

ClearAll[c, PotE, M]

v[x_] = c Sin[Pi x / L]

$$c \operatorname{Sin}\left[\frac{\pi x}{L}\right]$$

PotE[c_] = EI / 2 Integrate[v''[x]^2, {x, 0, L}] - Integrate[q v[x], {x, 0, L}] - F v[L / 2]

$$-c F + \frac{c^2 EI \pi^4}{4 L^3} - \frac{2 c L q}{\pi}$$

Solve[PotE'[c] == 0, c]

$$\left\{ \left\{ c \rightarrow \frac{2 L^3 (F \pi + 2 L q)}{EI \pi^5} \right\} \right\}$$

c = %[[1]][[1]][[2]]

$$\frac{2 L^3 (F \pi + 2 L q)}{EI \pi^5}$$

v[x] // Expand

$$\frac{2 F L^3 \operatorname{Sin}\left[\frac{\pi x}{L}\right]}{EI \pi^4} + \frac{4 L^4 q \operatorname{Sin}\left[\frac{\pi x}{L}\right]}{EI \pi^5}$$

v[0]

v[L]

0

0

v[L / 2] // Expand

$$\frac{2 F L^3}{EI \pi^4} + \frac{4 L^4 q}{EI \pi^5}$$

M[x_] = -EI v''[x] // Expand

$$\frac{2 F L \operatorname{Sin}\left[\frac{\pi x}{L}\right]}{\pi^2} + \frac{4 L^2 q \operatorname{Sin}\left[\frac{\pi x}{L}\right]}{\pi^3}$$

M[L / 2] // N

$$0.202642 F L + 0.129006 L^2 q$$

Error / difference from statical analysis:

L F / 4 + q L^2 / 8 - M[L / 2] // N

$$0.0473576 F L - 0.00400614 L^2 q$$