```
ClearAll[f1, f2, f3, L1, L2, Ym, A1, A2, c1, c2, c3, x]
f1[x_] := Piecewise[{{1 - x / L1, x < L1}, {0, x > L1}}]
f2[x_] := Piecewise[{{x / L1, x < L1}, {1 - (x - L1) / L2, x > L1}}]
f3[x_] := Piecewise[{{0, x < L1}, {(x - L1) / L2, x > L1}}]
u[x_] := c1 f1[x] + c2 f2[x] + c3 f3[x]
u[x]
```

$$c1 \left(\begin{cases} 1 - \frac{x}{L1} & x < L1 \\ 0 & \text{True} \end{cases} \right) + c3 \left(\begin{cases} 0 & x < L1 \\ \frac{-L1 + x}{L2} & x > L1 \\ 0 & \text{True} \end{cases} \right) + c2 \left(\begin{cases} \frac{x}{L1} & x < L1 \\ 1 - \frac{-L1 + x}{L2} & x > L1 \\ 0 & \text{True} \end{cases} \right)$$

```
u[0] // Evaluate
```

$$c1 \left(\begin{cases} 1 & 0 < L1 \\ 0 & \text{True} \end{cases} \right) + c2 \left(\begin{cases} 0 & 0 < L1 \\ 1 + \frac{L1}{L2} & 0 > L1 \\ 0 & \text{True} \end{cases} \right) + c3 \left(\begin{cases} 0 & 0 < L1 \\ -\frac{L1}{L2} & 0 > L1 \\ 0 & \text{True} \end{cases} \right)$$

```
Pote = Ym A1 / 2 Integrate[u'[x]^2, {x, 0, L1}, Assumptions → L1 > 0] +
   Ym A2 / 2 Integrate[u'[x]^2, {x, L1, L1 + L2}, Assumptions → {L1 > 0, L2 > 0}] -
   Q u[0] - F u[L1 + L2] // Simplify
```

$$\frac{1}{2} \left(\frac{A1\,(c1 - c2)^2\,Ym}{L1} + \frac{A2\,(c2 - c3)^2\,Ym}{L2} - 2F\left(c3 \left(\begin{cases} 0 & L2 \le 0 \\ 1 & \text{True} \end{cases} \right) + c1 \left(\begin{cases} -\frac{L2}{L1} & L2 < 0 \\ 0 & \text{True} \end{cases} \right) + c2 \left(\begin{cases} \frac{L1+L2}{L1} & L2 < 0 \\ 0 & \text{True} \end{cases} \right) \right) - 2Q\left(c1 \left(\begin{cases} 1 & 0 < L1 \\ 0 & \text{True} \end{cases} \right) + c3 \left(\begin{cases} -\frac{L1}{L2} & L1 < 0 \\ 0 & \text{True} \end{cases} \right) + c2 \left(\begin{cases} \frac{L1+L2}{L2} & L1 < 0 \\ 0 & \text{True} \end{cases} \right) \right) \right)$$

```
D1 = D[Pote, c1];
D2 = D[Pote, c2];
D3 = D[Pote, c3];
Res = Assuming[L1 > 0 && L2 > 0, Solve[{D2 == 0, D3 == 0}, {c2, c3}]] // Simplify
```

$$\Bigg\{\Bigg\{c2 \to \frac{1}{A1\,Ym} \left(A1\,c1\,Ym + F\,L1 \left(\begin{cases} 0 & L2 \le 0 \\ 1 & \text{True} \end{cases} \right) + L1\,Q \left(\begin{cases} -\frac{L1}{L2} & L1 < 0 \\ 0 & \text{True} \end{cases} \right) + F\,L1 \left(\begin{cases} \frac{L1+L2}{L1} & L2 < 0 \\ 0 & \text{True} \end{cases} \right) + L1\,Q \left(\begin{cases} \frac{L1+L2}{L2} & L1 < 0 \\ 0 & \text{True} \end{cases} \right) \right),$$

$$c3 \to \frac{1}{A1\,A2\,Ym} \left(F\,(A2\,L1 + A1\,L2) \left(\begin{cases} 0 & L2 \le 0 \\ 1 & \text{True} \end{cases} \right) + (A2\,L1 + A1\,L2)\,Q \left(\begin{cases} -\frac{L1}{L2} & L1 < 0 \\ 0 & \text{True} \end{cases} \right) + A2 \left(A1\,c1\,Ym + F\,L1 \left(\begin{cases} \frac{L1+L2}{L1} & L2 < 0 \\ 0 & \text{True} \end{cases} \right) + L1\,Q \left(\begin{cases} \frac{L1+L2}{L2} & L1 < 0 \\ 0 & \text{True} \end{cases} \right) \right) \right)\Bigg\}\Bigg\}$$

```
c2 = Res[[1]][[1]][[2]]
c3 = Res[[1]][[2]][[2]]
```

$$\frac{1}{\text{A1 Ym}}\left(\text{A1 c1 Ym} + \text{F L1}\left(\begin{cases}0 & \text{L2} \le 0\\ 1 & \text{True}\end{cases}\right) + \text{L1 Q}\left(\begin{cases}-\frac{\text{L1}}{\text{L2}} & \text{L1} < 0\\ 0 & \text{True}\end{cases}\right) + \text{F L1}\left(\begin{cases}\frac{\text{L1}+\text{L2}}{\text{L1}} & \text{L2} < 0\\ 0 & \text{True}\end{cases}\right) + \text{L1 Q}\left(\begin{cases}\frac{\text{L1}+\text{L2}}{\text{L2}} & \text{L1} < 0\\ 0 & \text{True}\end{cases}\right)\right)$$

$$\frac{1}{\text{A1 A2 Ym}}\left(\text{F}\,(\text{A2 L1}+\text{A1 L2})\left(\begin{cases}0 & \text{L2} \le 0\\ 1 & \text{True}\end{cases}\right) + (\text{A2 L1}+\text{A1 L2})\,\text{Q}\left(\begin{cases}-\frac{\text{L1}}{\text{L2}} & \text{L1} < 0\\ 0 & \text{True}\end{cases}\right) + \text{A2}\left(\text{A1 c1 Ym} + \text{F L1}\left(\begin{cases}\frac{\text{L1}+\text{L2}}{\text{L1}} & \text{L2} < 0\\ 0 & \text{True}\end{cases}\right) + \text{L1 Q}\left(\begin{cases}\frac{\text{L1}+\text{L2}}{\text{L2}} & \text{L1} < 0\\ 0 & \text{True}\end{cases}\right)\right)\right)$$

```
Solve[D1 == 0, Q] // Simplify
```

$$\left\{\left\{\text{Q} \to \begin{cases}-\text{F} & \text{L1} \ne 0\ \&\&\ \text{L2} \ne 0\\ \text{Indeterminate} & \text{L2} == 0\ \&\&\ \text{L1} == 0\\ \text{ComplexInfinity} & \text{L1} == 0\ \&\&\ \text{L2} \ne 0\\ 0 & \text{True}\end{cases}\right\}\right\}$$

```
Solve[u[0] == 0, c1] // Simplify
```

$$\left\{\left\{\text{c1} \to \begin{cases}\text{Indeterminate} & \text{L1} == 0\\ -\frac{\text{L1}\,(\text{A1 L1}+\text{A2 L2})\,\text{Q}}{\text{A1 A2 L2 Ym}} & \text{L1} < 0\ \&\&\ \text{L2} == 0\\ -\frac{\text{L1}\,(\text{A2 L2}\,(\text{F}+\text{Q})+\text{A1}\,(-\text{F L2}+\text{L1 Q}))}{\text{A1 A2 L2 Ym}} & \text{L1} < 0\ \&\&\ \text{L2} > 0\\ -\frac{\text{A1 L1}^2\,\text{Q}+\text{A2 L2}\,(\text{F}\,(\text{L1}+\text{L2})+\text{L1 Q})}{\text{A1 A2 L2 Ym}} & \text{L1} < 0\ \&\&\ \text{L2} < 0\\ 0 & \text{True}\end{cases}\right\}\right\}$$

```
c1 = %[[1]][[1]][[2]]
```

$$\begin{cases}\text{Indeterminate} & \text{L1} == 0\\ -\frac{\text{L1}\,(\text{A1 L1}+\text{A2 L2})\,\text{Q}}{\text{A1 A2 L2 Ym}} & \text{L1} < 0\ \&\&\ \text{L2} == 0\\ -\frac{\text{L1}\,(\text{A2 L2}\,(\text{F}+\text{Q})+\text{A1}\,(-\text{F L2}+\text{L1 Q}))}{\text{A1 A2 L2 Ym}} & \text{L1} < 0\ \&\&\ \text{L2} > 0\\ -\frac{\text{A1 L1}^2\,\text{Q}+\text{A2 L2}\,(\text{F}\,(\text{L1}+\text{L2})+\text{L1 Q})}{\text{A1 A2 L2 Ym}} & \text{L1} < 0\ \&\&\ \text{L2} < 0\\ 0 & \text{True}\end{cases}$$

u[x]

$$\left(\begin{cases} 1-\frac{x}{L1} & x<L1 \\ 0 & \text{True}\end{cases}\right)\left(\begin{cases} \text{Indeterminate} & L1==0 \\ -\frac{L1\,(A1\,L1+A2\,L2)\,Q}{A1\,A2\,L2\,Ym} & L1<0\,\&\&\,L2==0 \\ -\frac{L1\,(A2\,L2\,(F+Q)+A1\,(-F\,L2+L1\,Q))}{A1\,A2\,L2\,Ym} & L1<0\,\&\&\,L2>0 \\ -\frac{A1\,L1^2\,Q+A2\,L2\,(F\,(L1+L2)+L1\,Q)}{A1\,A2\,L2\,Ym} & L1<0\,\&\&\,L2<0 \\ 0 & \text{True}\end{cases}\right)+\frac{1}{A1\,Ym}$$

$$\left(\begin{cases} \frac{x}{L1} & x<L1 \\ 1-\frac{-L1+x}{L2} & x>L1 \\ 0 & \text{True}\end{cases}\right)\Bigg(F\,L1\left(\begin{cases} 0 & L2\le 0 \\ 1 & \text{True}\end{cases}\right)+L1\,Q\left(\begin{cases} -\frac{L1}{L2} & L1<0 \\ 0 & \text{True}\end{cases}\right)+F\,L1\left(\begin{cases} \frac{L1+L2}{L1} & L2<0 \\ 0 & \text{True}\end{cases}\right)+$$

$$L1\,Q\left(\begin{cases} \frac{L1+L2}{L2} & L1<0 \\ 0 & \text{True}\end{cases}\right)+A1\,Ym\left(\begin{cases} \text{Indeterminate} & L1==0 \\ -\frac{L1\,(A1\,L1+A2\,L2)\,Q}{A1\,A2\,L2\,Ym} & L1<0\,\&\&\,L2==0 \\ -\frac{L1\,(A2\,L2\,(F+Q)+A1\,(-F\,L2+L1\,Q))}{A1\,A2\,L2\,Ym} & L1<0\,\&\&\,L2>0 \\ -\frac{A1\,L1^2\,Q+A2\,L2\,(F\,(L1+L2)+L1\,Q)}{A1\,A2\,L2\,Ym} & L1<0\,\&\&\,L2<0 \\ 0 & \text{True}\end{cases}\right)\Bigg)+$$

$$\frac{1}{A1\,A2\,Ym}\left(\begin{cases} 0 & x<L1 \\ \frac{-L1+x}{L2} & x>L1 \\ 0 & \text{True}\end{cases}\right)\Bigg(F\,(A2\,L1+A1\,L2)\left(\begin{cases} 0 & L2\le 0 \\ 1 & \text{True}\end{cases}\right)+$$

$$(A2\,L1+A1\,L2)\,Q\left(\begin{cases} -\frac{L1}{L2} & L1<0 \\ 0 & \text{True}\end{cases}\right)+A2\,\Bigg(F\,L1\left(\begin{cases} \frac{L1+L2}{L1} & L2<0 \\ 0 & \text{True}\end{cases}\right)+$$

$$L1\,Q\left(\begin{cases} \frac{L1+L2}{L2} & L1<0 \\ 0 & \text{True}\end{cases}\right)+A1\,Ym\left(\begin{cases} \text{Indeterminate} & L1==0 \\ -\frac{L1\,(A1\,L1+A2\,L2)\,Q}{A1\,A2\,L2\,Ym} & L1<0\,\&\&\,L2==0 \\ -\frac{L1\,(A2\,L2\,(F+Q)+A1\,(-F\,L2+L1\,Q))}{A1\,A2\,L2\,Ym} & L1<0\,\&\&\,L2>0 \\ -\frac{A1\,L1^2\,Q+A2\,L2\,(F\,(L1+L2)+L1\,Q)}{A1\,A2\,L2\,Ym} & L1<0\,\&\&\,L2<0 \\ 0 & \text{True}\end{cases}\right)\Bigg)\Bigg)$$

L1 = 2; L2 = 2

2

f1[x]

$$\begin{cases} 1-\frac{x}{2} & x<2 \\ 0 & \text{True}\end{cases}$$

f1[0]

1

```
f1[L1]
```

0

```
Pote // Simplify
```

$$-\frac{(A1 + A2)\, F^2}{A1\, A2\, Ym}$$

```
c1
c2
c3
```

0

$$\frac{2\, F}{A1\, Ym}$$

$$\frac{(2\, A1 + 2\, A2)\, F}{A1\, A2\, Ym}$$