ClearAll[c, PotE, M]

v[x_] = c Sin[Pi x / L]

$c \sin\left[\dfrac{\pi x}{L}\right]$

PotE[c_] = EI / 2 Integrate[v''[x]^2, {x, 0, L}] - M v'[0] + M v'[L]

$-\dfrac{2\,c\,M\,\pi}{L} + \dfrac{c^2\,EI\,\pi^4}{4\,L^3}$

Solve[PotE'[c] == 0, c]

$\left\{\left\{c \to \dfrac{4\,L^2\,M}{EI\,\pi^3}\right\}\right\}$

c = %[[1]][[1]][[2]]

$\dfrac{4\,L^2\,M}{EI\,\pi^3}$

v[x] // Expand

$\dfrac{4\,L^2\,M\,\sin\left[\frac{\pi x}{L}\right]}{EI\,\pi^3}$

v[0]

v[L]

0

0

v[L / 2] // Expand

$\dfrac{4\,L^2\,M}{EI\,\pi^3}$

Mf[x_] = -EI v''[x] // Expand

$\dfrac{4\,M\,\sin\left[\frac{\pi x}{L}\right]}{\pi}$

Mf[L / 2] // N

1.27324 M

Error / difference from statical analysis:

M - Mf[L / 2] // N

−0.27324 M

(M L^2 / 4 - M L^2 / 8) / EI - v[L / 2]

$\dfrac{L^2\,M}{8\,EI} - \dfrac{4\,L^2\,M}{EI\,\pi^3}$

% // Simplify // N

$-\dfrac{0.00400614\,L^2\,M}{EI}$