# C h a p t e r   10

Stability and Ductility of Steel Frames with Semi-Rigid Connections

10.1. Introduction

10.1.1 Load-Deformation Response of Frames

Frame behaviour is characterised by the relationship between the loads, as they vary during the loading history, and the resulting deformations. A typical load-deflection curve is shown in the following Figure.

(A) Interactive Plastic Hinge. The traditional concept of plastic design of steel structures is based on the assumption that under gradually increasing static loads plastic zones develop and grow in size and number, and eventually cause unstricted, increasing deflections; thus loading to the onset of ultimate limit state of the structure. The concept was first introduced by *Kazinczy* by establishing concept of the   plastic hinge".

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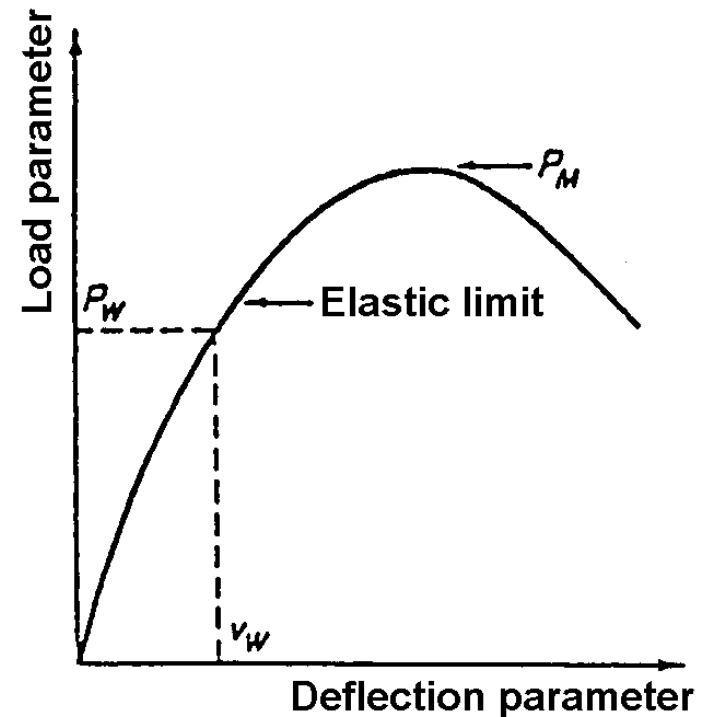
(B) Semi-rigid Connection. The type of beam-to-column connection used is a primary determinant of the behaviour of the frame. The construction types are defined in terms of connection rotational stiffness and moment resistance as represented by a moment-rotation (*M*-θ) diagram (Figure). Generally, accurate *M*-θ diagrams can only be obtained experimentally by tests.
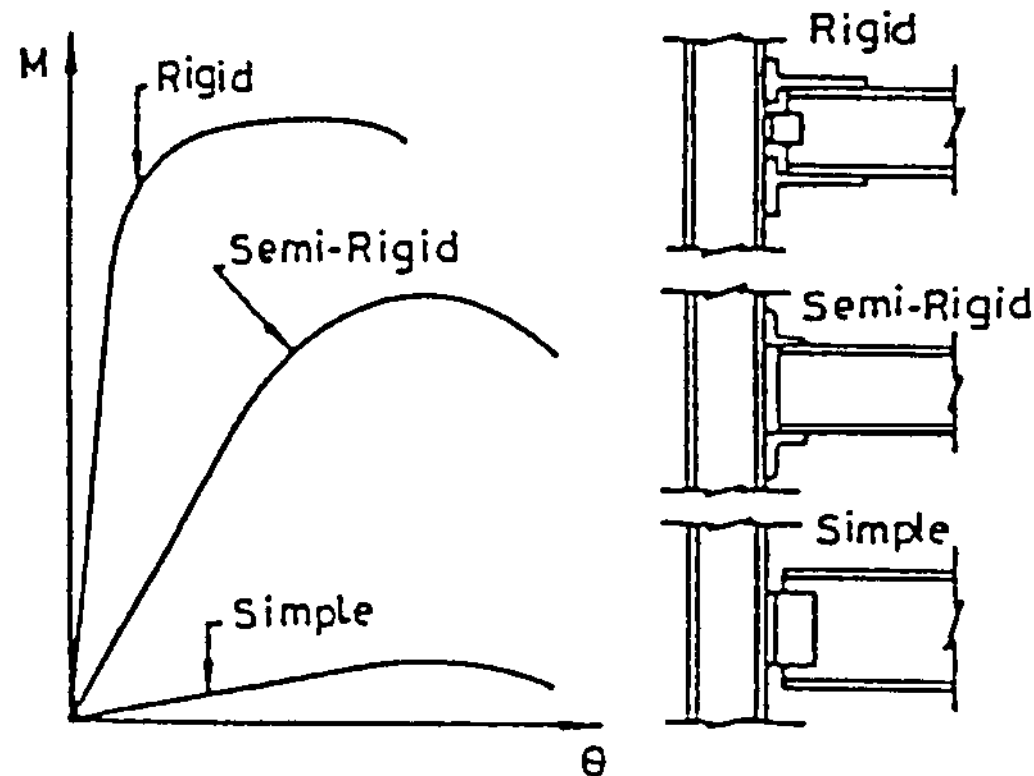
Local and lateral buckling are so diverse and so complex that large amounts of experimental and analytical data must be collected and systematized thus reliable frame analysis become practice.

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10.1.2 Effect of Softening Phenomenon

Studying the effect of softening phenomenon one should keep in mind that the load-displacement diagram of the structure may be of an ascending type even if the of the given member section or semi-rigid connections are of a descending type.

In the theory of plasticity, when deriving the condition of plasticity or some other physical relationships, *Drucker's* postulate for stability is applied, by assuming stable materials.

It should be noted that *Drucker's* postulate is not a natural law but a criterion of classification, the materials very often do not correspond to the assumptions of stable materials, or structural elements may behave in an unstable way, while, at the same time, their material is of a stable state.

Maier and *Drucker* re-examined the original *Drucker* postulate applied when determining the condition of plasticity since the original postulate is suitable for the determination of the convexity and normality of the condition of plasticity in case of stable materials only.

When studying the load bearing capacity of steel structures, the problem of unstable material or softening material, according to *Drucker's* postulate does not appear since the strain-hardening of the steel material may increase in a major way the plastic load bearing capacity of steel structure. However, as it has been known for a long time, the final collapse of steel structures is caused – in a high percentage of cases – by instability (plate buckling, flexural-torsional buckling), or semi-rigid phenomena that may occur in the cross section or in a structural unit.

The purpose of this article is to describe (1) how the load-deflection curve of frames may be constructed in as exact a manner as possible and (2) to describe approximate methods whereby the load-deflection curve, and particularly P_M, can be estimated.

10.2. Investigation of Plate Buckling with the Aid of Yield Mechanism

In the course of plate experiments, if the thickness/width ratio is small the plate does not lose its load-bearing capacity with the development of plastic deformation but is able to take the load causing yield until a deformation characteristic of the plate occurs; it is even able to take a small increase in load. In the course of the process „crumplings" (buckling) can be observed on the plate surface. These „crumplings" are formed by a yield mechanism, with the plastic moments acting in the linear plastic hinges (peaks of waves) not constant but ever-increasing due to strain-hardening. The yield mechanism performed by „crumpling" extends to the component plates of the bar. The description of its behaviour is obtained, from among the extreme-value theorems of plasticity, with the aid of the theorem of kinematics.

Thus, in the course of our investigations, an upper limit of load bearing has been determined. However, to be able to assess the results, the following have to be considered: on one hand, the yield mechanisms are taken into account through the „crumpling" forms determined experimentally; and on the other hand, the results of theoretical investigations are compared with the experimental ones.

10.2.1 Yield mechanism forms based on experimental results

The different forms of yield mechanisms can be determined on the basis of experimental results. The yield mechanism forms of an I-section bar can be classified according to the following criteria.

(a) According to the manner of loading.

(b) According to the positions of the intersecting lines of the web and the flanges, the so-called throat-lines"; thus,

(i) the evolving formation is called a planar yield mechanism if the two throat-lines" are in the same plane after the development of the yield mechanism.

(ii) the evolving formation is called a spatial yield mechanism if the two throat-lines" are not in the same plane after the development of the yield mechanism.

10.2.1.1 Bending moment constant along the members axis

(a) Planar yield mechanism: The buckled form of the bent specimen and the chosen yield mechanism formation are shown in the following Figure. As an effect of moment *M*, a rotation θ develops.
As an effect of *M*, tension and compression regions develop.
The symbol of the yield mechanism is *(MC)$_P$*, where *C* stands for the constant bending moment.
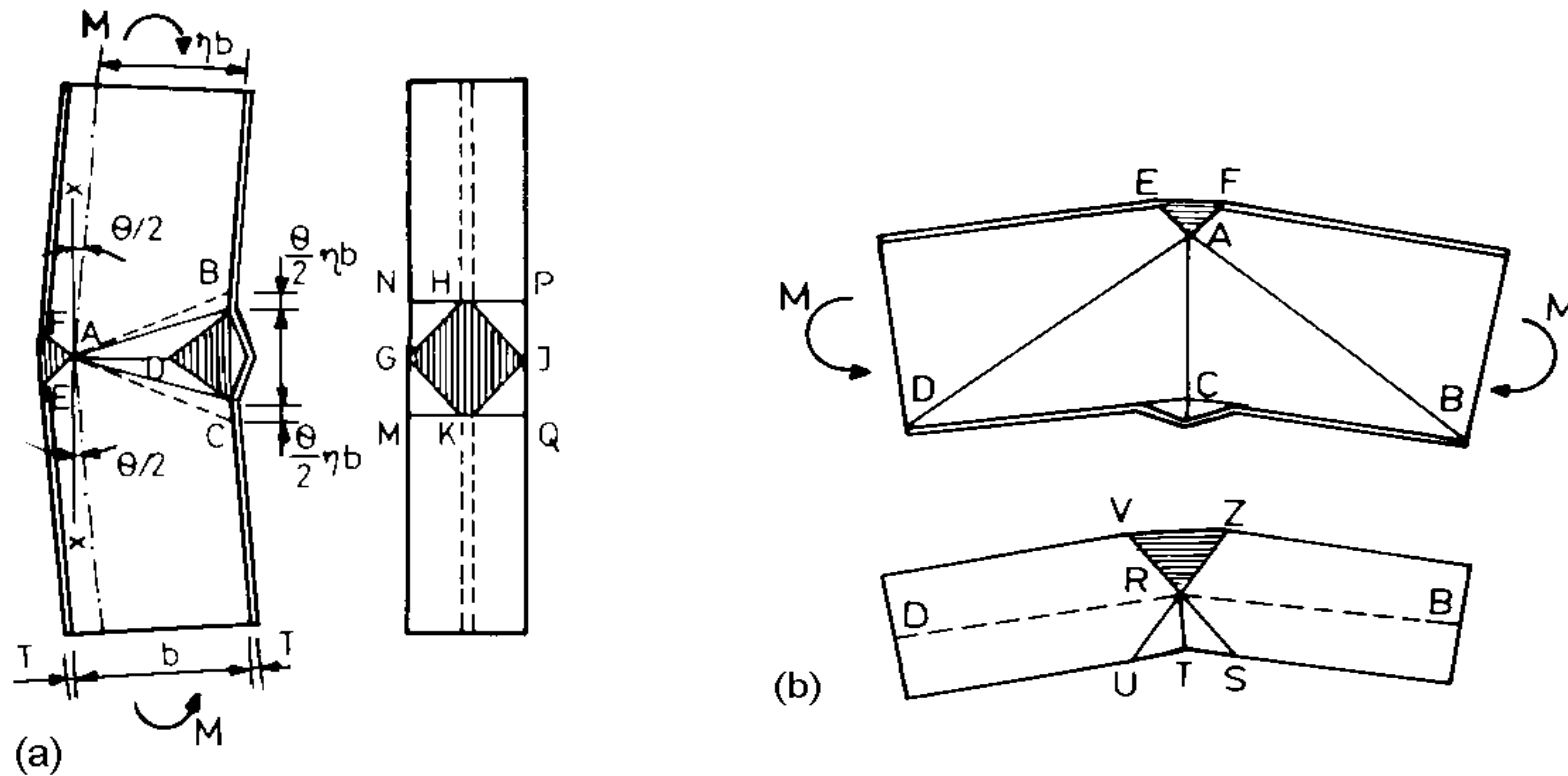
Planar *(a)* and spatial *(b)* yield mechanism of bent member with bending moment constant along the member axis

(b) Spatial yield mechanism: The form of the spatial yield mechanism in the case of a bent rod is indicated in the Figure. The rod ends are assumed to be hinge-supported in both main inertia directions. The yield mechanism models the buckling of the component plates of the bent member, the lateral buckling of the beams as well as their interaction. The symbol of this yield mechanism is $(MC)_S$.

10.2.1.2 Yield mechanism of the component plates of an I-section member

Yield mechanism formations have been determined for different stresses. On the basis of the experimental results it is expedient to decompose these yield mechanism formations into the yield mechanism formations of the component plates of an I-section rod, as certain component plate formations appear in other yield mechanisms too.

To classify the yield mechanisms of component plates, the following division has been used.

(a) Flange plate, if the plate is supported along one line.
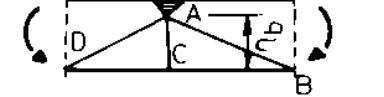
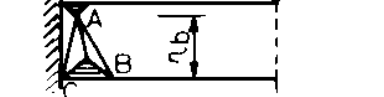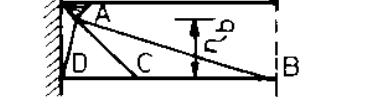(b) Web plate, if the plate is supported at the unloaded ends.
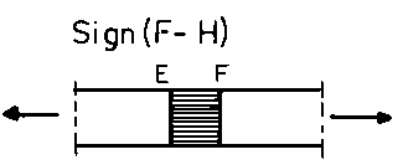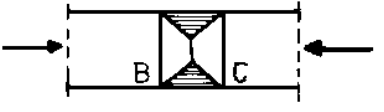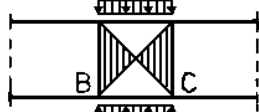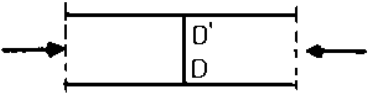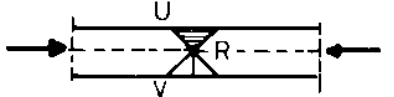
bi) axial forces and bending (W-1) – (W-6)
(bii) transverse forces transmitted directly through the web (W-11-12-13)
(bii) transverse forces only on one side of the web panel (W-21-22-23)
(biii) tension fields on the web panel (W-30; W-40)

Next Figure shows the yield mechanisms of the component plates where F is the flange plate, W is the web plate; the odd numbers refer to the planar yield mechanisms and the even ones to the spatial yield mechanisms.

Yield mechanisms of the component plate elements of an I-section member and beam-to-column joints

10.2.1.3 Yield Mechanism of Joint Configurations

Yield Mechanism of Single-sided Joint Configurations. The main sources of deformability of joint configuration which must be contemplated in a beam-to-column major joint are:
– the connection deformability M_b–θ_c characteristic;
– the column web panel shear deformability V_{wp}–γ characteristic;
– the local buckling of column web panel.
In the case of the yield mechanism formations in the next Figure, the effect of the beam local buckling cross-section, column web shear panel and patch loading has also been taken into account.
The symbol of this yield mechanism is *(SSJ)$_P$.*

Yield Mechanism of Double-sided Joint Configurations. The main sources of deformability of joint configuration which must be contemplated in a beam-to-column major joint are:
– the left hand side connection deformability M_{b1}–θ_{c1} characteristic;
– the right hand side connection deformability M_{b2}–θ_{c2} characteristic;
– the column web panel shear deformability V_{wp}–γ characteristic;
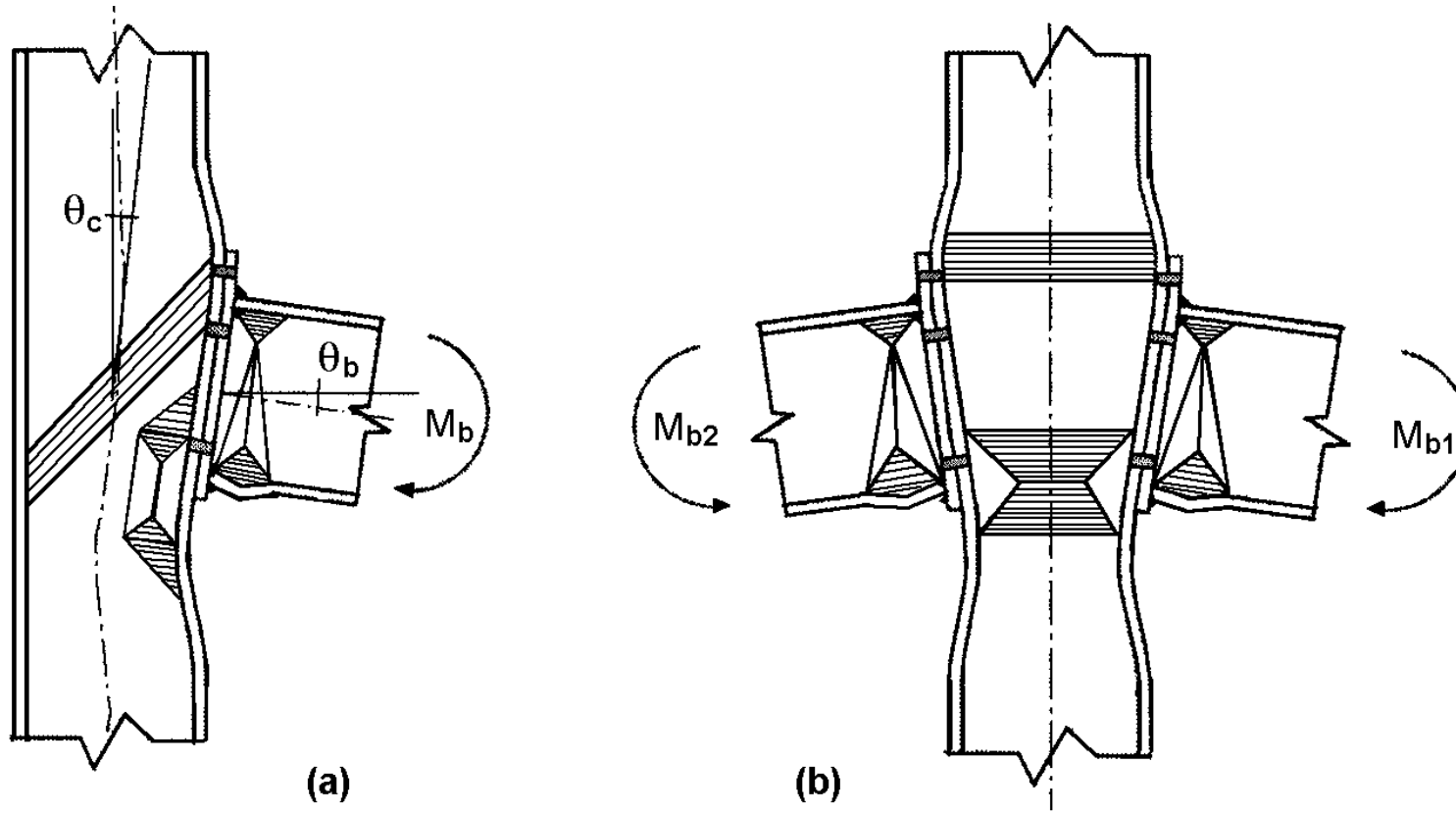– the local buckling of column web panel.
The yield mechanism formation is in the next Figure. The symbol of this yield mechanism is *(DSJ)$_P$.*

Single-sided *(a)* and double-sided *(b)* joint configuration

10.2.2 "Joining" the yield mechanisms of component plates

The "joining" of the yield mechanisms of component plates depends on the positions of the so-called "throat-lines" of the yield mechanism chosen on the basis of the experimental results.

In cases pertaining to planar yield mechanisms, this "joining" is to be realised in a linear manner, with a linear plastic hinge: the length of the linear plastic hinge is governed – due to the properties of the chosen yield mechanism – by the length of the yield mechanism of the compression flange plate (F-1). In the case of spatial yield mechanisms, the "joining" should be realised at one or more points.

The relationships between the component plate yield mechanisms and the "joining" of the component plates have been given by *Ivanyi* who later gave the basic relationships of partial cases.

10.2.3 Model of the interactive hinge

The model of the interactive hinge taking into consideration the effect of rigid – residual stress – strain-hardening – plate buckling can be described with the aid of the "equivalent beam length" suggested by *Horne* (next Figure). The material model employed in the investigations is shown in the Figure. The effect of the residual stresses and deformations is substituted by a straight line. The effect of strain-hardening can be determined with the help of the rigid-hardening (R-S) model. The buckling of the I-section member component plates is described by the yield mechanism curve, which is substituted by a straight line.

The Figure indicates the load-displacement relationship of the (R-O-S-L) interactive hinge.

The substitution by straight lines is justified to simplify the investigations. In the (R-O-S) sections the intersections are connected while in section L the moment-rotation relationship is substituted by a tangent that can be drawn at the apex.
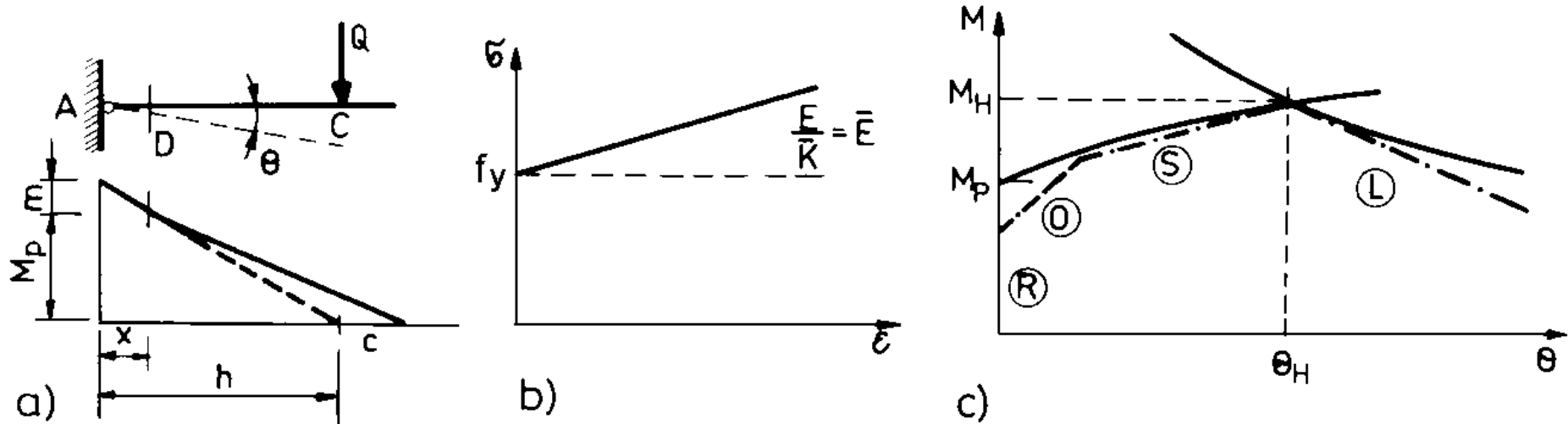
Model of interactive hinge

10.3. Analysis of Steel Frames with Global Bar Elements

Matrix methods are available for computer determination of stresses in plane bar systems. Our goal seemed to be better achieved by applying complex bar element (next Figure).
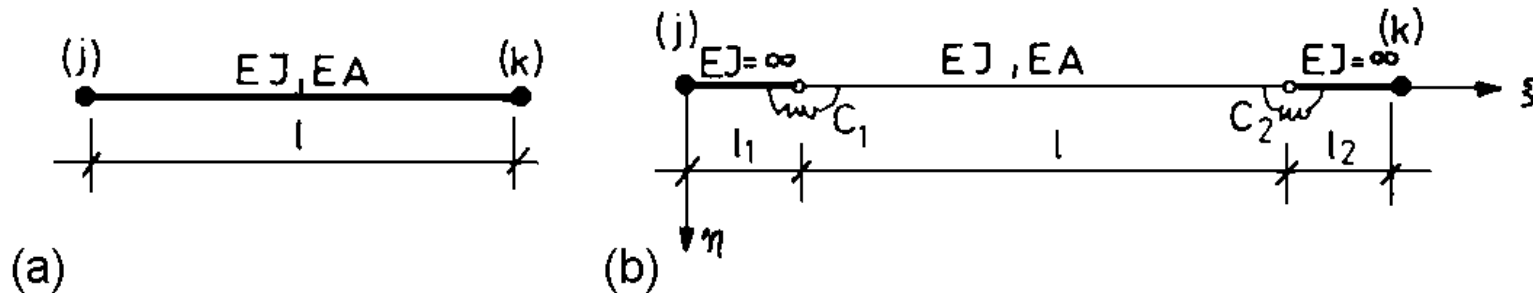
Simple *(a)* and complex *(b)* bar element

10.4. A Simple Approximate Method

Numerous approximate engineering methods are introduced in the literature, from which as one of the possibilites we are going to deal with the extension of the *Mechanism Curve Method*. The *Mechanism Curve Method* – above the determination of the plastic load bearing capacity – can be applied to take the effect of finite deformations and strain hardening of steel into consideration.

10.5. Evaluation of Load Bearing Capacity of Steel Frames

10.5.1 Test Program

The experimental research project was carried out in the Laboratory of the Department of Bridges and Structures, Budapest University of Technology and Economics.

An overall view of the testing arrangement is shown in the following Figure. The frames examined are two-storey single-bay ones. Both the columns and the beams are welded I sections. Pairs of test frames are identical. Columns are connected to a rigid steel base element by two bolts through an end plate (layout generally regarded as pinned joint in the practice).

2 x 1645

3500

(a)

(b)

OTKA–1

Beams (flange/web):
130-8 / 260-8

Columns (flange/web):
200-12 / 260-8
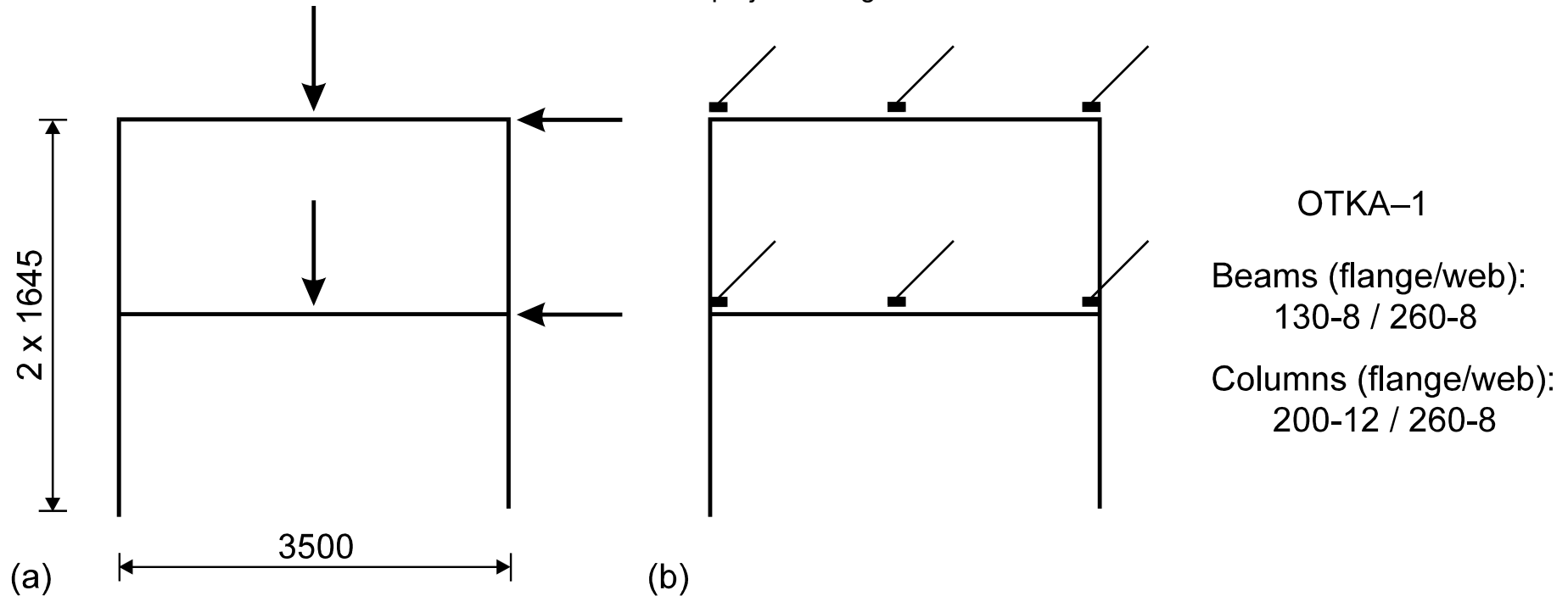
The tested frame: *(a)* main geometry and *(b)* lateral restrains

Beams and columns are connected with flush end plate joints (next Figure). In frame OTKA-1, the connections are strengthened with single-sided additional web plates. These were found to be necessary on the basis of an analysis of joint behaviour according to Revised Annex J of Eurocode 3.
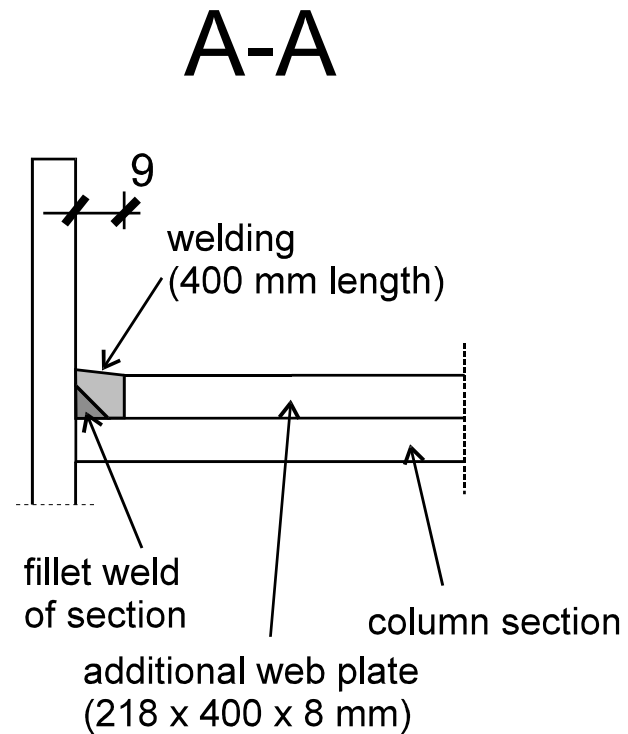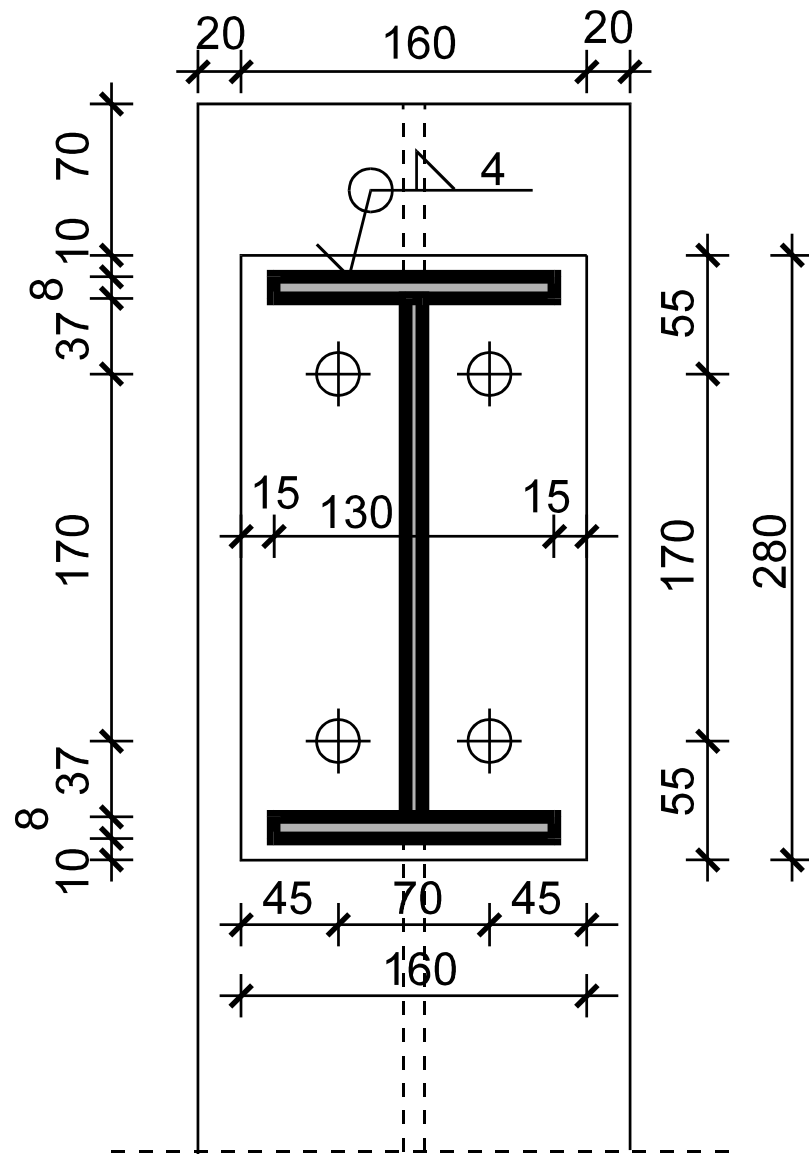
**Beam-to-column connection and the detail of
the additional web plate**

In order to avoid lateral-torsional buckling, lateral restraints are applied to the frame at the beam-to-columnjoint locations and at the mid-spans of the beams, see the Figures.

The frame is loaded by two vertical concentrated loads at the mid-spans of the beams, and two horizontal loads applied at one side of the frame in the levels of the beams (Figure). The two vertical loads are increased and decreased proportionally using three hydraulic jacks (one larger to the lower beam and two smaller and identical to the upper) connected into one oil circuit. Because of the slight difference between the pressure surfaces of the larger jack on one hand and the two smaller jacks on the other, the lower beam was loaded by a concentrated load 89% in magnitude of the load on the upper beam. The vertical loads are applied through so-called gravity load simulators, devices which ensure the verticality of the loads within certain limits of lateral displacements of the points of application of the loads. The horizontal loads are applied using one hydraulic jack through a simply supported vertical beam, which ensures the applied load to be equally distributed between the two beam levels. The direction of these horizontal loads is reversible.

10.5.2 Results of Theoretical and Experimental Investigation

Concerning the experimental frame OTKA-1, the relation of load-deflection curve developes according to the following Figure. The Approximate Engineering Method is presented on test frame OTKA-1. The comparison shows that the *Approximate Engineering Method* gives a satisfactory results for the maximum loads and the descending state path of whole structure as well; and at the same time the analysis can be done at the   desk of the designer   .
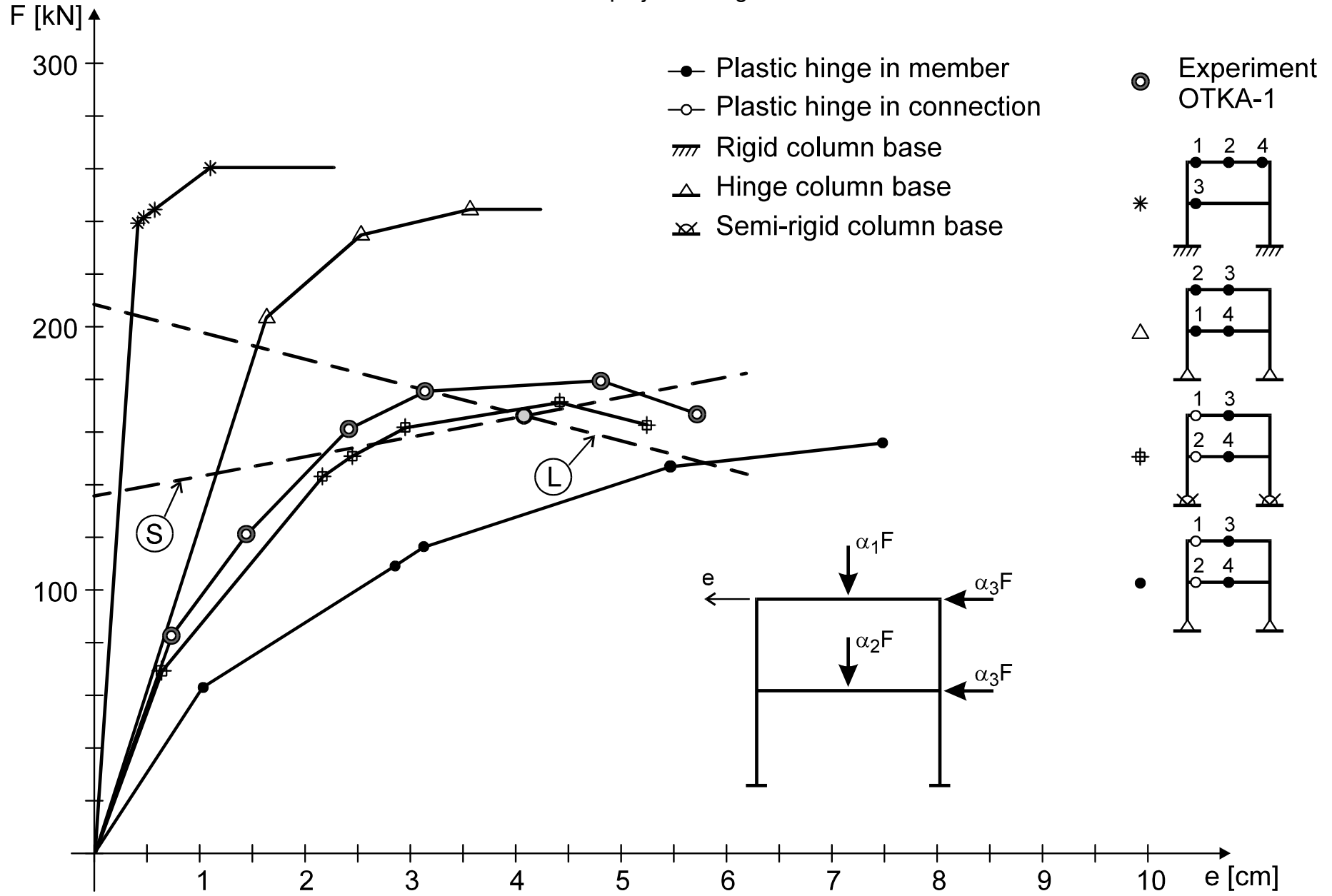
Load–displacement curve of experiment OTKA-1