{\Sigma \mathrm{M}_{res,i}^{\max} \cdot \left|\vartheta_i^+\right| - \Sigma \mathrm{M}_{res,i}^{\min} \cdot \left|\vartheta_i^-\right|}$$

For test frame COST 3B: shakedown load is P=62.4kN, test showed P=65.0kN

For test frame COST 4B: shakedown load is P=68.3kN, test showed P=70.2kN

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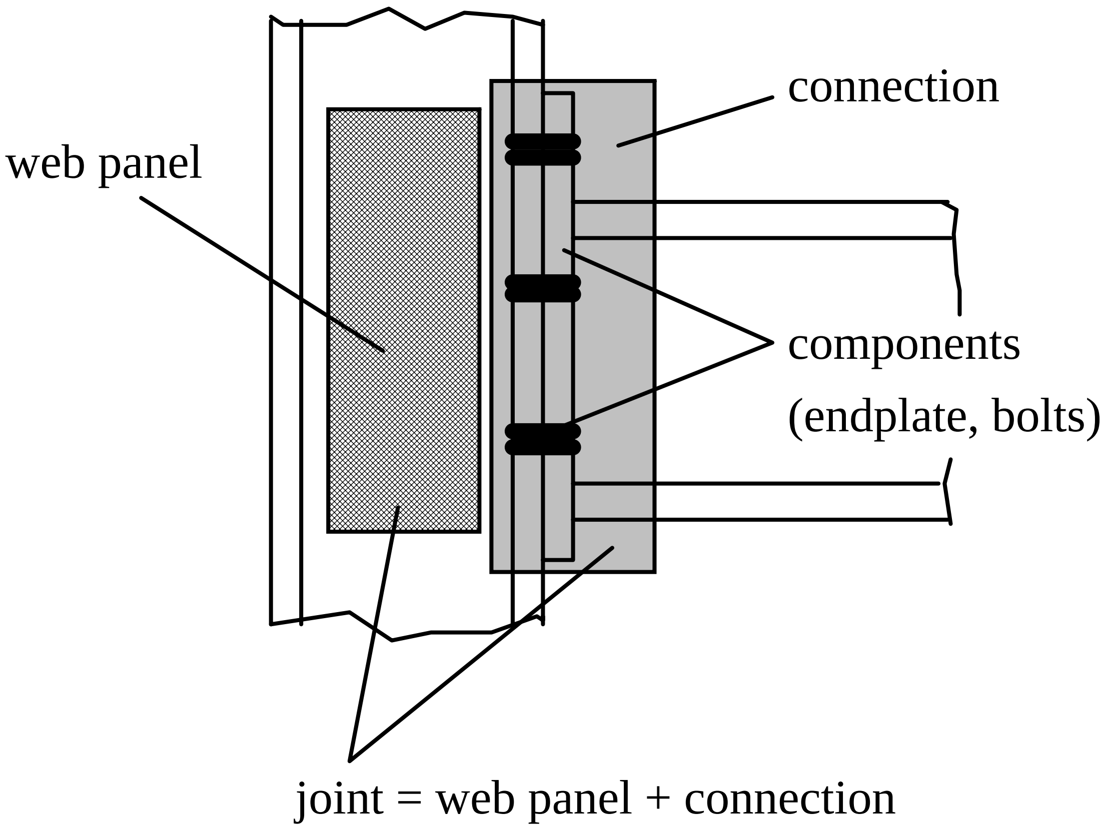
9.3.5 Conclusions

- Extensive experimental study has been carried out to analyse the effects of semi-rigid connections
- Four full-scale, three-dimensional multi-storey frames have been tested
- A condensed overview of the features of the experimental set-up has been given and it has been briefly explained how the complexities of the 3D nature of the tests were addressed
- Comparison of theoretical and experimental results showed reasonable agreement

9.4. Use of approximate engineering methods [Iványi, Skaloud,1995]

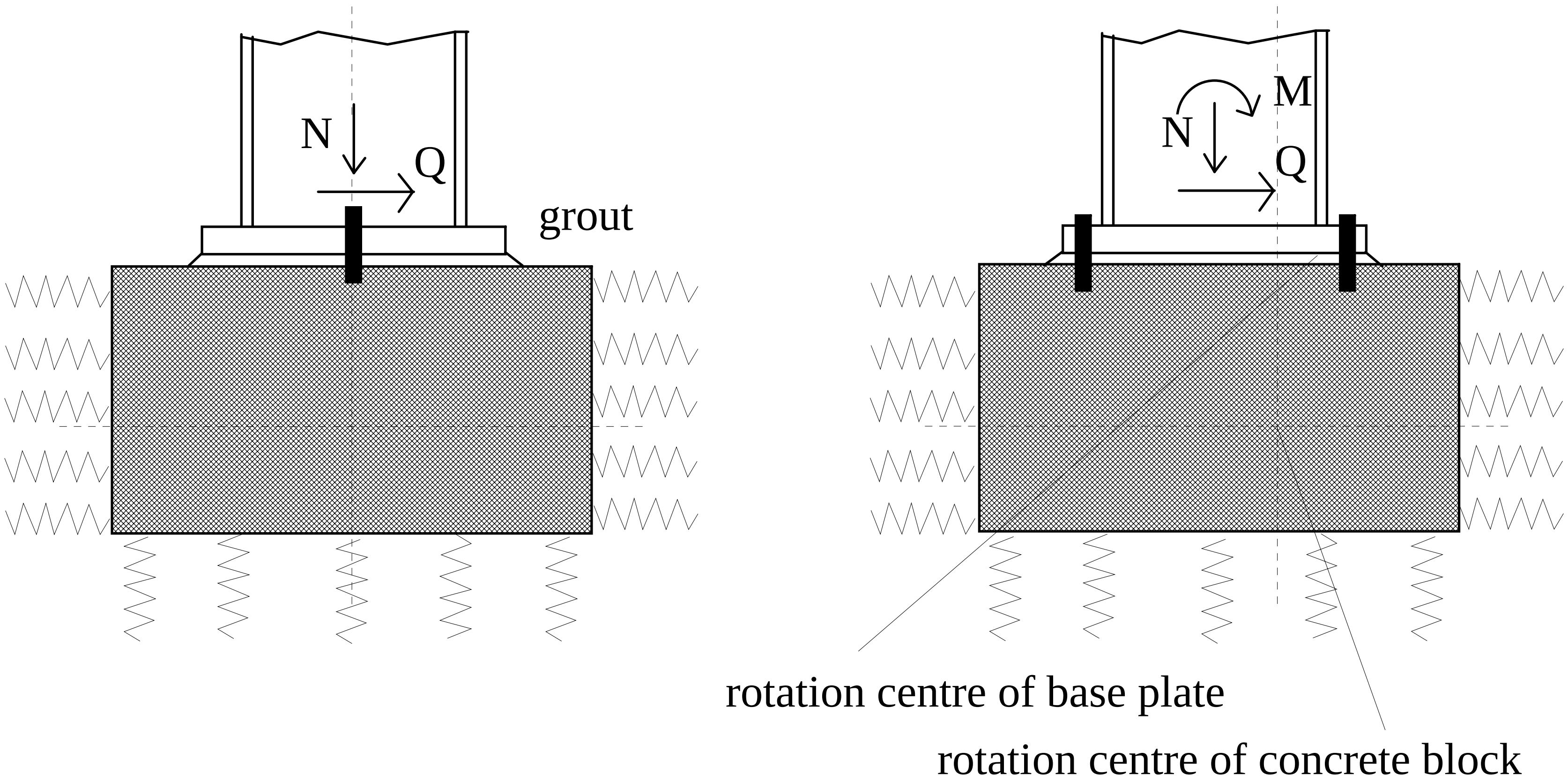
9.4.1 Introduction

9.4.2 Modelling joints in frames



Joint model of Annex JJ, Eurocode 3

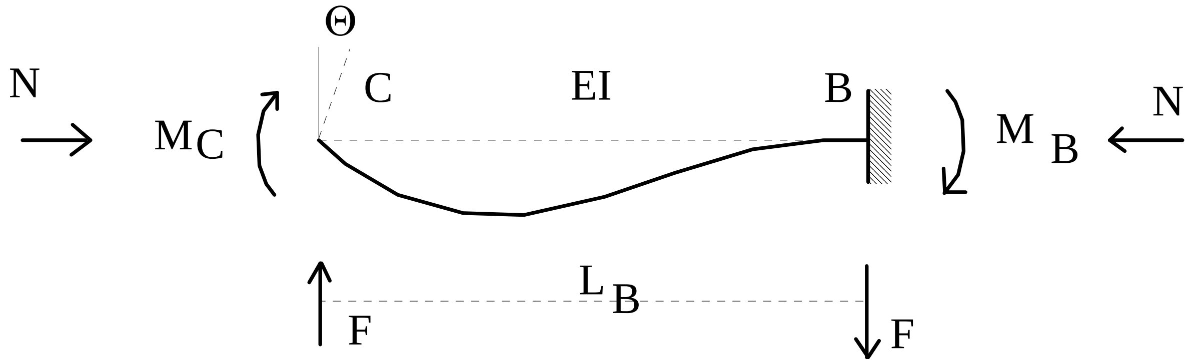
9.4.3 Influences at column bases



Column bases

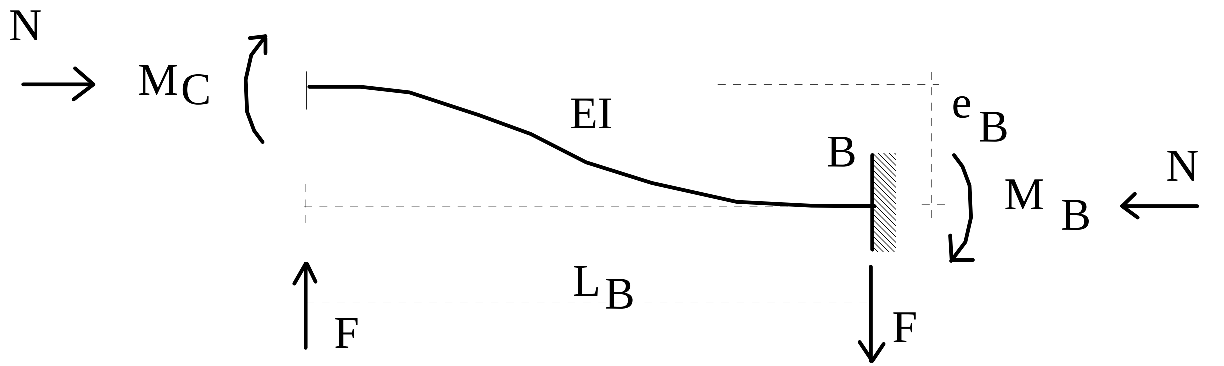
9.4.4 Use of Stability Functions

[Horne, Merchant, 1965] [Horne, Morris, 1981] [Majid, 1972]



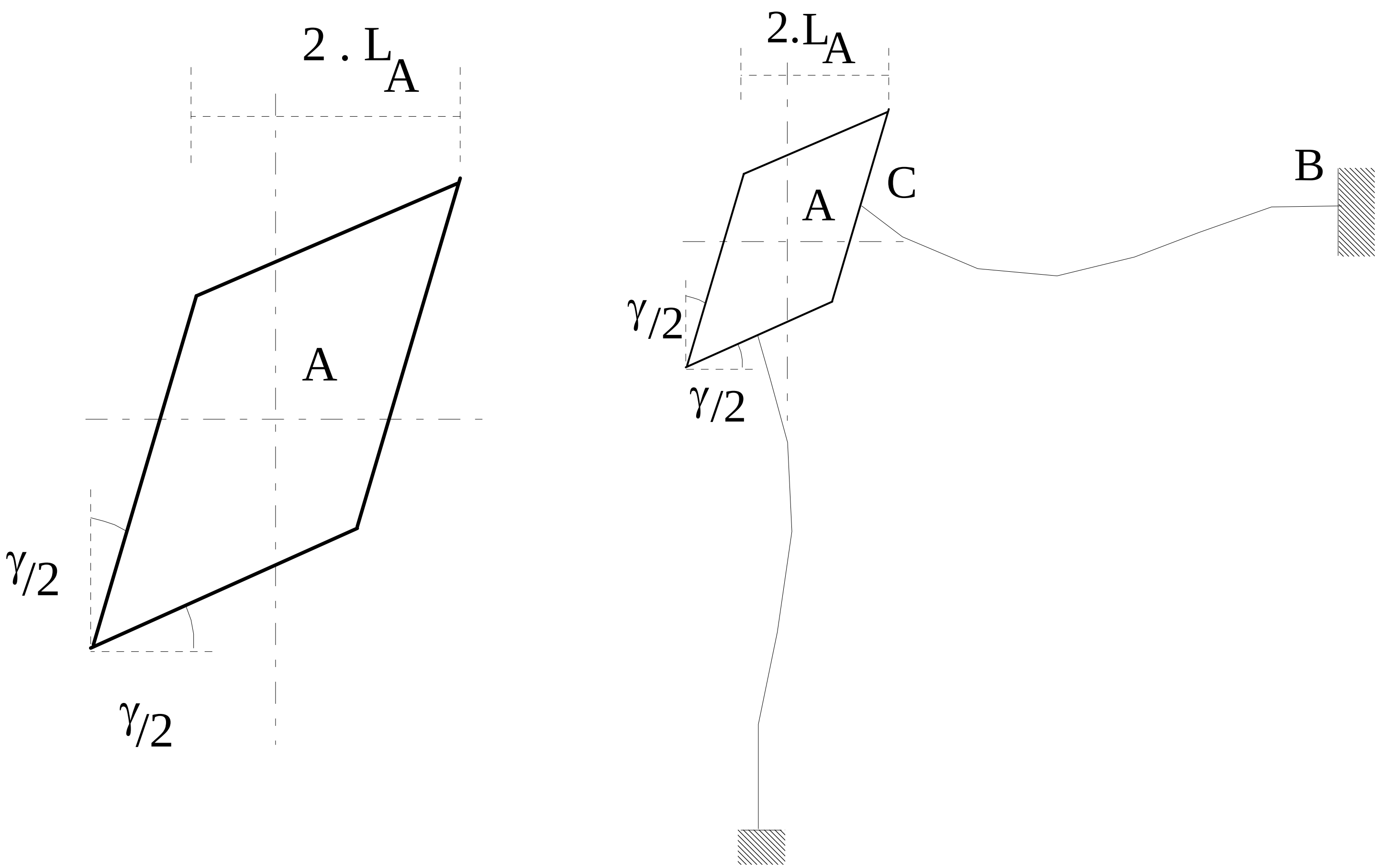
$$M_A = s.k.\Theta = S.\Theta; \quad M_B = s.c.k.\Theta = T.\Theta; \quad F = -s(1+c).k.\Theta/L = -U.\Theta/L$$

Stability functions: rotation at left end



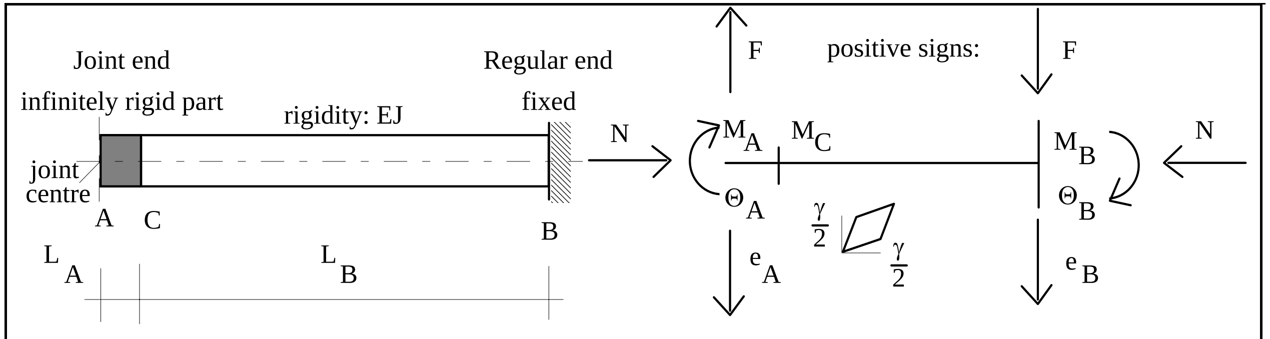
$$M_A = M_B = -u.k.e_B/L = -\,U.e_B/L; \quad F = (2.u-^{2}).k.e_B/L^2 = V.e_B/L^2$$

Stability functions: sway at right end



Shear deformation of web panel and its influence for deformation of members

Table 1. Stability functions including shear deformation of web panel – fixed bar

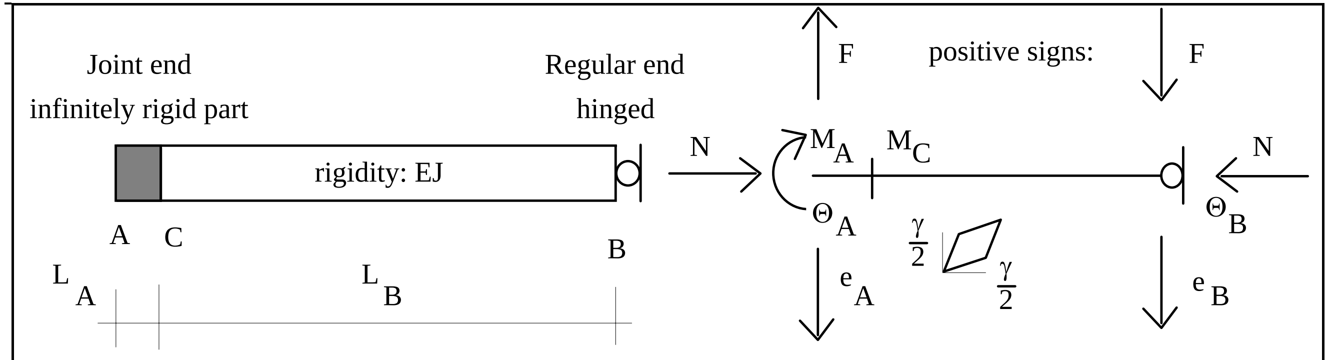


Basic notation: $s.k = S$ $\quad s.(1+c).k = U$ $\quad k = EJ / L_B$

$s.c.k = T$ $\quad (2.s.(1+c) - \pi^2\rho).k = V$ $\quad \rho = N.L_B / (\pi^2.k)$

	M_A	M_B	M_C	F
Θ_A	$\left[S+\frac{L_A}{L_B}\left(1+\frac{L_A}{L_B}\right)V\right]=S_k$	$\left[T+\frac{L_A}{L_B}U\right]=T_k$	$\left[S+\frac{L_A}{L_B}U\right]=S_c$	$-\left[U+\frac{L_A}{L_B}V\right]\frac{1}{L_B}=-U_k\frac{1}{L_B}$
Θ_B	$\left[T+\frac{L_A}{L_B}U\right]=T_k$	S	T	$-U\frac{1}{L_B}$
e_A	$\left[U+\frac{L_A}{L_B}V\right]\frac{1}{L_B}=U_k\frac{1}{L_B}$	$U\frac{1}{L_B}$	$U\frac{1}{L_B}$	$-V\frac{1}{L_B{}^2}$
e_B	$-\left[U+\frac{L_A}{L_B}V\right]\frac{1}{L_B}=-U_k\frac{1}{L_B}$	$-U\frac{1}{L_B}$	$-U\frac{1}{L_B}$	$V\frac{1}{L_B{}^2}$
$\frac{\gamma}{2}$	$\left[S-2U\left(\frac{L_A}{L_B}\right)^2+\pi^2\rho k\frac{L_A}{L_B}\left(1+\frac{L_A}{L_B}\right)\right]=S_\gamma$	$\left[T-\frac{L_A}{L_B}U\right]=T_\gamma$	$\left[S-\frac{L_A}{L_B}U\right]=S_{\gamma c}$	$-\left[U-\frac{L_A}{L_B}V\right]\frac{1}{L_B}=-U_\gamma\frac{1}{L_B}$

Table 2. Stability functions including shear deformation of web panel – hinged bar



Basic notation: $s.k = S$ $\quad s.c.k = T$ $\quad s.(1+c).k = U$

$(2.s.(1+c) - \pi^2\rho).k = V$ $\quad s.(1-c^2).k = S''$ $\quad S'' - \pi^2\rho.k = V''$

$k = EJ / L_B$ $\quad \rho = N.L_B / (\pi^2.k)$

	M_A	M_C	F
Θ_A	$\left[S_k - \frac{T_k^2}{S}\right] = S_k''$	$\left[S_c - \frac{T}{S}T_k\right] = S_c''$	$-\left[U_k - \frac{U}{S}T_k\right]\frac{1}{L_B} = -U_k''\frac{1}{L_B}$
e_A	$\left[U_k - \frac{U}{S}T_k\right]\frac{1}{L_B} = U_k''\frac{1}{L_B}$	$U\left(1-\frac{T}{S}\right)\frac{1}{L_B} = S''\frac{1}{L_B}$	$-\left[V - \frac{U^2}{S}\right]\frac{1}{L_B^2} = -V''\frac{1}{L_B^2}$
e_B	$-\left[U_k - \frac{U}{S}T_k\right]\frac{1}{L_B} = -U_k''\frac{1}{L_B}$	$-U\left(1-\frac{T}{S}\right)\frac{1}{L_B} = -S''\frac{1}{L_B}$	$\left[V - \frac{U^2}{S}\right]\frac{1}{L_B^2} = V''\frac{1}{L_B^2}$
$\frac{\gamma}{2}$	$\left[S_\gamma - \frac{T_k}{S}T_\gamma\right] = S_\gamma''$	$\left[S_{\gamma c} - \frac{T}{S}T_\gamma\right] = S_{\gamma c}''$	$-\left[U\left(1-\frac{T_\gamma}{S}\right) - \frac{L_A}{L_B}V\right]\frac{1}{L_B} = -U_\gamma''\frac{1}{L_B}$

Displacement Method:

$$[\underline{q}^*] = [\underline{\underline{K}}^*][\underline{u}^*]$$

q* is the load vector,
K* is the stiffness matrix and
u* is the displacement vector

In general form the *M–functions* (moment - relative rotation correlation at the local flexibilities) can be written:

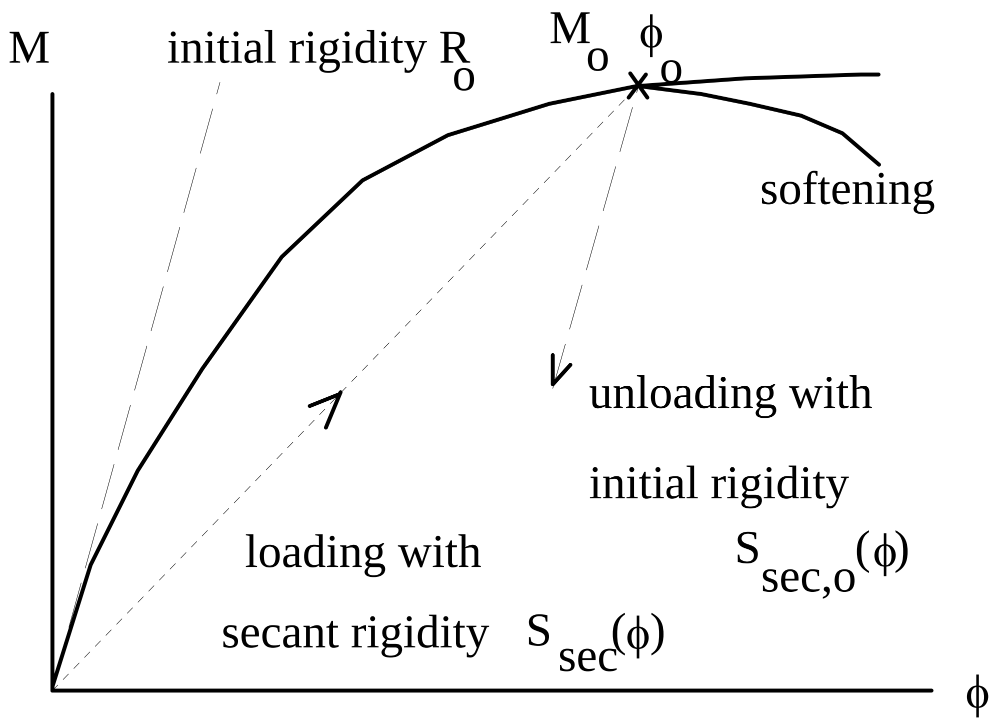
$$M = R \cdot \phi + M_1$$

Loading: $R = S_{sec}(\phi)$

$$M_1 = 0$$

Unloading: $R = S_{sec,0}$

$$M_1 = M_0 - S_{sec,0} \cdot \phi_0$$



Spring characteristic

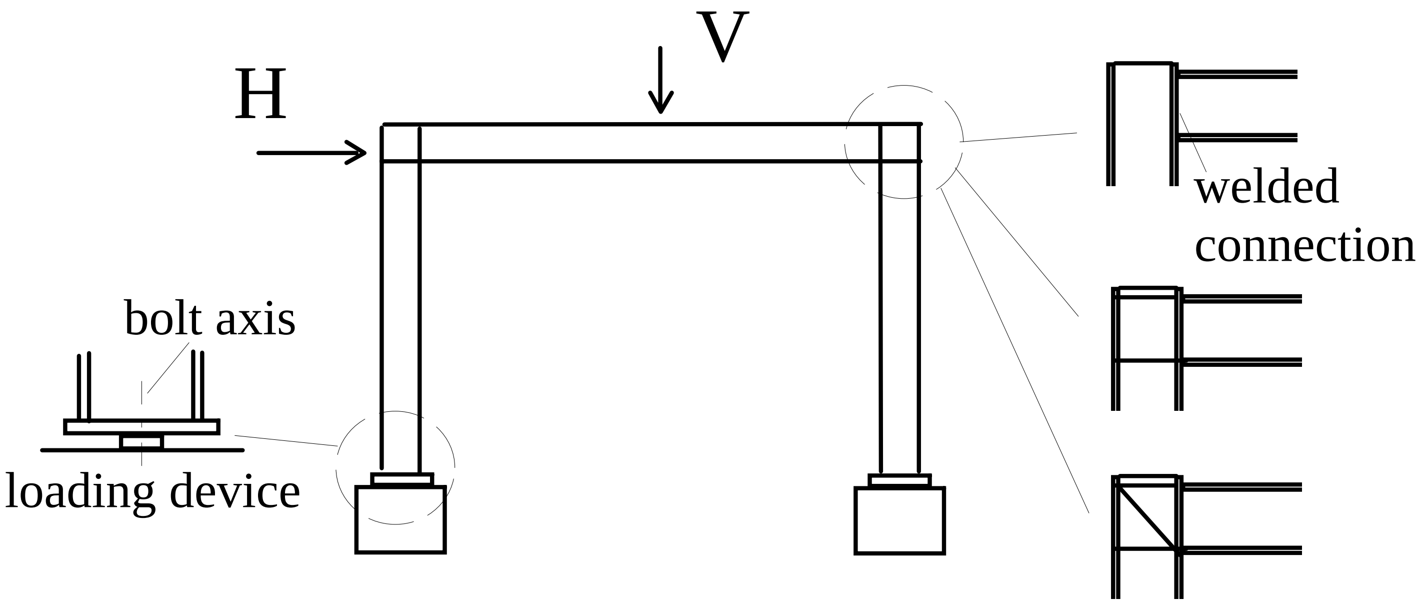
9.4.5 Extended Use of Stability Functions and Springs

- This above detailed method is suitable to analyse elastic members, taking allowance for geometric non-linearities (second order effects).
- As the spring rigidity functions theoretically can be of any shape, with this set of expressions those cases also can be treated, when the spring rigidities involve some plastic, hardening or softening phenomena.
- The use the stability functions together with all of those possibilities, which were detailed before, can be extended to carry out an elastic - plastic hinge analysis.
- When the frame to be analysed is carrying distributed loads, it is preferable to substitute it with concentrated ones, because those sections, in which plastic hinges develop in a member of distributed loading, are generally not known in advance
- Either non-linear behaviour of connection or plastic behaviour of the material is the reason of the relative deformations, it should be always kept in mind, that loading (increase of deformations accompanied by either increase or decrease of load intensity) follows the mentioned curves, while unloading (decrease of deformation and loading together) is always elastic (follows the initial rigidity).

9.4.6 Demonstrative Example

[Ivanyi Jr., 1993]

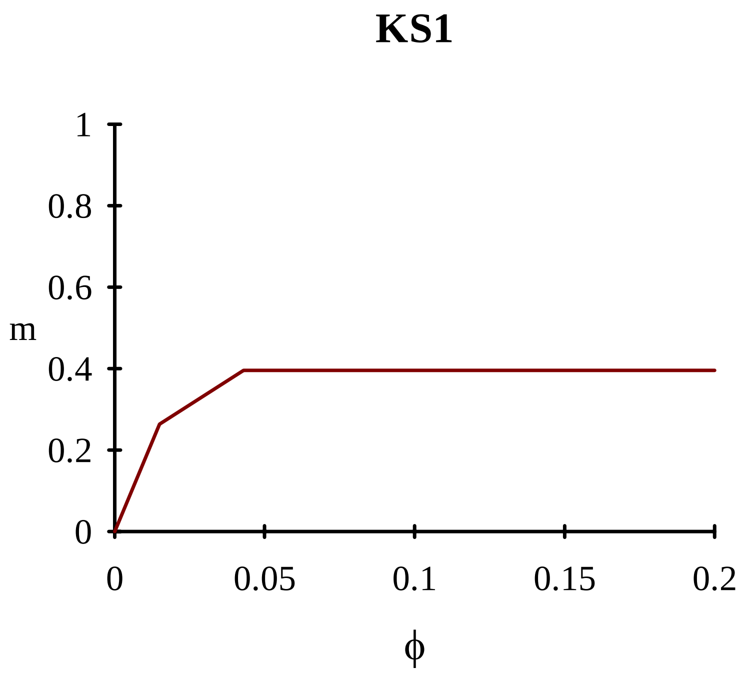
Three types of frame knees were tested: *(i)* no stiffeners, *(ii)* horizontal stiffeners, *(iii)* horizontal + diagonal stiffeners. The ratio of *H*:*V*=0.5:1.

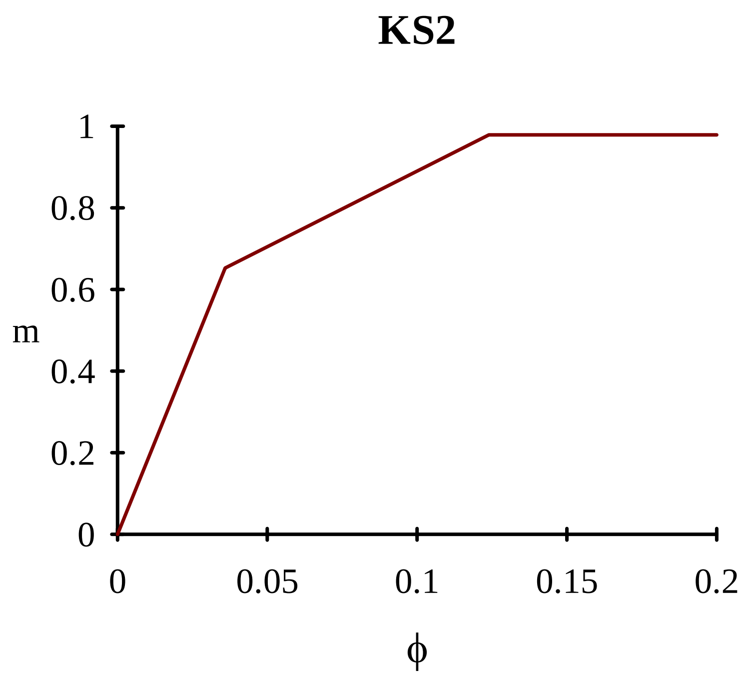


Test model built up from hot rolled sections

Allowing for the real conditions, some simplifications were made, as:

1. a strong steel device was constructed to transfer the loading and to support the test frame – there were no foundation displacements;
2. flexibility of welded connection on the interface of beam-to-column is small – there is no need for connection springs at beam ends,
3. beam-to-column connections are not stronger than the sections themselves – no need for plastic hinge springs around the joints.





Moment – rotation curves by EC 3

Stiffness Matrix

	1	2	3	4	5	6	7	8	9	10
1	$(S_k)_1$ $(S_k)_2$	$(T_k)_2$	0	$-\left(\frac{U_k}{L_B}\right)_1$	$-\left(\frac{U_k}{L_B}\right)_2$	$(T_k)_1$	$-(S_\gamma)_1$ $(S_\gamma)_2$	0	0	0
2	$(T_k)_2$	$(S_k)_2$ $(S_k)_3$	$(T_k)_3$	0	$-\left(\frac{U}{L_B}\right)_2$ $\left(\frac{U}{L_B}\right)_3$	0	$(T_\gamma)_2$	$(S)_3$	$(T_\gamma)_3$	0
3	0	$(T_k)_3$	$(S_k)_3$ $(S_k)_4$	$-\left(\frac{U_k}{L_B}\right)_4$	$\left(\frac{U_k}{L_B}\right)_3$	0	0	$(T_k)_3$	$(S_\gamma)_3$ $-(S_\gamma)_4$	$(T_k)_4$
4	$-\left(\frac{U_k}{L_B}\right)_1$	0	$-\left(\frac{U_k}{L_B}\right)_4$	$\left(\frac{V}{L_B^2}\right)_1$ $\left(\frac{V}{L_B^2}\right)_4$	0	$-\left(\frac{U}{L_B}\right)_1$	$\left(\frac{U_\gamma}{L_B}\right)_1$	0	$\left(\frac{U_\gamma}{L_B}\right)_4$	$-\left(\frac{U}{L_B}\right)_4$
5	$-\left(\frac{U_k}{L_B}\right)_2$	$-\left(\frac{U}{L_B}\right)_2$ $\left(\frac{U}{L_B}\right)_3$	$\left(\frac{U_k}{L_B}\right)_3$	0	$\left(\frac{V}{L_B^2}\right)_2$ $\left(\frac{V}{L_B^2}\right)_3$	0	$-\left(\frac{U_\gamma}{L_B}\right)_2$	$\left(\frac{U}{L_B}\right)_3$	$\left(\frac{U_\gamma}{L_B}\right)_3$	0
6	$(T_k)_1$	0	0	$-\left(\frac{U}{L_B}\right)_1$	0	$(S)_1$ Spring_1	$-(T_\gamma)_1$	0	0	0
7	$(S_k)_2$	$(T_k)_2$	0	0	$-\left(\frac{U_k}{L_B}\right)_2$	0	$(S_\gamma)_2$ Spring_2	0	0	0
8	0	$(S)_3$	$(T_k)_3$	0	$\left(\frac{U}{L_B}\right)_3$	0	0	$(S)_3$ Spring_3	$(T_\gamma)_3$	0
9	0	$(T_k)_3$	$(S_k)_3$	0	$\left(\frac{U_k}{L_B}\right)_3$	0	0	$(T_k)_3$	$(S_\gamma)_3$ Spring_4	0
10	0	0	$(T_k)_4$	$-\left(\frac{U}{L_B}\right)_4$	0	0	0	0	$-(T_\gamma)_4$	$(S)_4$ Spring_5

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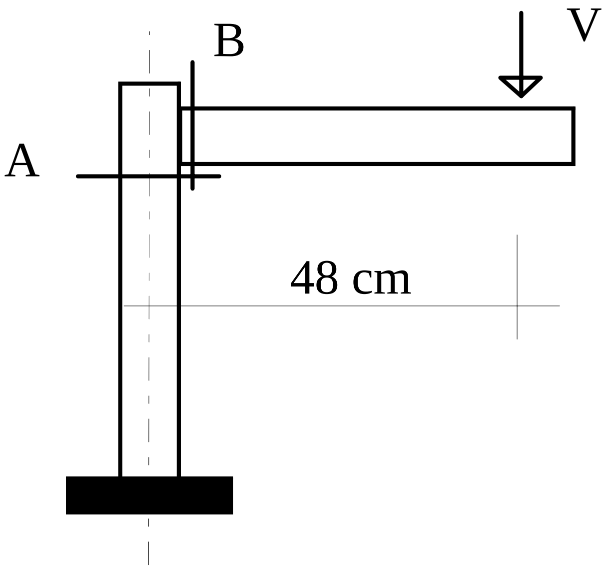
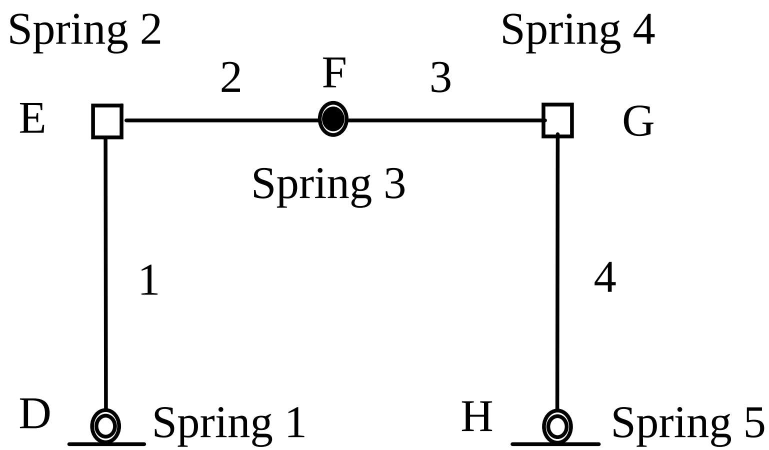
Load Vector

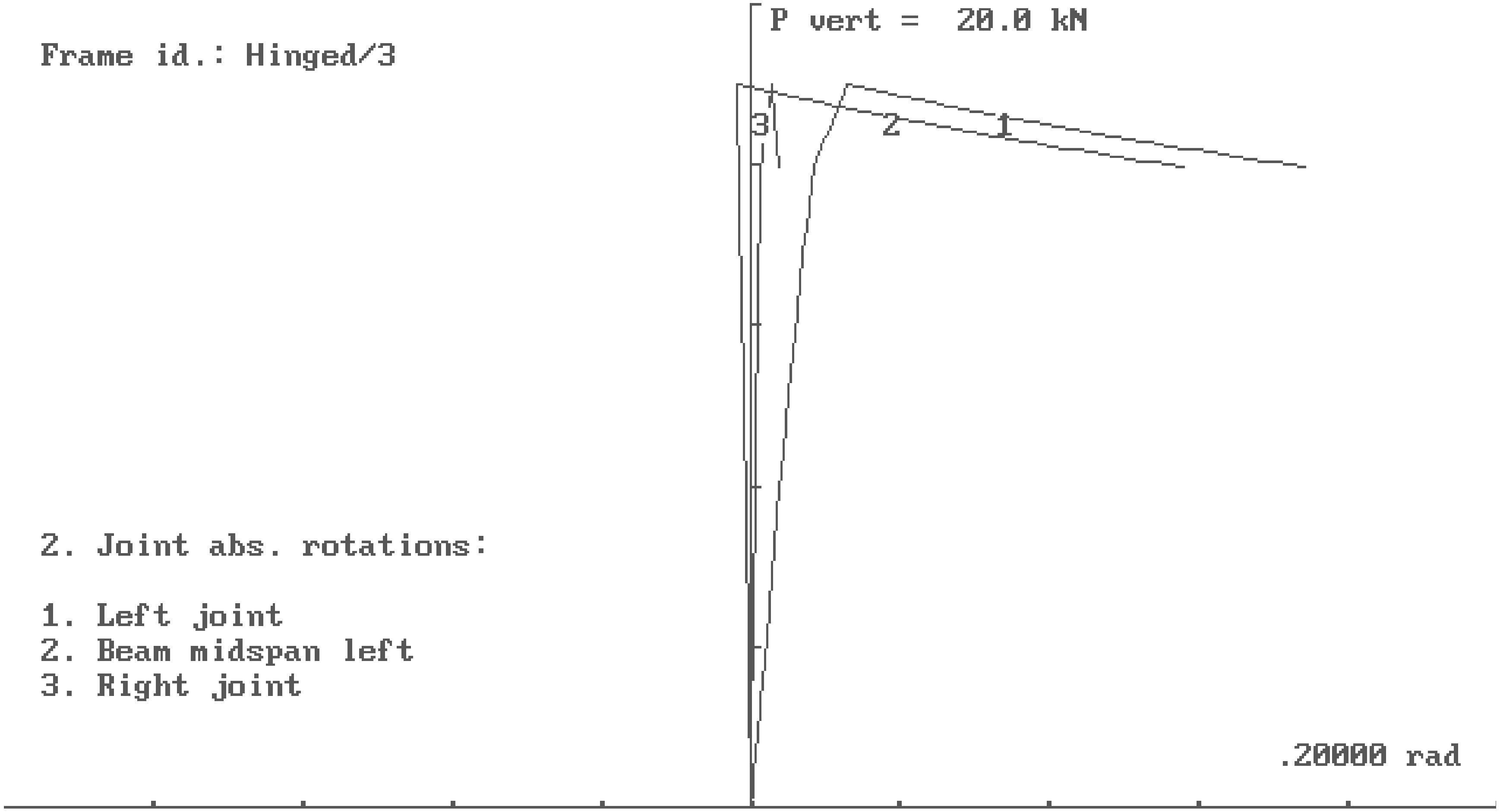
0	0	0	*H*	*V*	0	0	0	0	0



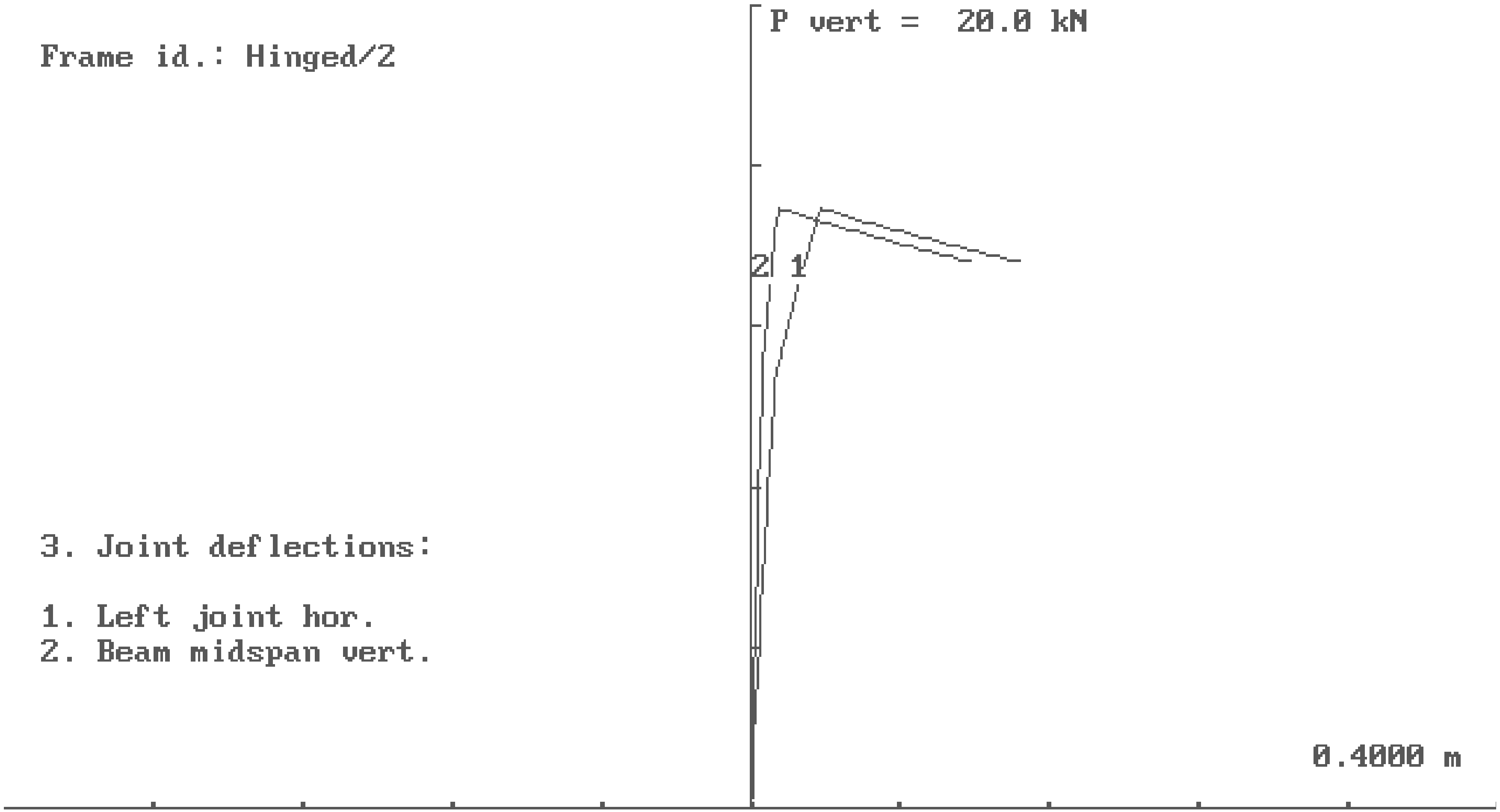
Frame knee models



Frame model built up from springs and elastic members



For example:
Vertical load and joint absolute rotation curves



For example:
Vertical load and joint deflection curves

9.4.7 Summary

This subdivision gave a short summary about the possibilities of constructing more precise, but not too difficult connection models for frame analysis.

It has been dealing with those simple methods, which can be useful tools for

– either to construct envelop curves for the elastic and for the plastic behaviour separately,

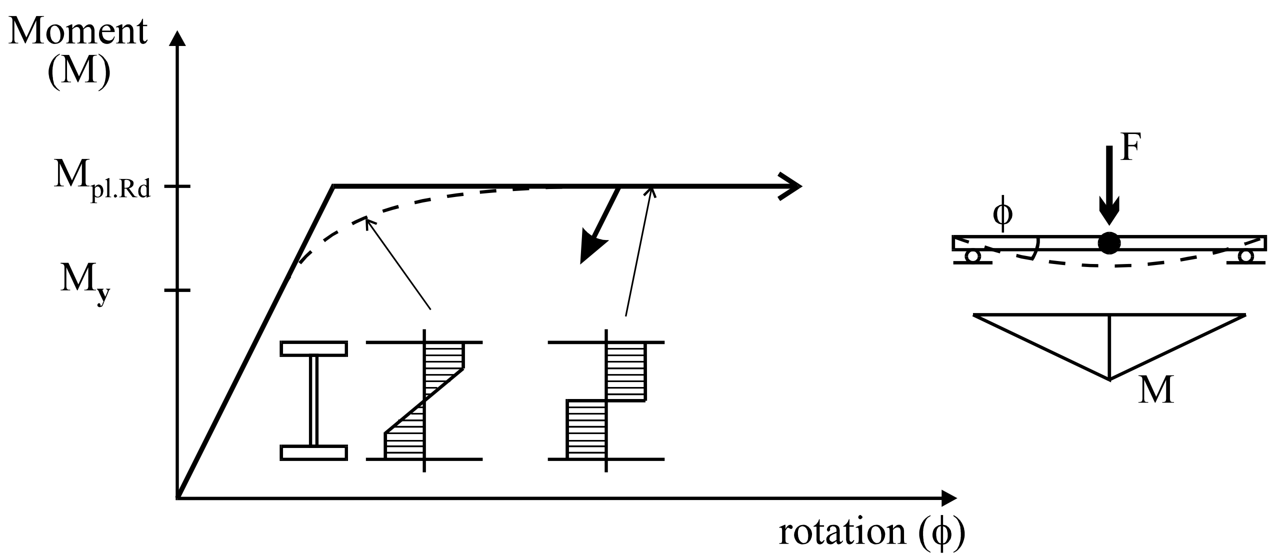
– or to determine the elastic-plastic response of the frame by concentrating all of non-linearities into real or pseudo connection springs, which are connecting the elastic members.

9.5. Direct Design Method of Steel Frames with Semi-Rigid Connections

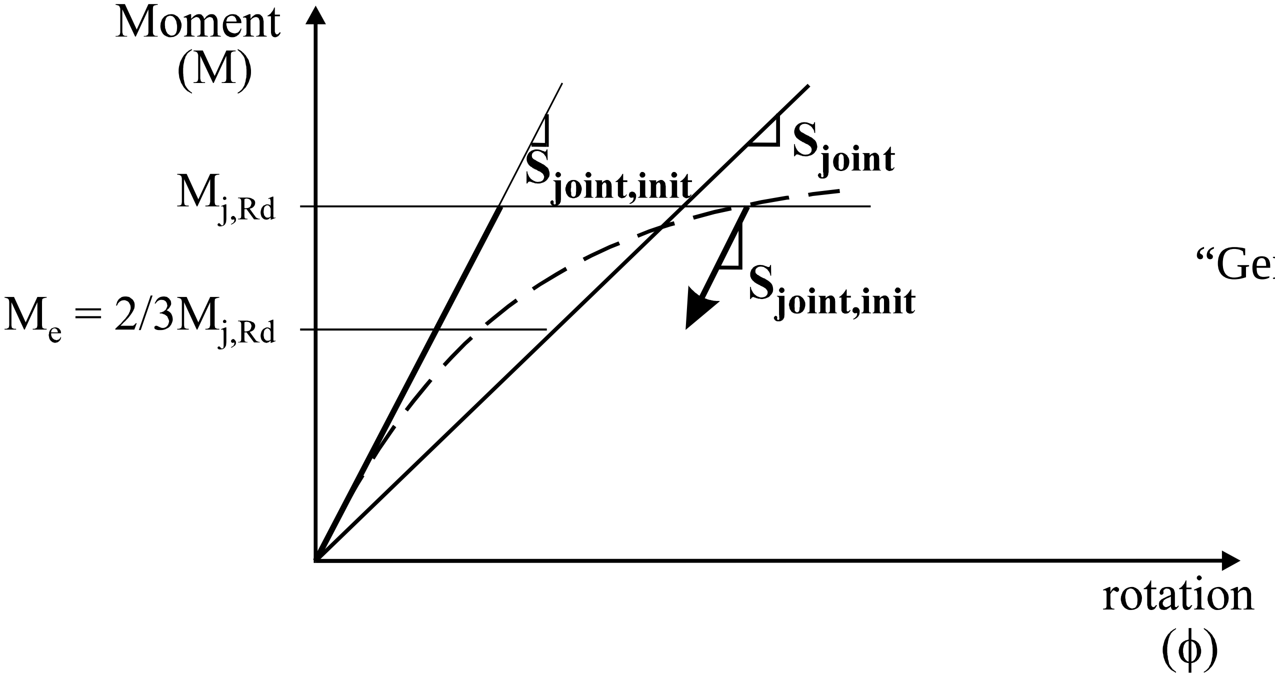
9.5.1 Introduction

The most important parts in steel frames are the beam-to-column connections and the connections between columns and foundation, since their behaviour greatly influences the whole structural behaviour (distribution of moments and forces, displacements, overall stability, etc.).

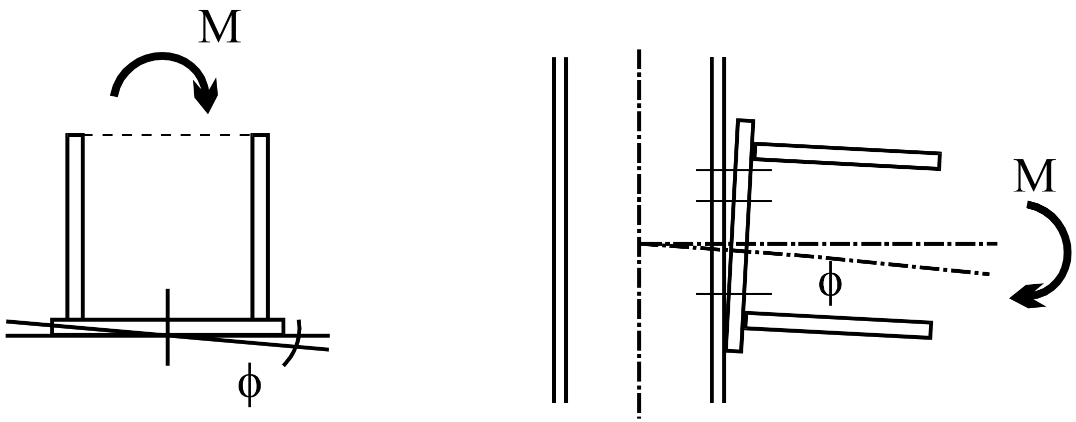
9.5.2 Behaviour of Beam-to-Column Connections and Column Bases



The classical plastic hinge [Kazinczy, 1914]



"Generalised Plastic Hinge"
[Ivanyi, 1983]



The beam-to-column connection and the column base

9.5.3 Effect of beam-to-column connections and column bases on the behaviour of steel frames

– *Modified first order approach:* [Prager, 1951]

$$F_0^{(I)} = f_I\left(M_{pl}; M_{Rd}\right)$$

$M_{pl.Rd}$ – the plastic moment of the cross section,
$M_{j.Rd}$ – the moment resistance of the semi-rigid connection or column base.

The virtual work equation furnishes:

$$\sum_i \alpha_i F_0^{(I)} u_i = \sum_j \left|M_{pl}\kappa_j\right|$$

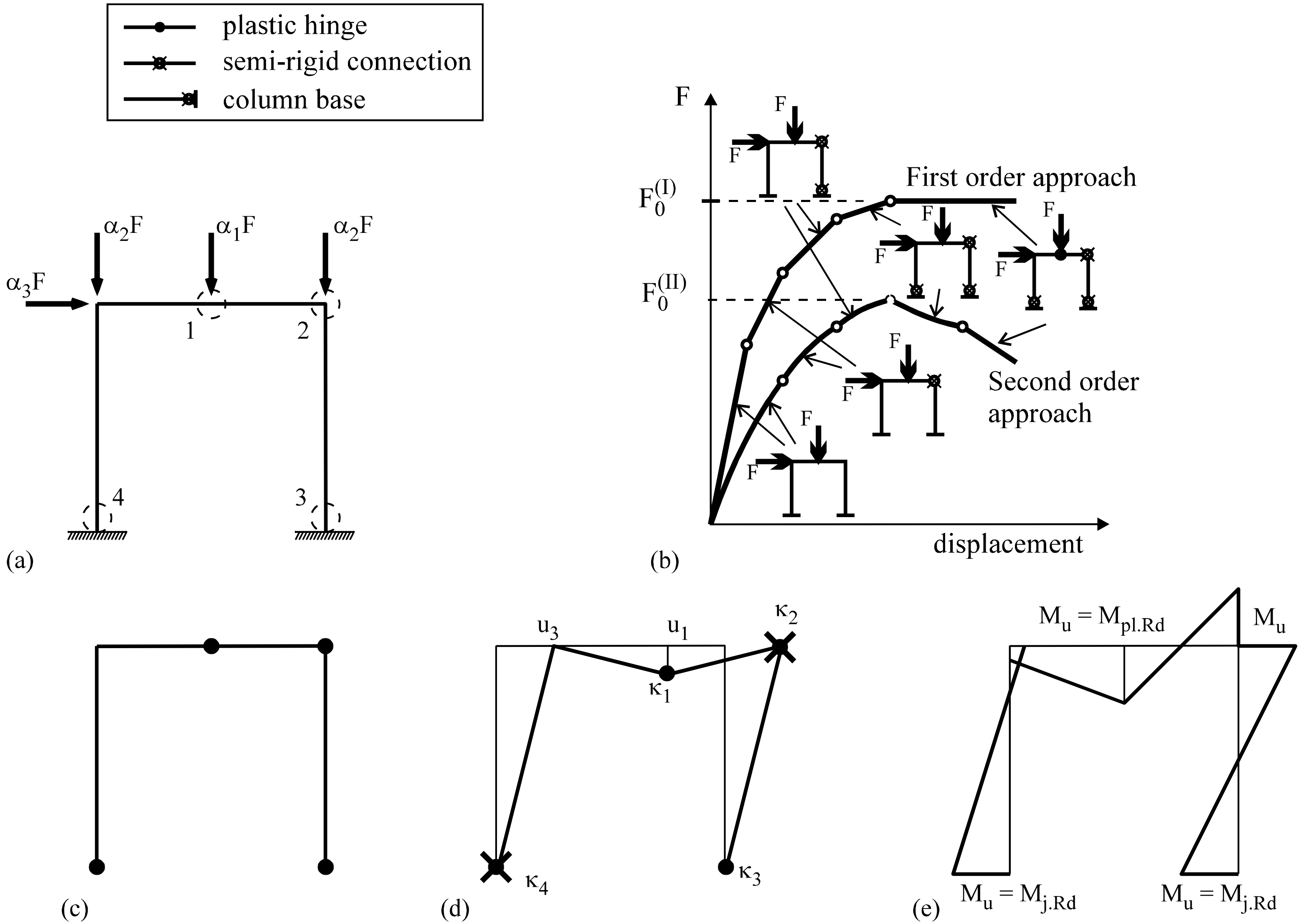
The required value of M_{pl} will be:

$$M_{pl} = F_0^{(I)} \cdot \frac{\sum_i \alpha_i u_i}{\sum_j \left|\kappa_j\right|}$$

– *Modified second order approach:* [Halasz, 1969]

$$F_0^{(II)} = f_I\left(M_{pl}; EI; M_{Rd}; S_{joint}\right)$$

$M_{pl.Rd}$ – as above,
EI – the elastic stiffness of the cross sections,
$M_{j.Rd}$ – as above,
S_{joint} – the rotational stiffness of the semi-rigid connection or column base.



First and second order approach

The second order load-deflection curve differs basically from that based on the first order approach as follows:

(i) the branches are curvilinear;

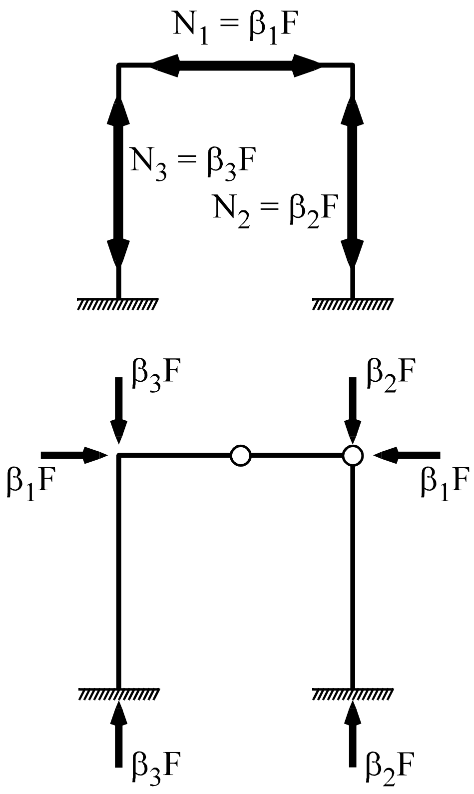
(ii) the failure load (or peak load) is lower than in the case of the simple plastic (first order) theory;

(iii) the failure may occur before the complete yield mechanism has developed and is followed by unstable behaviour.

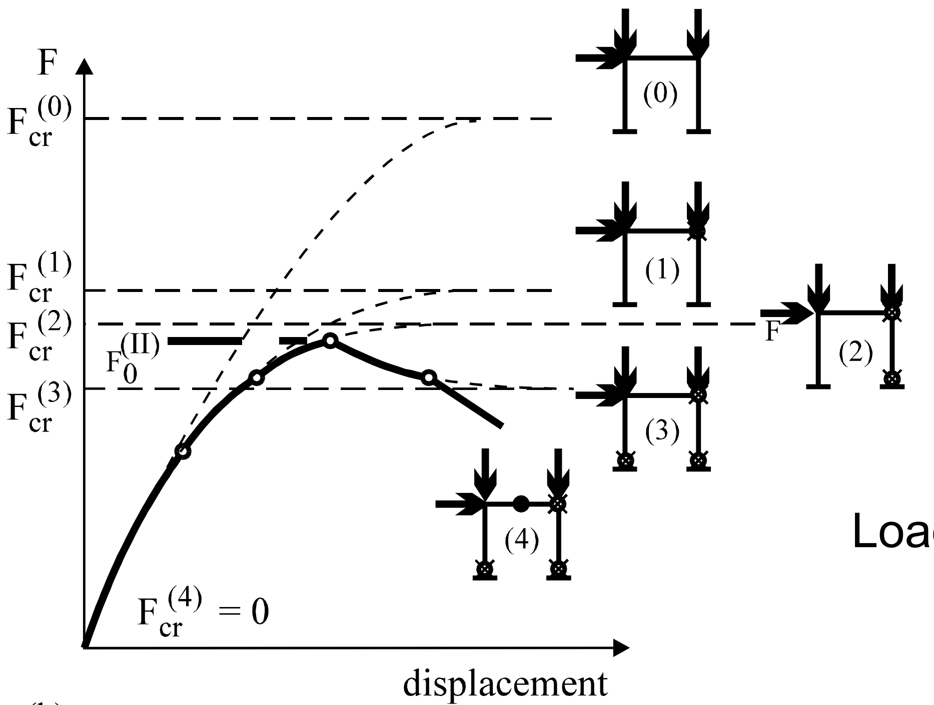
In addition, the location and sequence of occurrence of the generalised hinges do not necessarily coincide with those in the case of first order theory

9.5.4 Assumptions

Let us express the axial forces in the form: $N_k = \beta_k \cdot F$



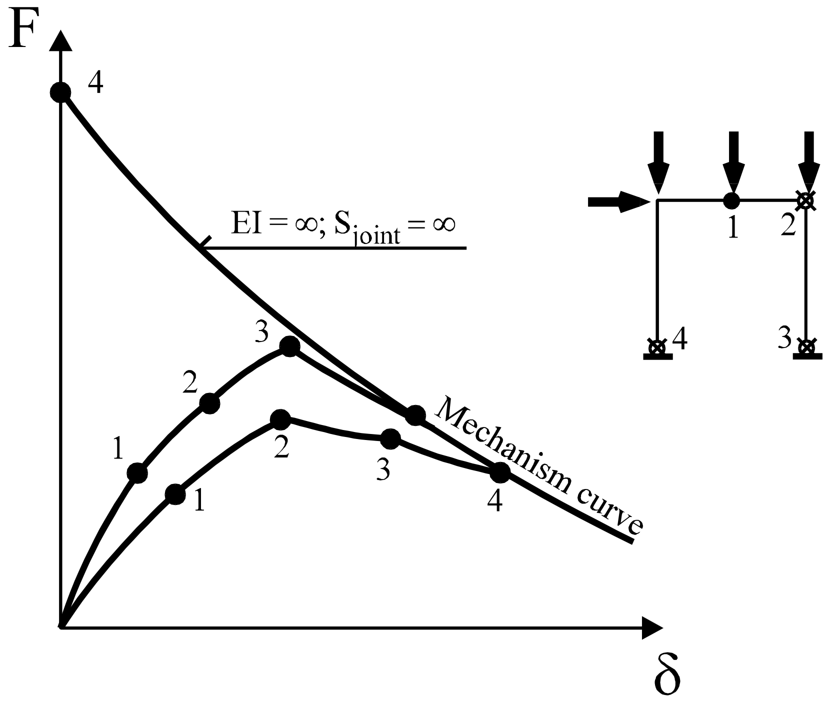
(a)



(b)

Load-displacement curve

9.5.5 The case of the “direct method of design”



Load-deflection curves for different stiffness values

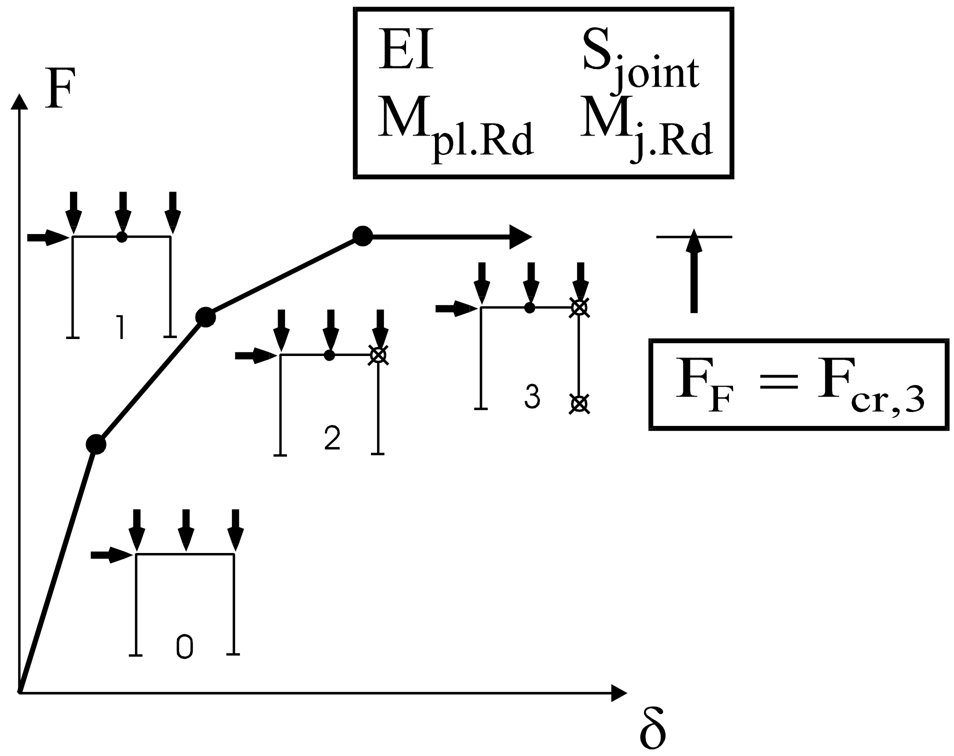
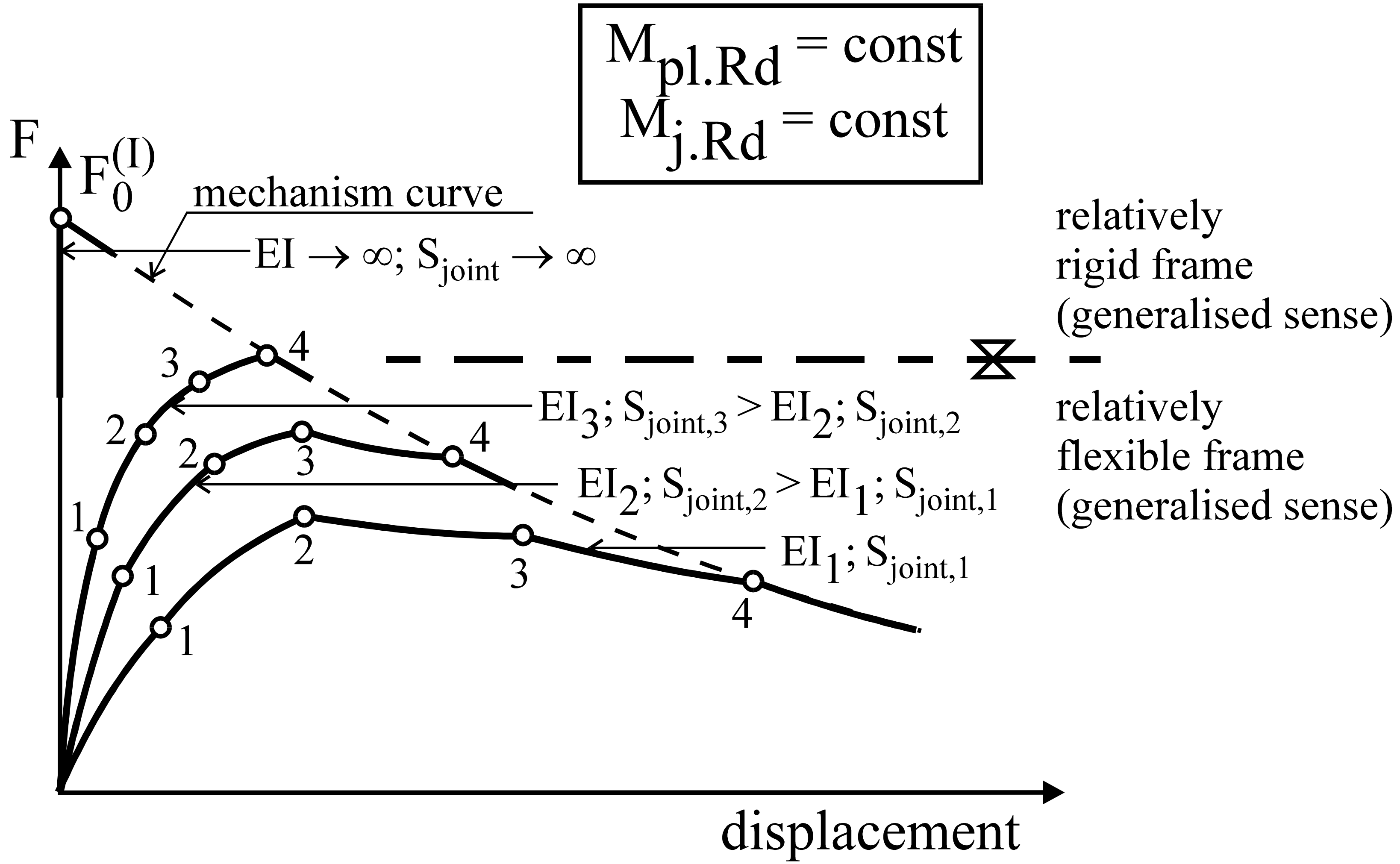


Illustration of the direct method of design



Relatively rigid and relatively flexible frames
in the generalised sense

We can distinguish two frame types:

- "relatively rigid" frame: the failure should take place when the number of generalized hinges has reached the number *n* of hinges necessary for the complete mechanism, thus *i*=*n*;
- "relatively flexible" frame: the failure load is reached in the presence of a lower number of generalized hinges than that transforming the structure into a complete mechanism (*i*<*n*).

The "deteriorated" critical load : $F_{cr,3} = \dfrac{cEI}{L^2}$ $\qquad F_F = F_{cr,3}$

The "required value of the frame rigidity: $EI = \dfrac{L^2}{c} F_F$ $\qquad F_{cr} = F_{cr,3}$

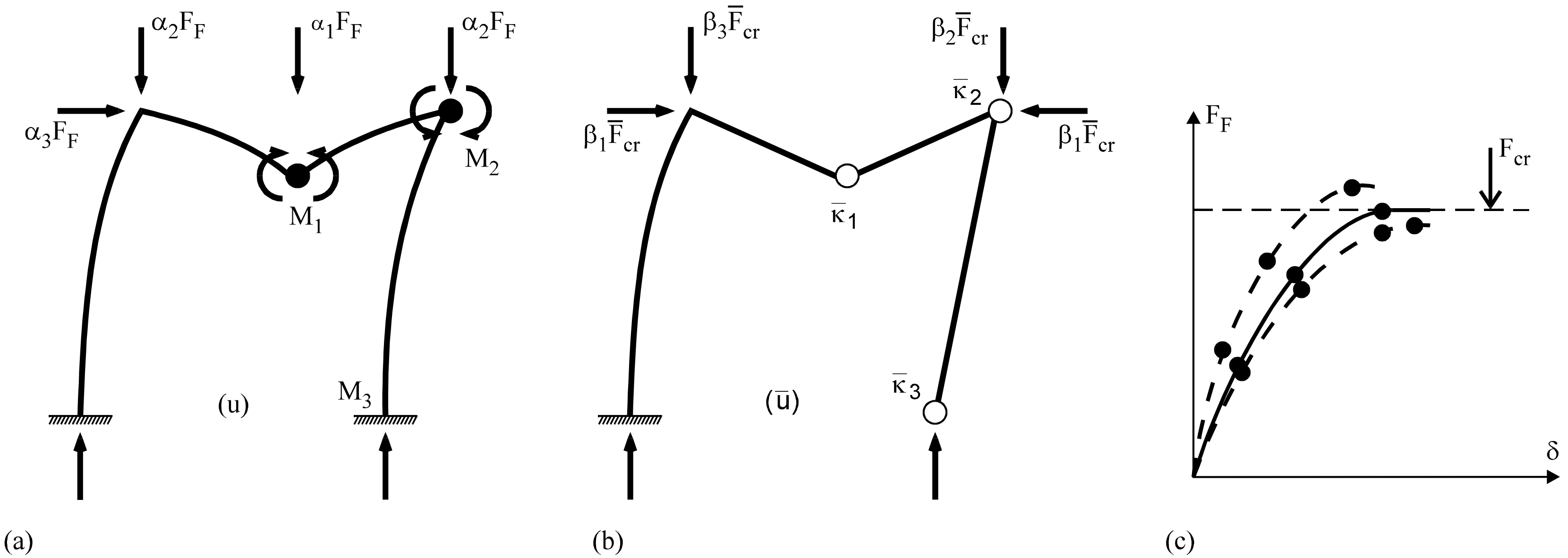
$$\sum_i \alpha_i F_F \overline{u}_i - \sum_j M_j \overline{\kappa}_j + \sum_k \beta_k F_F \int_l EIu'\overline{u}'\,\mathrm{d}x = \int_l EIu''\overline{u}''\,\mathrm{d}x$$

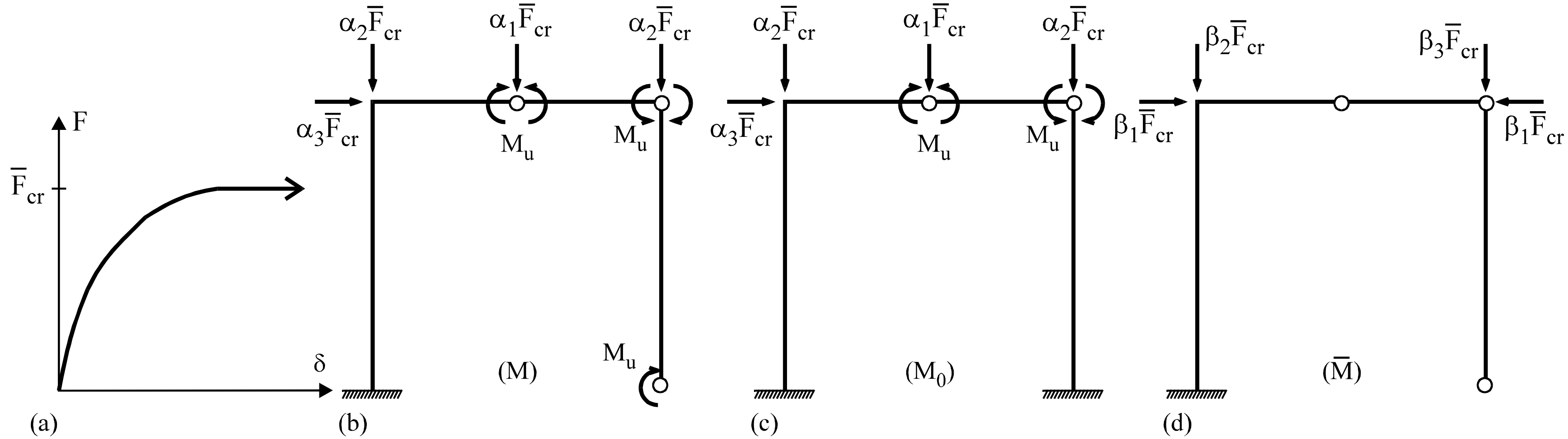
$$\sum_k \beta_k P_{cr,3} \int_l u'\overline{u}'\,\mathrm{d}x = \int_l EIu''\overline{u}''\,\mathrm{d}x \qquad u_3 = \overline{u}_3 = 0$$

$$\sum_i \alpha_i F_F \overline{u}_i - \sum_j M_j \overline{\kappa}_j = \sum_k \beta_k (F_{cr,3} - F_F) \int_l u'\overline{u}'\,\mathrm{d}x$$

$$F_F \sum_i \alpha_i \overline{u}_i = \sum_j M_j \overline{\kappa}_j \qquad F_F \sum_i \alpha_i \overline{u}_i = \sum_j \left| M_{pl} \overline{\kappa}_j \right| \qquad \boxed{M_{pl} = F_F \frac{\sum_i \alpha_i \overline{u}_i}{\sum_j \left| \overline{\kappa}_j \right|}}$$

Készült az ERFP – DD2002 – HU – B – 01 szerzősésszámú projekt támogatásával





$$M = M_0 + a \cdot \overline{M}; \quad \kappa = \kappa_0 + a \cdot \overline{\kappa}; \quad a \to \operatorname{sign} \kappa = \operatorname{sign} \overline{\kappa}; \quad |\kappa|_{min} = 0$$

Calculation of the required value of the generalised plastic moment M_{pl}

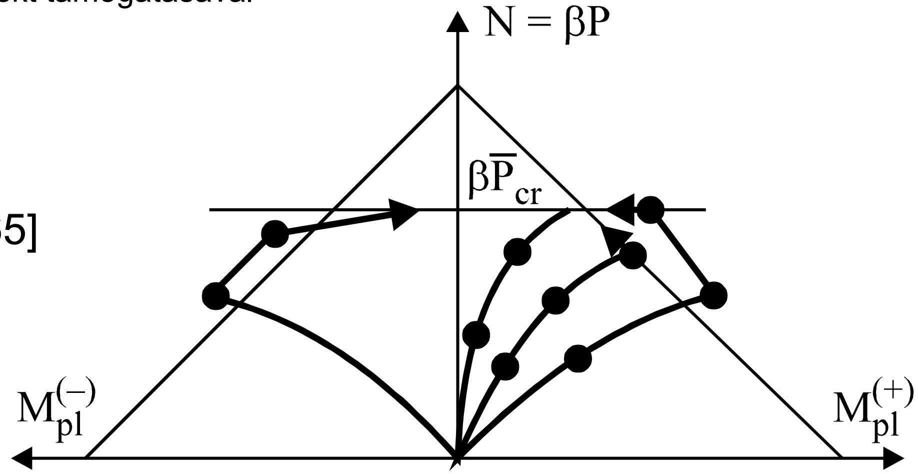
$$F_F = \overline{F}_{cr} = F_{cr,3}$$

$$M = M_0 + a\overline{M}$$ [Halasz, 1969]

$$\kappa_j = \kappa_{0j} + a\overline{\kappa}_j$$ [Horne, Merchant, 1965]

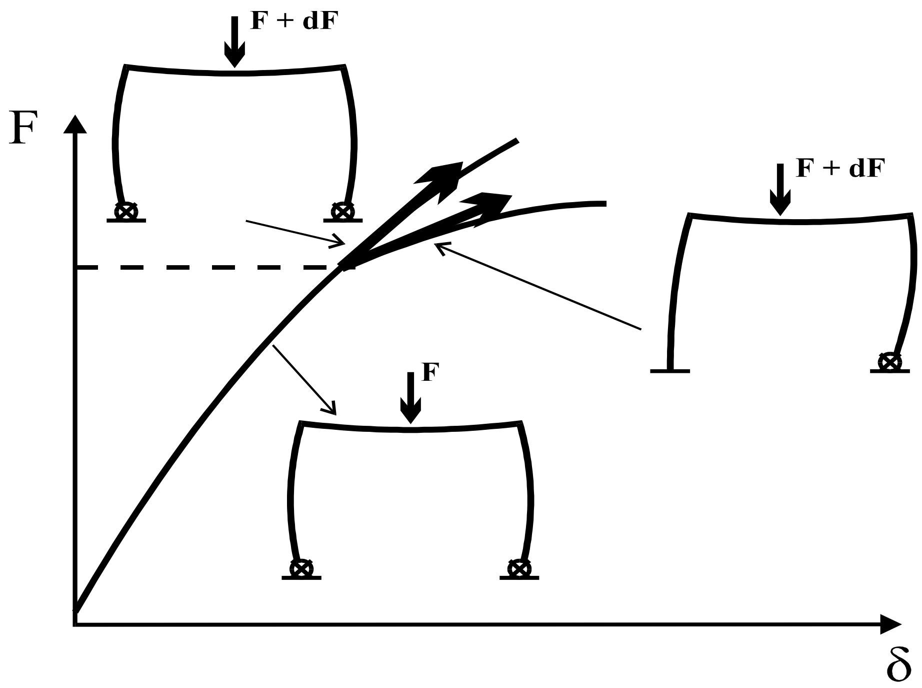
$$\operatorname{sign}\kappa_j = \operatorname{sign}\overline{\kappa}_j$$

$$\left|\kappa_j\right|_{\min} = 0$$



Interaction between axial force and full plastic moment

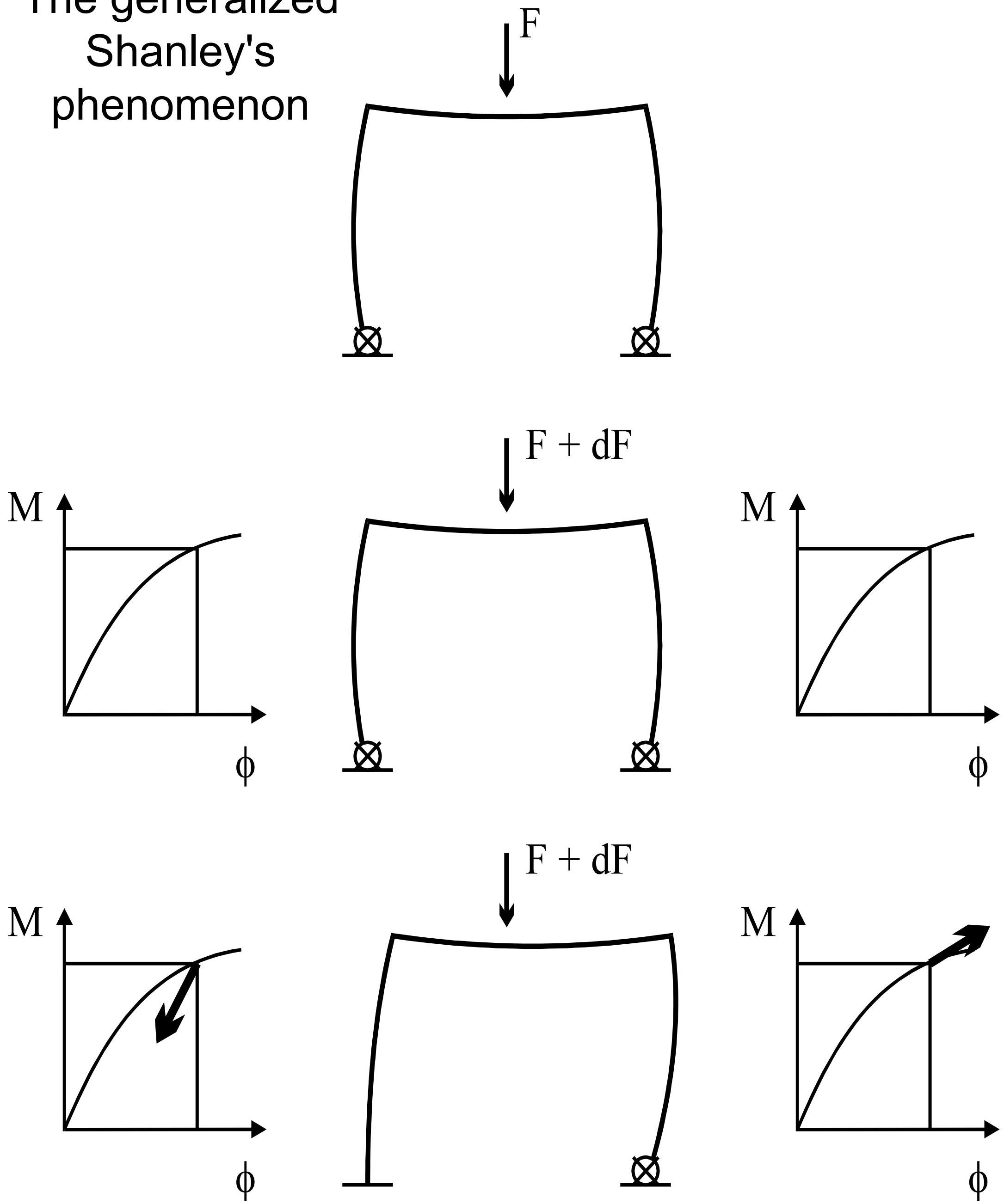
9.5.6 A generalisation of Shanley's phenomenon



$$\sum \alpha_i \overline{u}_i = 0$$ [Hill, 1958]

[Halasz, 1967]

The generalized Shanley's phenomenon



Bifurcation under stable conditions