

```
ClearAll[c1, c2, c3, c4, L, M, EI, u, v, f1, f2, f3, f4]
```

```
f1[x_] := Sin[Pi x / L]
```

```
f2[x_] := Cos[Pi x / L]
```

```
u[x_] := c1 f1[x] + c2 f2[x]
```

```
PotE = EI / 2 Integrate[u''[x]^2, {x, 0, L}] - M u'[0] + M u'[L]
```

$$-\frac{2 c_1 M \pi}{L} + \frac{(c_1^2 + c_2^2) EI \pi^4}{4 L^3}$$

```
Solve[{D[PotE, c1] == 0, D[PotE, c2] == 0}, {c1, c2}]
```

$$\left\{ \left\{ c_1 \rightarrow \frac{4 L^2 M}{EI \pi^3}, c_2 \rightarrow 0 \right\} \right\}$$

```
c1 = %[[1]][[1]][[2]]
```

```
c2 = %%[[1]][[2]][[2]]
```

$$\frac{4 L^2 M}{EI \pi^3}$$

$$0$$

```
0
```

```
u[x]
```

$$\frac{4 L^2 M \sin\left[\frac{\pi x}{L}\right]}{EI \pi^3}$$

```
u[0]
```

```
0
```

```
u[L / 2]
```

$$\frac{4 L^2 M}{EI \pi^3}$$

$$0$$

```
u[L]
```

```
0
```

```
(M L^2 / 4 - M L^2 / 8) / EI - u[L / 2]
```

$$\frac{L^2 M}{8 EI} - \frac{4 L^2 M}{EI \pi^3}$$

```
% // Simplify // N
```

$$-\frac{0.00400614 L^2 M}{EI}$$

```
f3[x_] := x - x^2 / 2 / L
```

```
f4[x_] := x - x^3 / 3 / L^2
```

```
v[x_] := c3 f3[x] + c4 f4[x]
```

```
PotE2 = EI / 2 Integrate[v''[x]^2, {x, 0, L}] - M v'[0] + M v'[L]
```

$$\frac{1}{2} EI \left(\frac{c_3^2}{L} + \frac{2 c_3 c_4}{L} + \frac{4 c_4^2}{3 L} \right) - (c_3 + c_4) M$$

```
Solve[{D[PotE2, c3] == 0, D[PotE2, c4] == 0}, {c3, c4}]
```

$$\left\{ \left\{ c_3 \rightarrow \frac{L M}{EI}, c_4 \rightarrow 0 \right\} \right\}$$

```

c3 = %[[1]][[1]][[2]]
c4 = %%[[1]][[2]][[2]]

L M
---
EI
0

(M L^2 / 4 - M L^2 / 8) / EI - v[L / 2]

- L^2 M
- ---
4 EI

% // Simplify // N

0.25 L^2 M
---
EI

u[x]
v[x]

4 L^2 M Sin[ $\frac{\pi x}{L}$ ]
---
EI  $\pi^3$ 

L M  $\left(x - \frac{x^2}{2L}\right)$ 
---
EI

L = 10;
EI = 1000;
M = 10;

Plot[{u[x], v[x]}, {x, 0, L}]

```

