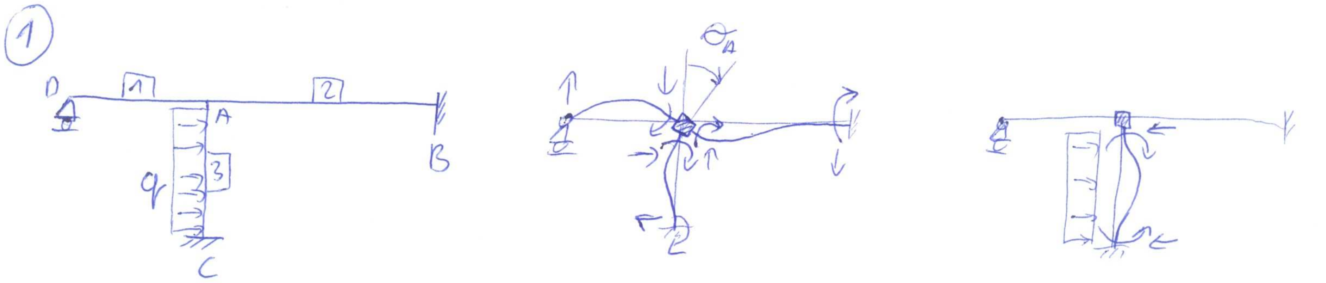
1



$$\Theta_A \to \sum M_A = (s_1'' k_1 + s_2 k_2 + s_3 k_3)\Theta_A + f_3 \cdot \frac{q \cdot l_3^2}{12} = \emptyset$$

$$S_1 = S_3 = \emptyset$$

$$N_2 = \frac{q \cdot l_3}{2} = \frac{40}{3} \cdot \frac{3}{2} = \underline{20 \text{ kN}}$$

$$P_{E2} = \frac{\pi^2 \cdot 1000}{5^2} = \underline{400 \text{ kN}} \quad \to S_2 = \frac{20}{400} = \underline{0{,}05}$$

$$k_1 = \frac{1000}{3} = \underline{333{,}3} \quad ; \quad k_2 = \frac{1000}{5} = 200 \; ; \quad k_3 = \frac{2000}{3} = \underline{666{,}6}$$

$$s_1'' = 3 \qquad s_2 = 3{,}934 \qquad s_3 = 4 \qquad s_2 c_2 = 2{,}017 \qquad s_3 c_3 = 2$$

$$s_2(1+c_2) = 5{,}950 \qquad s_3(1+c_3) = 6 \qquad f_3 = 1$$

$$\sum M_A = (3 \cdot 333{,}3 + 3{,}934 \cdot 200 + 4 \cdot 666{,}6) \cdot \Theta_A + 1 \cdot \frac{40}{3} \cdot \frac{3^2}{12} = \emptyset \quad \to \Theta_A = \underline{-0{,}002245} \; (\circlearrowleft)$$

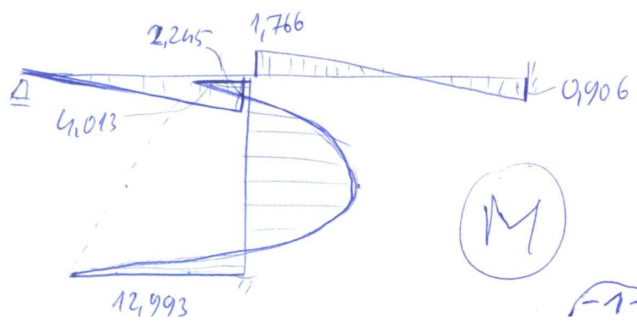
$$M_{1A} = s_1'' k_1 \cdot \Theta_A = 3 \cdot 333{,}3 \cdot (-0{,}002245) = \underline{-2{,}245 (\circlearrowleft)} \text{ kNm}$$

$$M_{2A} = s_2 k_2 \cdot \Theta_A = 3{,}934 \cdot 200 \cdot (-0{,}002245) = \underline{-1{,}766 (\circlearrowleft)} \text{ kNm}$$

$$M_{2B} = s_2 \cdot c_2 \cdot k_2 \cdot \Theta_A = 2{,}017 \cdot 200 (-0{,}002245) = \underline{-\emptyset{,}906 (\circlearrowleft)} \text{ kNm}$$

$$M_{3A} = s_3 \cdot k_3 \cdot \Theta_A + \frac{q \cdot l_3^2}{12} = 4 \cdot 666{,}6 \cdot (-0{,}002245) + \frac{40}{3} \cdot \frac{3^2}{12} = \underline{4{,}013 (\circlearrowright)} \text{ kNm}$$

$$M_{3C} = s_3 c_3 k_3 \Theta_A - \frac{q \cdot l_3^2}{12} = 2 \cdot 666{,}6 \cdot (-0{,}002245) - \frac{40}{3} \cdot \frac{3^2}{12} = -\underline{12{,}993 (\circlearrowleft)} \text{ kNm}$$



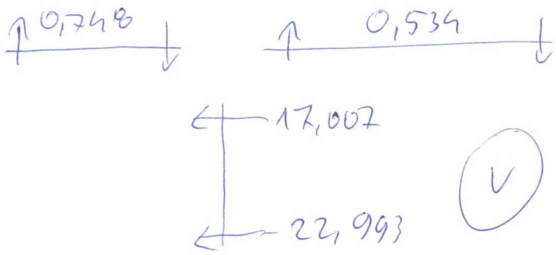
Nyíróerők:

$$V_{1A} = -s_1''\frac{k_1}{l_1}\Theta_A = -3\cdot\frac{333,3}{3}(-0,002245) = \underline{\underline{0,748}}\ kN(\downarrow) = V_{1B}(\uparrow)$$

$$V_{2A} = s_2(1+c_2)\cdot\frac{k_2}{l_2}\Theta_A = 5,95\ \frac{200}{5}(-0,002245) = \underline{\underline{0,534}}\ kN(\uparrow) = V_{2B}(\downarrow)$$

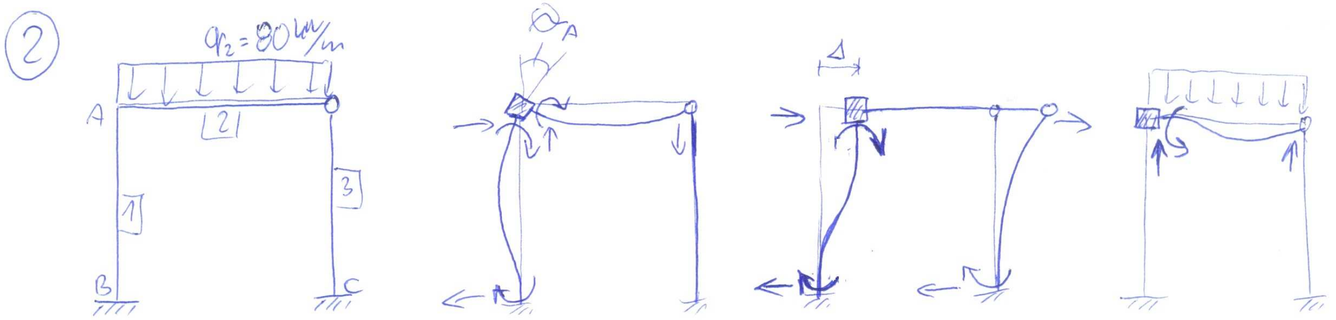
$$V_{3A} = -s_3(1+c_3)\frac{k_3}{l_3}\cdot\Theta_A - \frac{q\cdot l_3}{2} = -6\cdot\frac{666,6}{3}\cdot(-0,002245) - \frac{40}{3}\cdot\frac{3}{2} = \underline{\underline{-17,007(\leftarrow)\ kN}}$$

$$V_{3C} = s_3(1+c_3)\frac{k_3}{l_3}\cdot\Theta_A - \frac{q\cdot l_3}{2} = +6\cdot\frac{666,6}{3}(-0,002245) - \frac{40}{3}\cdot\frac{3}{2} = \underline{\underline{-22,993\ kN(\leftarrow)}}$$



$N_2^{új} = 17,007$ lenne.

2



$l_1 = l_2 = l_3 = 4\,m$; $EJ_1 = 3200\ kNm^2$; $EJ_2 = 6000\ kNm^2$; $EJ_3 = 1920\ kNm^2$

$\pi^2 \simeq 10$

$$\left.\begin{aligned}\Theta_A &\to \Sigma M_A = (s_1k_1 + s_2''k_2)\Theta_A - s_1(1+c_1)k_1\frac{\Delta}{l_1} - \frac{q_2\cdot l_2^2}{8} = \emptyset\\ \Delta &\to \Sigma V_x = \left(-s_1(1+c_1)\frac{k_1}{l_1}\right)\Theta_A + \left[\frac{2s_1(1+c_1)}{m_1}\cdot\frac{k_1}{l_1^2} + (s_3'' - \pi^2\cdot\rho_3)\cdot\frac{k_3}{l_3^2}\right]\cdot\Delta = \emptyset\end{aligned}\right\}\rightarrow \Theta_A, \Delta$$

$N_1 = \frac{5}{8}q_2 l_2 = \frac{5}{8}\cdot 80\cdot 4 = \underline{\underline{200\ kN}}$; $P_{E1} = \frac{\pi^2\cdot 3200}{4^2} = \underline{\underline{2000\ kN}}$; $\rho_1 = \frac{200}{2000} = \underline{\underline{0,1}}$

$N_2 \simeq \emptyset$; $P_{E2} = \frac{\pi^2\,6000}{4^2} = \underline{\underline{3750\ kN}}$; $\rho_2 = \frac{\emptyset}{3750} = \underline{\underline{\emptyset}}$

$N_3 = \frac{3}{8}q_2\cdot l_2 = \frac{3}{8}\cdot 80\cdot 4 = \underline{\underline{120\ kN}}$; $P_{E3} = \frac{\pi^2\cdot 1920}{4^2} = \underline{\underline{1200\ kN}}$; $\rho_3 = \frac{120}{1200} = \underline{\underline{\emptyset,1}}$

$k_1 = \frac{3200}{4} = \underline{\underline{800\ kNm}}$; $k_2 = \frac{6000}{4} = \underline{\underline{1500\ kNm}}$; $k_3 = \frac{1920}{4} = \underline{\underline{480\ kNm}}$

$$\Sigma M_A = (3,867 \cdot 800 + 3 \cdot 1500)\Theta_A - 5,901 \cdot \frac{800}{4}\Delta = \frac{80 \cdot 4^2}{8}$$

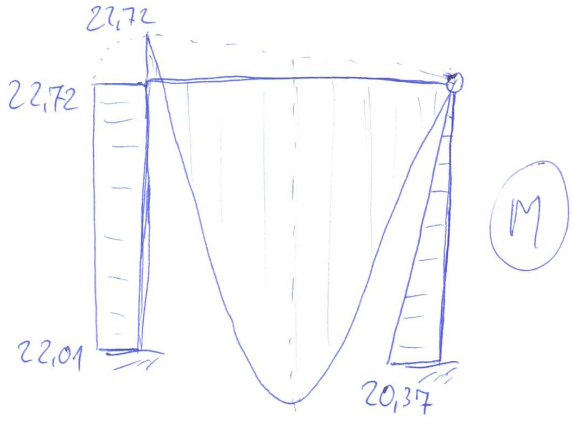
$$V_{AC} = \left(-5,901 \cdot \frac{800}{4}\right)\Theta_A + \left(10,814 \cdot \frac{800}{4^2} + 1,81 \cdot \frac{480}{4^2}\right)\Delta = \emptyset$$

$$\begin{bmatrix} 7593,36 & -1180,2 \\ -1180,2 & 593,1 \end{bmatrix} \begin{bmatrix} \Theta_A \\ \Delta \end{bmatrix} = \begin{bmatrix} 160 \\ \emptyset \end{bmatrix} \Rightarrow \underline{\underline{\Theta_A = 0,0305}} \ ; \ \underline{\underline{\Delta = 0,0607\ m}}$$

$$M_{1A} = M_{2A} = s_1 \cdot k_1 \cdot \Theta_A - s_1(1+c_1)k_1 \cdot \frac{\Delta}{l_1} = 3,867 \cdot 800 \cdot 0,0305 - 5,901 \cdot \frac{800}{4} \cdot 0,0607 = \underline{\underline{22,72\ kNm}}$$

$$M_{1B} = s_1 \cdot c_1 \cdot k_1 \cdot \Theta_A - s_1 \cdot (1+c_1)k_1 \cdot \frac{\Delta}{l_1} = 2,034 \cdot 800 \cdot 0,0305 - 5,901 \cdot \frac{800}{4} \cdot 0,0607 = \underline{\underline{-22,01\ kNm}}$$

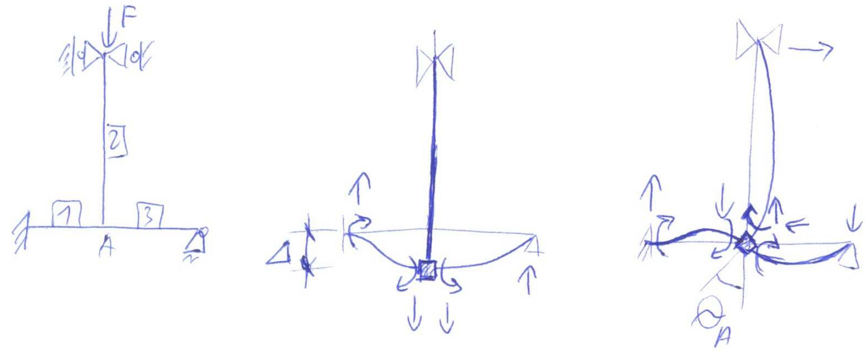
$$M_{3C} = -s_3'' k_3 \cdot \frac{\Delta}{l_3} = -2,797 \cdot \frac{480}{4} \cdot 0,0607 = \underline{\underline{-20,37\ kNm}}$$



$$N_2^{új} = V_3 = (s_3'' - \pi^2 \rho_3)\frac{k_3}{l_3^2} \cdot \Delta = \left(1,810 \cdot \frac{480}{4^2}\right) \cdot 0,0607 = \underline{\underline{3,296\ kN}} \text{ (nyomás)}$$

$$\rho_2^{új} = \frac{3,296}{3750} = \underline{\underline{0,00088 \cong \emptyset}} \ \checkmark$$

3) Kritikus erő



$l_1 = l_3 = 2\,m$

$l_2 = 4\,m$

$EJ_1 = EJ_3 = 200\,kNm^2$

$EJ_2 = 600\,kNm^2$

$$\Delta \to \Sigma V_y = \left[\frac{2s_1(1+c_1)}{m_1}\frac{k_1}{l_1^2} + \left(s_3'' - \pi^2\rho_3\right)\frac{k_3}{l_3^2}\right]\Delta + \left[-s_1(1+c_1)\frac{k_1}{l_1} - \left(-s_3''\frac{k_3}{l_3}\right)\right]\varphi_A = 0$$

$$\varphi_A \to \Sigma M_A = \left[-s_1(1+c_1)\cdot\frac{k_1}{l_1} - \left(-s_3''\frac{k_3}{l_3}\right)\right]\Delta + \left[s_1\cdot k_1 + s_3'' k_3 + s_2'' k_2\right]\cdot\varphi_A = 0$$

$$\rho_1 = \rho_3 \approx 0 \quad ; \quad \rho_2 \neq 0$$

$$\left[\begin{array}{c|c} \left(12\cdot\frac{100}{2^2} + 3\cdot\frac{100}{2^2}\right) & -6\cdot\frac{100}{2} + 3\cdot\frac{100}{2} \\ \hline -6\cdot\frac{100}{2} + 3\cdot\frac{100}{2} & 4\cdot 100 + 3\cdot 100 + s_2''\cdot 150 \end{array}\right]\left[\begin{array}{c} \Delta \\ \hline \varphi_A \end{array}\right] = \left[\begin{array}{c} 0 \\ \hline 0 \end{array}\right]$$

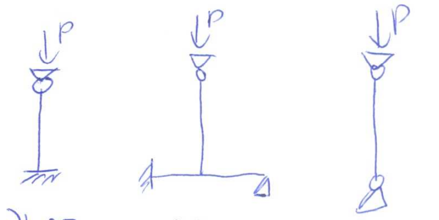
$$\det\begin{vmatrix} 375 & -150 \\ -150 & 700+150\cdot s_2'' \end{vmatrix} = 0 \implies 375(700 + 150\cdot s_2'') - 150^2 = 0$$

$$s_2'' = \left(\frac{150^2}{375} - 700\right)\frac{1}{150} = \underline{\underline{-4,2667}}$$

$$\left.\begin{array}{l} \rho_2 = 1,5 \to s_2'' = -4,215 \\ \rho_2 = 1,6 \to s_2'' = -6,032 \end{array}\right\} \rho_2 = 1,5 + \frac{1,6-1,5}{6,032-4,215}\left(4,2667 - 4,215\right) = \underline{\underline{1,503}}$$

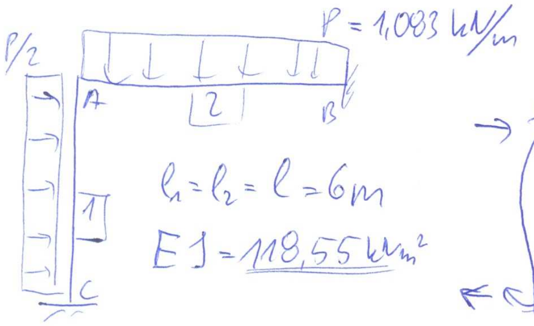
$$\rho_2 = \frac{P_{cr}}{P_{E_2}} \to P_{cr} = 1,503\cdot\frac{\pi^2\cdot EJ_2}{l_2^2} = 1,503\cdot\frac{\pi^2\cdot 600}{4^2} = \underline{\underline{556,3\,kN}}$$

$$\rho_2 = \frac{1}{\nu_2^2} \to \nu_2 = \frac{1}{\sqrt{\rho_2}} = \frac{1}{\sqrt{1,503}} = \underline{\underline{0,816}}$$



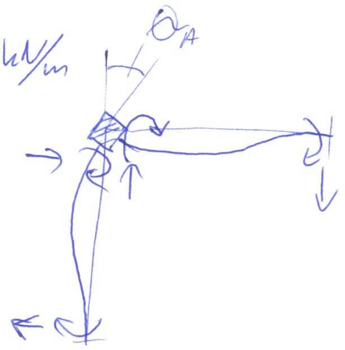
$\nu = 0,7$ $\quad$ $\nu = 0,816$ $\quad$ $\nu = 1$

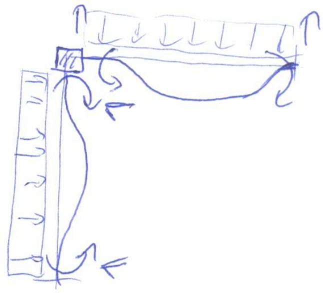
4



$l_1 = l_2 = l = 6\,m$

$EI = 118{,}55\ kNm^2$





$$k_1 = k_2 = k = \frac{EI}{l} = \frac{118{,}55}{6} = 19{,}76\ kNm$$

$$P_E = \frac{\pi^2 EI}{l^2} = \frac{\pi^2 \cdot 118{,}55}{6^2} = 32{,}50\ kN$$

$$N_1 = \frac{p \cdot l}{2} = \frac{1{,}083 \cdot 6}{2} = 3{,}249\ kN \rightarrow \rho_1 = \frac{3{,}249}{32{,}50} = 0{,}1$$

$$N_2 = \frac{p/2 \cdot l}{2} = \frac{1{,}083/2 \cdot 6}{2} = 1{,}625\ kN \rightarrow \rho_2 = \frac{1{,}625}{32{,}50} = 0{,}05$$

$$\Theta_A \rightarrow \Sigma M_A = \emptyset = (s_1 + s_2)\, k \cdot \Theta_A + f_1 \cdot \frac{p/2 \cdot l^2}{12} - f_2 \cdot \frac{p \cdot l^2}{12} = \emptyset$$

$$(3{,}867 + 3{,}934) \cdot 19{,}76 \cdot \Theta_A + 1{,}017 \cdot \frac{1{,}083/2 \cdot 6^2}{12} - 1{,}008 \cdot \frac{1{,}083 \cdot 6^2}{12} = \emptyset$$

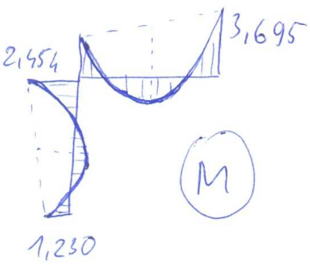
$$\Theta_A = 0{,}0105 \ (\curvearrowright)$$

$$M_{1A} = s_1 \cdot k \cdot \Theta_A + \frac{f_1}{2} \cdot \frac{p \cdot l^2}{12} = 3{,}867 \cdot 19{,}76 \cdot 0{,}0105 + \frac{1{,}017}{2} \cdot \frac{1{,}083 \cdot 6^2}{12} = 2{,}454\ kNm$$

$$M_{1C} = s_1 \cdot c_1 \cdot k \cdot \Theta_A - f_1 \cdot \frac{p/2 \cdot l^2}{12} = 2{,}034 \cdot 19{,}76 \cdot 0{,}0105 - 1{,}017 \cdot \frac{1{,}083 \cdot 6^2}{2 \cdot 12} = -1{,}230\ kNm$$

$$M_{2A} = s_2 \cdot k \cdot \Theta_A - f_2 \cdot \frac{p \cdot l^2}{12} = 3{,}934 \cdot 19{,}76 \cdot 0{,}0105 - 1{,}008 \cdot \frac{1{,}083 \cdot 6^2}{12} = -2{,}459\ kNm$$

$$M_{2B} = s_2 \cdot c_2 \cdot k \cdot \Theta_A + f_2 \cdot \frac{p \cdot l^2}{12} = 2{,}017 \cdot 19{,}76 \cdot 0{,}0105 + 1{,}008 \cdot \frac{1{,}083 \cdot 6^2}{12} = 3{,}695\ kNm$$

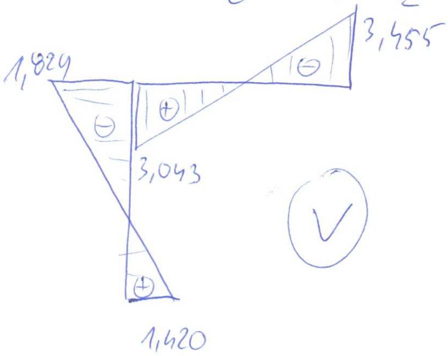


$$V_{1A} = -s_1(1+c_1)\frac{k}{l}\cdot\Theta_A - \frac{P/2\cdot l}{2} = -5{,}901\cdot\frac{19{,}76}{6}\cdot 0{,}0105 - \frac{1{,}083/2\cdot 6}{2} = -1{,}829\ kN\ (\leftarrow)$$

$$V_{1C} = -\left[-s_1(1+c_1)\frac{k}{l}\cdot\Theta_A\right] - \frac{P/2\cdot l}{2} = -\left(-5{,}901\cdot\frac{19{,}76}{6}\cdot 0{,}0105\right) - \frac{1{,}083/2\cdot 6}{2} = -1{,}420\ kN\ (\leftarrow)$$

$$V_{2A} = -s_2(1+c_2)\frac{k}{l}\cdot\Theta_A + \frac{P\cdot l}{2} = -5{,}950\cdot\frac{19{,}76}{6}\cdot 0{,}0105 + \frac{1{,}083\cdot 6}{2} = -3{,}043\ kN\ (\uparrow)$$

$$V_{2B} = -\left(-s_2(1+c_2)\frac{k}{l}\cdot\Theta_A\right) + \frac{P\cdot l}{2} = 5{,}950\cdot\frac{19{,}76}{6}\cdot 0{,}0105 + \frac{1{,}083\cdot 6}{2} = 3{,}455\ kN\ (\uparrow)$$



$$N_1^{uj} = 3{,}044\ kN\ (-) \rightarrow g_1^{uj} = \frac{3{,}044}{32{,}50} = 0{,}094$$

$$N_2^{uj} = 1{,}829\ kN\ (-) \rightarrow g_2^{uj} = \frac{1{,}829}{32{,}50} = 0{,}056$$