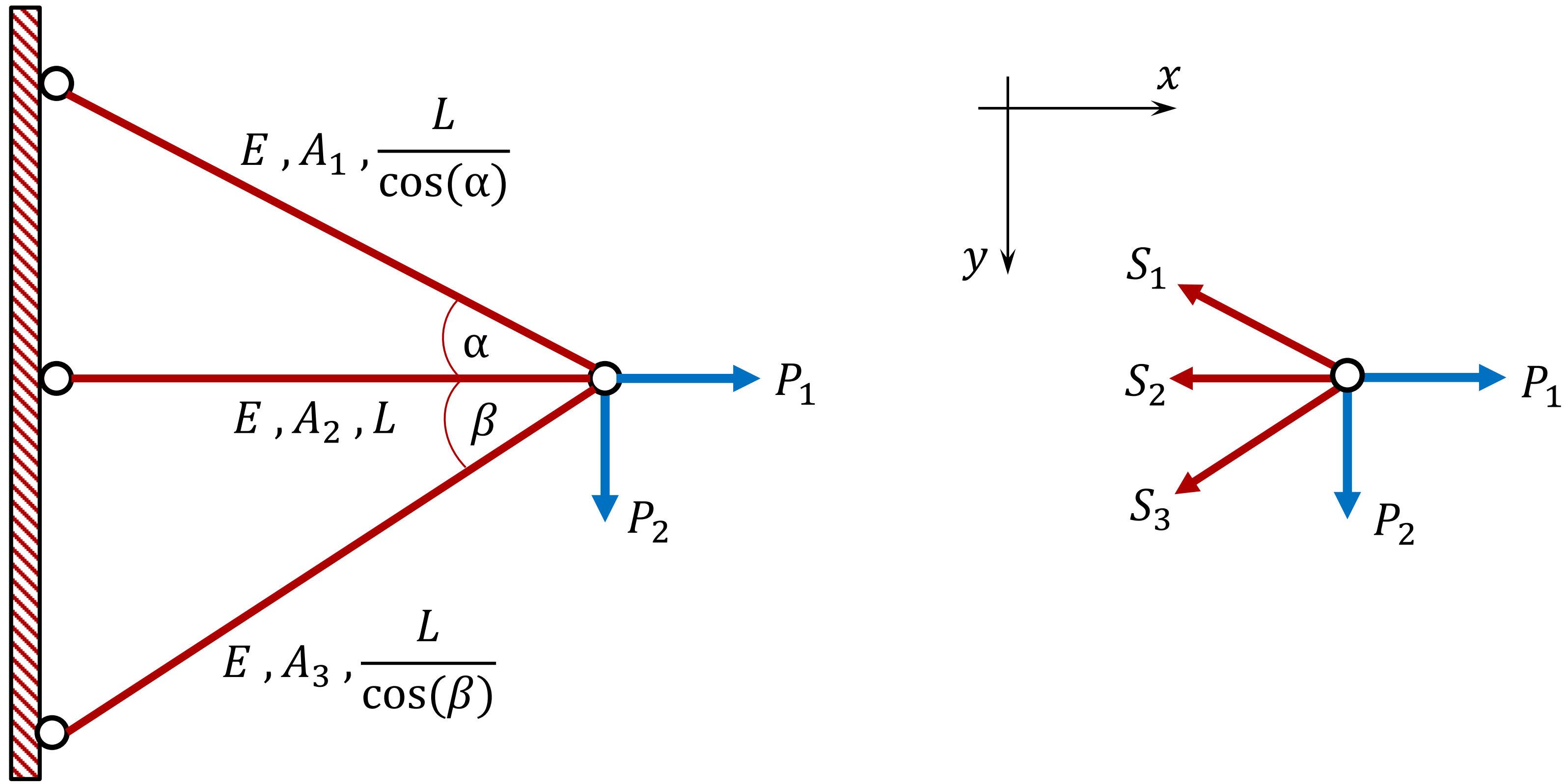
Optimális szerkezettervezés
Dr. Pomezanski Vanda

5. Gyakorlat: Statikailag határozatlan rácsos tartók számítása feszültségkorlát esetén

1. Példa: Három rúdból álló statikailag határozatlan rácsos tartó minimális súlyra való tervezése feszültség korlát esetén



- A terhelés függvényében a húzott és nyomott rudak száma változik, így a σ_H és σ_{EH} aktivitása is.

1. Példa: Három rúdból álló statikailag határozatlan rácsos tartó minimális súlyra való tervezése feszültség korlát esetén

- $\mathbf{v} = \mathbf{K}^{-1}\mathbf{q} = (\mathbf{G}\mathbf{F}^{-1}\mathbf{G}^{\mathrm{T}})^{-1}\mathbf{q}$
- $\mathbf{s} = -\mathbf{F}^{-1}\mathbf{G}^{\mathrm{T}}\mathbf{v}$

$$\mathbf{G} = \begin{bmatrix} -\cos(\alpha) & -1 & -\cos(\beta) \\ -\sin(\alpha) & 0 & \sin(\beta) \end{bmatrix}$$

$$\mathbf{F} = \begin{bmatrix} \frac{L}{EA_1\cos(\alpha)} & 0 & 0 \\ 0 & \frac{L}{EA_2} & 0 \\ 0 & 0 & \frac{L}{EA_3\cos(\beta)} \end{bmatrix}$$

$$\mathbf{q} = \begin{bmatrix} P_1 \\ P_2 \end{bmatrix}$$

- $\sigma_i = \frac{S_i}{A_i} \leq \sigma_H$
- $\sigma_i = \frac{S_i}{A_i} \leq \sigma_{EH}$

- $\rho L\left(\frac{A_1}{\cos(\alpha)} + A_2 + \frac{A_3}{\cos(\beta)}\right) = Min!$

1. Példa: Teheresetek

$L = 100\ \text{cm},\qquad E = 21000\ \text{kN/cm}^2,\qquad \sigma_H = 23{,}5\ \text{kN/cm}^2$

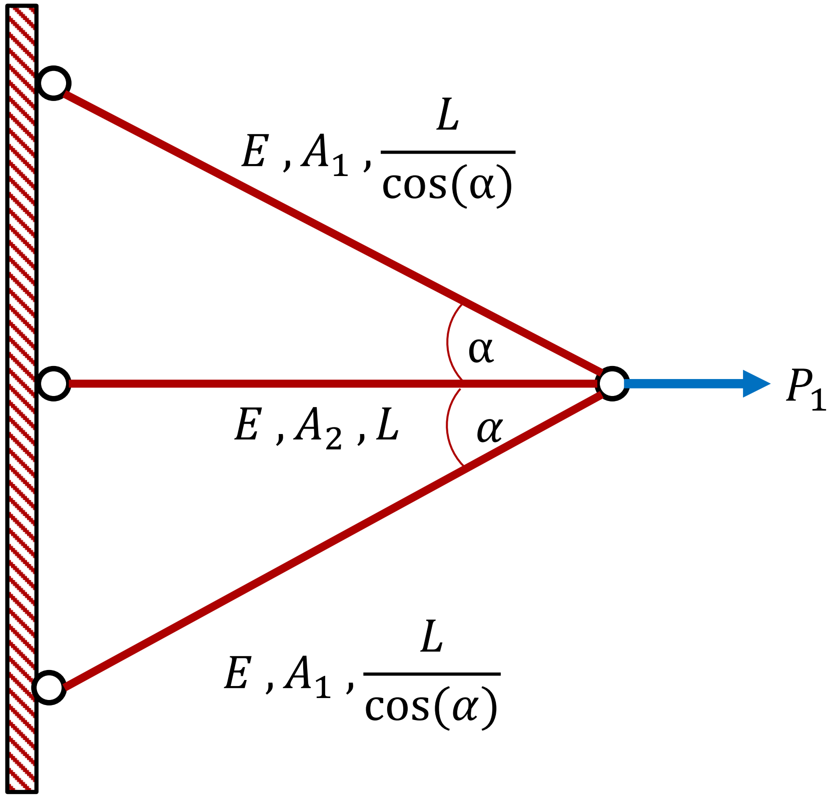
I. ESET:
- $\alpha = \beta = 45°$
- $A_1 = A_3$
- $P_1 = 400\ \text{kN}$
- $P_2 = 0\ \text{kN}$

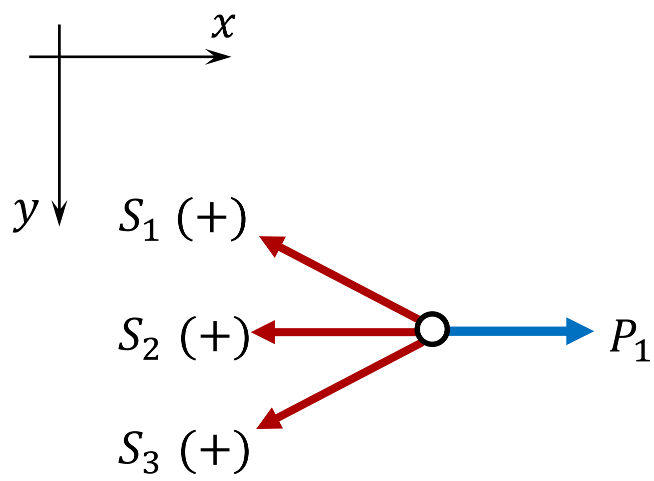
II. ESET:
- $\alpha = \beta = 45°$
- $A_1 = A_3$
- $P_1 = 0\ \text{kN}$
- $P_2 = 400\ \text{kN}$

III. ESET:
- $\alpha = \beta = 45°$
- $A_1 = A_3$
- $P_1 = 400\ \text{kN}$
- $P_2 = 400\ \text{kN}$

1. Példa/I. Eset:

- Szimmetrikus!





1. Példa/I. Eset : Megoldás

- $\mathbf{v} = \mathbf{K}^{-1}\mathbf{q} = (\mathbf{G}\mathbf{F}^{-1}\mathbf{G}^{\mathrm{T}})^{-1}\mathbf{q}$
- $\mathbf{s} = -\mathbf{F}^{-1}\mathbf{G}^{\mathrm{T}}\mathbf{v}$

$$\mathbf{v} = \begin{bmatrix} \dfrac{400\sqrt{2}}{A_1 + \sqrt{2}A_2} \\ 0 \end{bmatrix} \frac{L}{E} \qquad \mathbf{s} = \begin{bmatrix} \dfrac{200\sqrt{2}A_1}{A_1 + \sqrt{2}A_2} \\ \dfrac{400\sqrt{2}A_2}{A_1 + \sqrt{2}A_2} \\ \dfrac{200\sqrt{2}A_1}{A_1 + \sqrt{2}A_2} \end{bmatrix}$$

- $\sigma_1 = \dfrac{200\sqrt{2}}{A_1 + \sqrt{2}A_2} \leq \sigma_H$
- $\sigma_2 = \dfrac{400\sqrt{2}}{A_1 + \sqrt{2}A_2} \leq \sigma_H$
- $2\sqrt{2}A_1 + A_2 = Min!$

$$\mathbf{G} = \begin{bmatrix} -\dfrac{1}{\sqrt{2}} & -1 & -\dfrac{1}{\sqrt{2}} \\ -\dfrac{1}{\sqrt{2}} & 0 & \dfrac{1}{\sqrt{2}} \end{bmatrix}$$

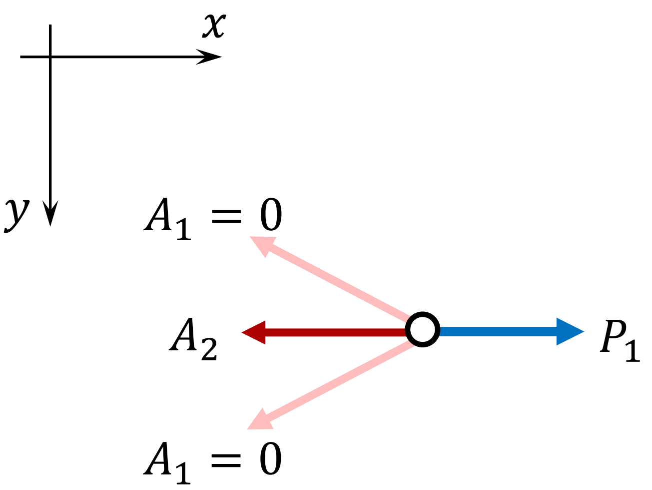
$$\mathbf{F} = \begin{bmatrix} \dfrac{L\sqrt{2}}{EA_1} & 0 & 0 \\ 0 & \dfrac{L}{EA_2} & 0 \\ 0 & 0 & \dfrac{L\sqrt{2}}{EA_1} \end{bmatrix}$$

$$\mathbf{K} = \begin{bmatrix} \dfrac{A_1}{\sqrt{2}} + A_2 & 0 \\ 0 & \dfrac{A_1}{\sqrt{2}} \end{bmatrix} \frac{E}{L}$$

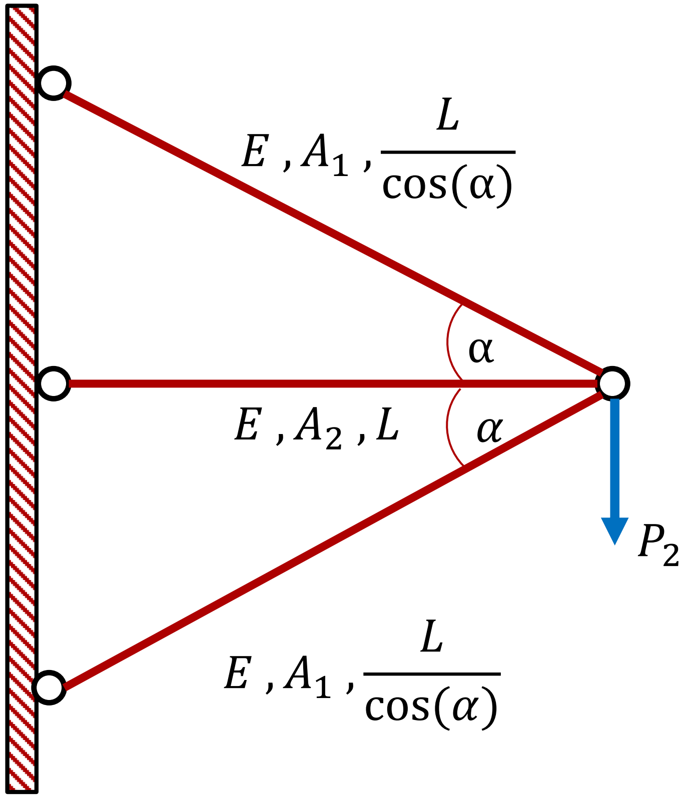
$$\mathbf{q} = \begin{bmatrix} 400 \\ 0 \end{bmatrix}$$

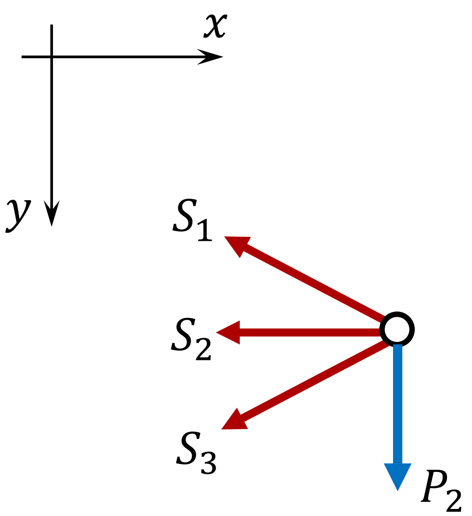
1. Példa/I. Eset : Megoldás

- $A_1 = 0$
- $A_2 = 17{,}0213\ \mathrm{cm}^2$
- $cf = 17{,}0213\ \mathrm{cm}^2$



1. Példa/II. Eset:





1. Példa/II. Eset: Megoldás

- $\mathbf{v} = \mathbf{K}^{-1}\mathbf{q} = (\mathbf{G}\mathbf{F}^{-1}\mathbf{G}^{\mathrm{T}})^{-1}\mathbf{q}$
- $\mathbf{s} = -\mathbf{F}^{-1}\mathbf{G}^{\mathrm{T}}\mathbf{v}$

$$\mathbf{G} = \begin{bmatrix} -\frac{1}{\sqrt{2}} & -1 & -\frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} & 0 & \frac{1}{\sqrt{2}} \end{bmatrix}$$

$$\mathbf{v} = \begin{bmatrix} 0 \\ \frac{400(\sqrt{2}A_1 + 2A_2)}{A_1(A_1 + \sqrt{2}A_2)} \end{bmatrix} \frac{L}{E} \qquad \mathbf{s} = \begin{bmatrix} \frac{200(\sqrt{2}A_1 + 2A_2)}{A_1 + \sqrt{2}A_2} \\ 0 \\ -\frac{200(\sqrt{2}A_1 + 2A_2)}{A_1 + \sqrt{2}A_2} \end{bmatrix}$$

$$\mathbf{F} = \begin{bmatrix} \frac{L\sqrt{2}}{EA_1} & 0 & 0 \\ 0 & \frac{L}{EA_2} & 0 \\ 0 & 0 & \frac{L\sqrt{2}}{EA_1} \end{bmatrix}$$

- $\sigma_i = \frac{200(\sqrt{2}A_1+2A_2)}{A_1(A_1+\sqrt{2}A_2)} \leq \sigma_H$

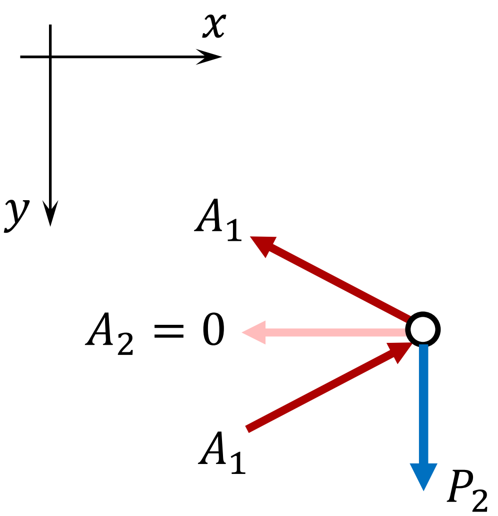
$$\mathbf{K} = \begin{bmatrix} \frac{A_1}{\sqrt{2}} + A_2 & 0 \\ 0 & \frac{A_1}{\sqrt{2}} \end{bmatrix} \frac{E}{L}$$

- $2\sqrt{2}A_1 + A_2 = Min!$

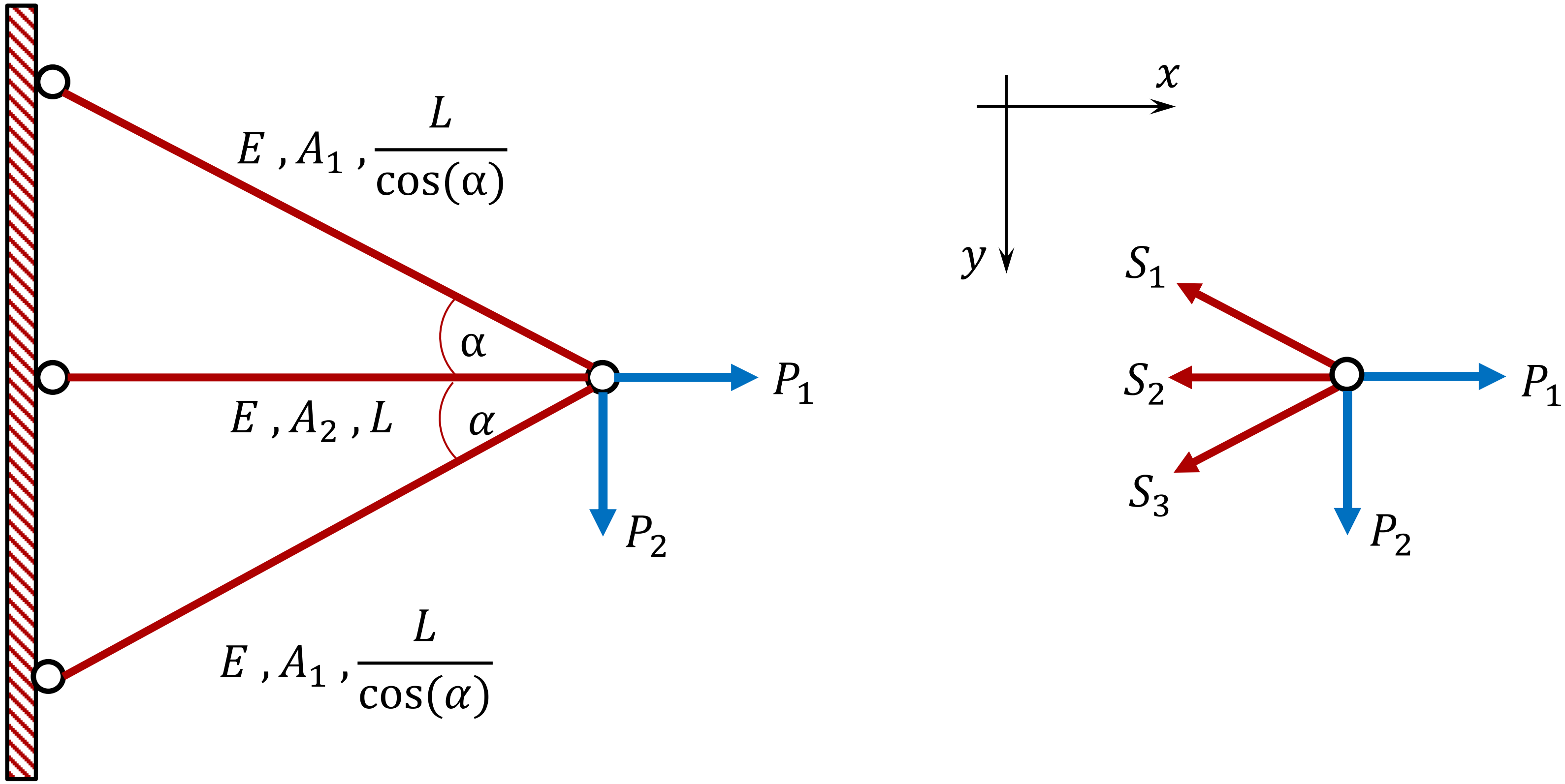
$$\mathbf{q} = \begin{bmatrix} 0 \\ 400 \end{bmatrix}$$

1. Példa/II. Eset: Megoldás

- $A_1 = 12{,}0359\ \text{cm}^2$
- $A_2 = 0$
- $cf = 34{,}0426\ \text{cm}^2$



1. Példa/III. Eset:



1. Példa/II. Eset: Megoldás

- $\mathbf{v} = \mathbf{K}^{-1}\mathbf{q} = (\mathbf{G}\mathbf{F}^{-1}\mathbf{G}^{\mathrm{T}})^{-1}\mathbf{q}$
- $\mathbf{s} = -\mathbf{F}^{-1}\mathbf{G}^{\mathrm{T}}\mathbf{v}$

$$\mathbf{G} = \begin{bmatrix} -\frac{1}{\sqrt{2}} & -1 & -\frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} & 0 & \frac{1}{\sqrt{2}} \end{bmatrix}$$

$$\mathbf{v} = \begin{bmatrix} \frac{400\sqrt{2}}{A_1 + \sqrt{2}A_2} \\ \frac{400(\sqrt{2}A_1 + 2A_2)}{A_1(A_1 + \sqrt{2}A_2)} \end{bmatrix}\frac{L}{E} \qquad \mathbf{s} = \begin{bmatrix} \frac{400(\sqrt{2}A_1 + A_2)}{A_1 + \sqrt{2}A_2} \\ \frac{400\sqrt{2}A_2}{A_1 + \sqrt{2}A_2} \\ -\frac{400\sqrt{2}A_2}{A_1 + \sqrt{2}A_2} \end{bmatrix}$$

$$\mathbf{F} = \begin{bmatrix} \frac{L\sqrt{2}}{EA_1} & 0 & 0 \\ 0 & \frac{L}{EA_2} & 0 \\ 0 & 0 & \frac{L\sqrt{2}}{EA_1} \end{bmatrix}$$

- $\sigma_1 = \frac{400(\sqrt{2}A_1 + A_2)}{A_1(A_1 + \sqrt{2}A_2)} \le \sigma_H$
- $\sigma_2 = \frac{400\sqrt{2}}{A_1 + \sqrt{2}A_2} \le \sigma_H$
- $\sigma_2 = \frac{400\sqrt{2}A_2}{A_1(A_1 + \sqrt{2}A_2)} \le \sigma_H$
- $2\sqrt{2}A_1 + A_2 = Min!$

$$\mathbf{K} = \begin{bmatrix} \frac{A_1}{\sqrt{2}} + A_2 & 0 \\ 0 & \frac{A_1}{\sqrt{2}} \end{bmatrix}\frac{E}{L}$$

$$\mathbf{q} = \begin{bmatrix} 400 \\ 400 \end{bmatrix}$$

Példa/III. Eset: Megoldás

- $A_1 = 18{,}985 \text{ cm}^2$
- $A_2 = 9{,}827 \text{ cm}^2$
- $cf = 63{,}524 \text{ cm}^2$