Highway and Railway Design 1 **EXERCISES 2.** (for Lecture 3. of **Dr. A. Timár)**

A. Ascending Grades

Example 1

A road engineer designs a road at design speed of 105 km/h, which has an inclined section with a 3% grade. How much can the elevation of the roadway increase before the speed of the larger vehicles is reduced to 90 km/h, assuming that a 3% grade causes a reduction in speed of 15 km/h after 425 m?

Solution

To find the exact increase in the elevation of the highway we would need to employ some simple trigonometry. But, since the angle of a 3% grade is small, we can just estimate the elevation increase by multiplying the length of the grade by the grade itself. This yields 425*0.03 = **13** m. The elevation of the roadway can only be increased by about 13 m before heavy vehicles are reduced to a speed of 90 km/h.

B. Crest Vertical Curves

Example

You have been instructed to design a crest vertical curve that will connect a road segment with a 3% grade to an adjoining segment with a -1% grade. Assume that the minimum stopping sight distance (*S* or D_S) for the road is 165 m. If the elevation of the VPC is 455 m, what will the elevation of the curve be at L/2?

Solution

The first step in the analysis is to find the length of the crest vertical curve. The grade changes from 3% to -1%, which is a change of -4% or A = |-4%|. In addition, for the stopping sight distance h_1 = 1.05 m and h_2 = 0.60 m. Since we know D_S = 165 m, we can go ahead and solve for the length of the crest vertical curve.

If S > L then (invalid because L > S)

$$L = 2S - \frac{200(\sqrt{h1} + \sqrt{h2})^2}{A}$$

If S < L then

$$L = \frac{AS^2}{100(\sqrt{2h1} + \sqrt{2h2})^2}$$

where:

L = length of the crest vertical curve (m)
S = sight distance, 165 m
A = the change in grades, |-4%|
h_1 = height of the driver's eyes above the ground, 1.05 m
h_2 = height of the object above the roadway, 0.60 m

The curve length calculated from the 'S < L' equation was L=336 m, which is greater than the sight distance of 165 m. To find the elevation of the curve at a horizontal distance of L/2 from the VPC, we need to use the equation below:

$$Y = VPCy + B*x + (A*x^2)/(200*L)$$

where:

Y = elevation of the curve at a distance x from the VPC (m)
VPCy = elevation of the VPC, 455 m
B = slope of the approaching roadway, or the roadway that intersects the VPC, 0.03
A = the change in grade between the disjointed segments, -4 (From 3% to -1% would be a change of -4%)
x = L/2 = 336 m /2 = **168** m

The equation above yields a curve elevation of

Y = 455 + 0.03*168 + (-4*28224) / 200*336 = 460 – 1.7 = **458.3** m

at a distance L/2 from the VPC.

Example

A -2.5% grade is connected to a +1.0% grade by means of a 180 m vertical curve. The P.I. station is 100+00 and the P.I. elevation is 100.0 m above sea level. What are the station and elevation of the lowest point on the vertical curve?

Solution

Rate of change of grade:

$$r = \frac{g_2 - g_1}{L} = \frac{1.0\% - (-2.5\%)}{1.8 \text{ sta}} = 1.944\%/\text{sta}$$

Station of the low point:

At low point, $g = 0$

$g = g_1 + rx = 0$

or

$$x = \frac{-g_1}{r} = -\left(\frac{-2.5}{1.944}\right) = 1.29 = 1 + 29 \text{ sta}$$

Station of BVC = (100 + 00) − (0 + 90) = 99 + 10

Station of low point = (99 + 10) + (1 + 29) = 100 + 39

Elevation of BVC:

$$y_0 = 100.0 \text{ m} + (-0.9 \text{ sta})(-2.5\%) = 102.25 \text{ m}$$

Elevation of low point:

$$y = y_0 + g_1 x + \frac{rx^2}{2}$$

$$= 102.25 \text{ m} + (-2.5\%)(1.29 \text{ sta}) + \frac{(1.944\%/\text{sta})(1.29 \text{ sta})^2}{2}$$

$$= 100.64 \text{ m}$$

Example

Determine the minimum length of a crest vertical curve between a 0.5% grade and a

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1.0% grade for a road with a 100-km/h design speed. The vertical curve must provide 190-m stopping sight distance and meet the California appearance criteria. Round up to the next greatest 20 m interval.

Solution

Stopping sight distance criterion:

Assume $S \leq L$

$$L = \frac{AS^2}{200(\sqrt{h_1} + \sqrt{h_2})^2} = \frac{[0.5 - (-1.0)](190^2)}{200(\sqrt{1.070} + \sqrt{0.150})^2} = 134.0 \text{ m}$$

134.0 m < 190 m, so $S > L$

$$L = 2S - \frac{200(\sqrt{h_1} + \sqrt{h_2})^2}{A} = 2(190) - \frac{200(\sqrt{1.070} + \sqrt{0.150})^2}{[0.5 - (-1.0)]}$$

$$= 380.0 - 269.5 = 110.5 \text{ m}$$

Appearance criterion:

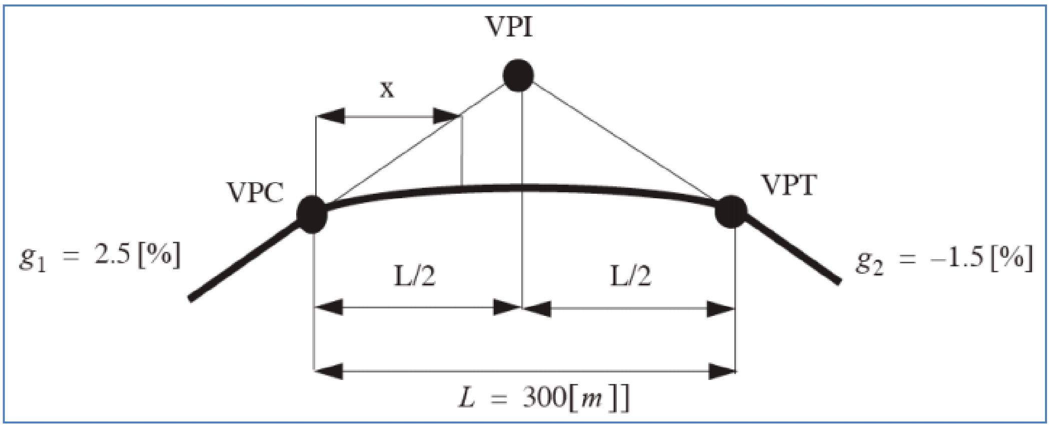
Design speed = 100 km/h > 60 km/h but grade break = 1.5% < 2%. Use 60 m.

Conclusion:

Sight distance criterion governs. Use 120 m vertical curve.

Example

The length of a tangent vertical curve equals 300 [m]. The initial and the final grades are known to be +2.5% and -1.5% respectively. The grades intersect at the station 3 + 650 and at an elevation of 210.500 m:

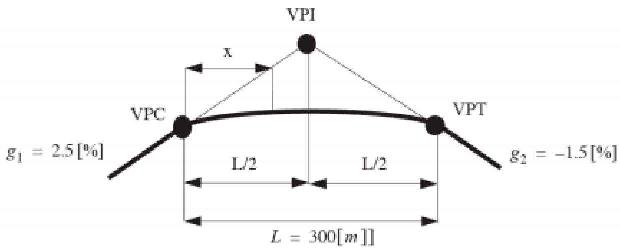


Determine the station and the elevation of the VPC and PVT points (b) Calculate the elevation of the point on the curve 100 meters from the VPC point (c) Determine the station and the elevation of the highest point on the curve.

Solution

The grades of the initial G_1 and final tangent G_2 are:

$$G_1 = \frac{2.5}{100} = 0.025\left[\frac{m}{m}\right]$$



$$G_2 = -\frac{1.5}{100} = -0.015\left[\frac{m}{m}\right]$$

The station of the VPC and VPT are:

$$VPCSta = VPISta - \frac{L}{2} = 3 + 650 - \frac{300}{2} = 3 + 650 - 150 = 3 + 500$$

$$VPTSta = VPCSta + L = 3 + 500 + 300 = 3 + 800$$

The elevation of the VPC and VPT. are:

$$E_{VPC} = E_{VPI} - \left(G_1 \cdot \frac{L}{2}\right) = 210.500 - \left(0.025 \cdot \frac{300}{2}\right) = 210.500 - 3.75 = 206.75[m]$$

$$E_{VPT} = E_{VPI} - \left(G_2 \cdot \frac{L}{2}\right) = 210.500 - \left(0.015 \cdot \frac{300}{2}\right) = 210.500 - 2.25 = 208.25[m]$$

The elevation y of any point on the curve located at distance x from VPC is described by the parabola used instead of the circle:

$$y = a \cdot x^2 + b \cdot x + c$$

After substituting the known values for the coefficients, we get:

$$y = \frac{G_2 - G_1}{2 \cdot L} \cdot x^2 + G_1 \cdot x + E_{VPC}$$

$$y = \frac{-0.015 - 0.025}{2 \cdot 300} \cdot x^2 + 0.025 \cdot x + 206.75$$

$$y = -0.000067 \cdot x^2 + 0.025 \cdot x + 206.75$$

For $x = 100$ we get:

$$y = -0.000067 \cdot 100^2 + 0.025 \cdot 100 + 206.75 = 208.58$$

The elevation of the point equals 208.58 [m].

(c)

Grades are opposite in sign. This means that the highest point can be estimated by setting the first derivative the equation of the parabola to zero, i.e.:

$$\frac{dy}{dx} = 2 \cdot a \cdot x + b = 0$$

$$2 \cdot (-0.000067) \cdot x + 0.025 = 0$$

$$x = 185.57[m]$$

The station of the highest point is:

$$HighSta = VPCSta + 185.57 = 3 + 500 + 185.57 = 3 + 685.57$$

The elevation of the highest point is:

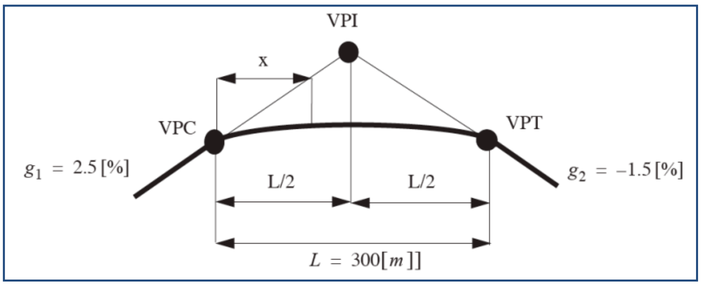
$$y = -0.000067 \cdot x^2 + 0.025 \cdot x + 206.75$$

For $x = 185.57$ we get:

$$y = -0.000067 \cdot 185.57^2 + 0.025 \cdot 185.57 + 206.75 = 209.08[m]$$

Example

Calculate minimum length of the vertical curve that satisfies the minimum stopping sight distance (D_{Smin}=210 m) corresponding to design speed of 100 km/h.



Determine the station and the elevation of the VPC and PVT points (b) Calculate the elevation of the point on the curve 100 meters from the VPC point (c) Determine the station and the elevation of the highest point on the curve.

Solution

The grades are: $g_1 = 2.5\%$ and $g_2 = -1.5\%$

The difference in grades is: A = -2.5-1.5 = -4; thus IAI = 4

The minimum length of crest curve is $L_{min} = L_{min}*K*IAI = 4K$

Values of K are given in the following table:

Design Speed [km/h]	Assumed Speed for Condition [km/h]	Coefficient of friction	Stopping Sight Distance Rounded for Design [m]	Rate of Vertical Curvature *K* [length[m] per % of A] Computed	Rate of Vertical Curvature *K* [length[m] per % of A] Rounded for Design
100	85-100	0.29	160-210	61.01-104.02	62-105
110	91-110	0.28	180-250	79.75-150.28	80-151
120	98-120	0.28	210-290	101.90-201.90	102-202

The minimum curve length that satisfy minimum stopping sight distance requirement is (for a design speed of 100 km/h):

$$\mathbf{L_{min}} = 4*K = 4*62 = \underline{\mathbf{248}} \text{ [m]}$$

C. Sag Vertical Curves

Example

If a stopping sight distance of 120 m is to be maintained on a sag vertical curve with tangent grades of -3% and 0%, what should the length of the curve be? Assume a headlight beam upward divergence angle of 1°.

Solution

Since we know everything that we need to know to solve this problem, we'll jump straight into the equations. If S > L then

$$L = 2S - \frac{200(H + S^{*}\tan(B))}{A}$$

If S < L then (invalid because L < S)

$$L = \frac{AS^2}{200(H + S^{*}\tan(B))}$$

where:

L = Curve length (m)
S = Sight distance, 120 m
B = Beam upward divergence, 1°
H = Height of the headlights, 0.6 m (assumed)
A = Change in grade, 3% ($|G_2-G_1|$ as a percent)

Solving the equations above results in a curve length of **61** m. You can find the elevation of any point along the curve once you have the curve length. See the crest vertical curve example problem.

Example

Determine the minimum length of a sag vertical curve between a -0.7% grade and a +0.5% grade for a road with a 110 km/h design speed. The vertical curve must provide 220 m stopping sight distance and meet the appearance criteria and the comfort standard. Round up to the next greatest 20 m interval.

Solution

Stopping sight distance criterion:

Assume $S \le L$

$$L = \frac{AS^2}{120 + 3.5S} = \frac{[0.5 - (-0.7)](220^2)}{120 + 3.5(220)} = 65.3 \text{ m}$$

65.3 m < 220 m, so $S > L$

$$L = 2S - \frac{120 + 3.5S}{A} = 2(220) - \frac{120 + 3.5(220)}{[0.5 - (-0.7)]}$$

$$= 440 - 741.7 = -301.7 \text{ m}$$

Since $L < 0$, no vertical curve is needed to provide stopping sight distance.

Comfort criterion:

$$L = \frac{AV^2}{395} = \frac{[0.5 - (-0.7)](110^2)}{395} = 36.8 \text{ m}$$

Appearance criterion:

Design speed = 110 km/h > 60 km/h but grade break = 1.2% < 2%. Use 60 m.

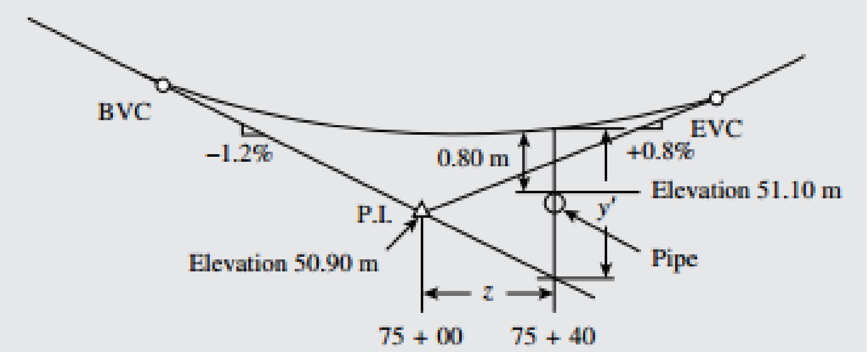
Conclusion:

Appearance criterion governs. Use 60 m vertical curve.

Example

A vertical curve joins a -1.2% grade to a +0.8% grade. The P.I. of the vertical curve is at station 75+00 and elevation 50.90 m above sea level. The centerline of the roadway must clear a pipe located at station 75+40 by 0.80 m. The elevation of the top of the pipe is 51.10 m above sea level. What is the minimum length of the vertical curve that can be used?

Solution



Determine z:

$$z = (75 + 40) - (75 + 00) = 0.40 \text{ sta.}$$

Determine y'

$$\text{Elevation of tangent} = 50.90 + (-1.2)(0.4) = 50.42 \text{ m}$$

$$\text{Elevation of roadway} = 51.10 + 0.80 = 51.90 \text{ m}$$

$$y' = 51.90 - 50.42 = 1.48 \text{ m}$$

Determine w:

$$A = g_2 - g_1 = (+0.8) - (-1.2) = 2.0$$

$$w = \frac{y'}{A} = \frac{1.48}{2} = 0.74$$

Determine L:

$$L = 4w - 2z + 4\sqrt{w^2 - wz}$$

$$= 4(0.74) - 2(0.4) + 4\sqrt{0.74^2 - (0.74)(0.4)} = 4.17 \text{ sta} = 417 \text{ m}$$

Check y':

$$x = \frac{4.17}{2} + 0.4 = 2.485 \text{ sta}$$

$$r = \frac{A}{L} = \frac{2}{4.17} = 0.48$$

$$y' = \frac{rx^2}{2} = \frac{(0.48)(2.485^2)}{2} = 1.48$$ *Check OK*

D. Horizontal alignment

Example

What is the minimum radius of curvature allowable for a roadway with a 100 km/h design speed, assuming that the maximum allowable superelevation rate is 0.12? Compare this with the minimum curve radius recommended (490 m). What is the actual maximum superelevation rate allowable under recommended standards for a 100 km/h design speed, if the value of f is the maximum allowed for this speed (f_{max}=0.12)? Round the answer down to the nearest whole percent.

Solution

Minimum radius of curvature for 100 km/h design speed:

$$R = \frac{V^2}{127(f + e)} = \frac{100^2}{127(0.12 + 0.12)} = 328 \text{ m}$$

Minimum radius recommended is 490 m. Actual maximum superelevation rate recommended according to relevant standards for 100 km/h is

$$e = \frac{V^2}{127R} - f = \frac{100^2}{127(490)} - 0.12 = 0.041$$

Rounding,

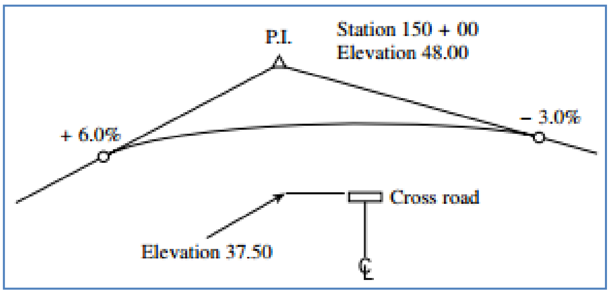
$$e_{max} = 0.04 = 4\%$$

Determine stations of critical points:

$$\begin{aligned}
\text{TS station} &= \text{P.I. station} - (T' + k) \\
&= (150 + 00) - [(0 + 96.1) + (0 + 30.0)] \\
&= 148 + 73.9 \\
\text{SC station} &= \text{TS station} + L_s \\
&= (148 + 73.9) + (0 + 60) \\
&= 149 + 33.9
\end{aligned}$$

Further examples for exercise (without solution)

1. Compute the minimum length of vertical curve to provide passing sight distance for a design speed of 100 km/h at the intersection of a +1.40% grade with a -0.60% grade.
2. Compute the minimum length of vertical curve that will provide 190 m stopping sight distance for a design speed of 100 km/h at the intersection of a +2.60% grade and a -2.40% grade.
3. Compute the minimum length of vertical curve that will provide 220 m stopping sight distance for a design speed of 110 km/h at the intersection of a +3.50% grade and a -2.70% grade.
4. Compute the minimum length of vertical curve that will provide 130 m stopping sight distance for a design speed of 80 km/h at the intersection of a +2.30% grade and a -4.80% grade.
5. Compute the minimum length of vertical curve that will provide 190 m stopping sight distance for a design speed of 100 km/h at the intersection of a -2.60% grade and a +2.40% grade.
6. Compute the minimum length of vertical curve that will provide 220 m stopping sight distance for a design speed of 110 km/h at the intersection of a -3.50% grade and a +2.70% grade.
7. Compute the minimum length of vertical curve that will provide 130 m stopping sight distance for a design speed of 80 km/h at the intersection of a -2.30% grade and a +4.80% grade.
8. A 350 m vertical curve connects a +3.00% grade with a -2.00% grade. If the station of the BVC is 150+00, what is the station of the highest point on the curve?
9. A 400 m vertical curve connects a -2.00% grade to a +4.00% grade. The P.I. is located at station 150+00 and elevation 60.00 m above sea level. A pipe is to be located at the low point on the vertical curve. The roadway at this point consists of two 3.6 m lanes with a normal crown slope of 2%. If the lowest point on the surface of the roadway must clear the pipe by 0.75 m, what is the station and maximum elevation of the pipe?
10. Given the profile below, determine: (a) The length of vertical curve needed to make the highest point on the vertical curve come out exactly over the centerline of the cross road at station 150+70. (b) The vertical clearance between the profile grade on the vertical curve and the centerline of the cross road.



11. A vertical curve joins a -0.5% grade to a +1.0% grade. The P.I. of the vertical curve is at station 200+00 and elevation 150.00 m above sea level. The centerline of the roadway must clear a pipe located at station 200+70 by 0.75 m. The elevation of the top of the pipe is 150.40 m above sea level. What is the minimum length of vertical curve that can be used?

12. A vertical curve joins a -2.0% grade to a +0.5% grade. The P.I. of the vertical curve is at station 100+00 and elevation 69.50 m above sea level. The centerline of the roadway must clear an overhead structure located at station 99+20 by 5.67 m. The elevation of the bottom of the structure is 77.45 m above sea level. What is the maximum length of vertical curve that can be used?
13. Compute the minimum radius of a circular curve for a highway designed for 110 km/h. The maximum superelevation rate is 12%.
14. Compute the minimum radius of a circular curve for a highway designed for 80 km/h. Because snow and ice are present, the maximum superelevation rate is 8%.
15. Compute the minimum radius of a circular curve for a highway designed for 100 km/h. The maximum superelevation rate is 12%.
16. A roadway goes from tangent alignment to a 250 m circular curve by means of an 80 m long spiral transition curve. The deflection angle between the tangents is 45°. Use formulas to compute Xs, Ys, p, and k. Assume that the station of the P.I., measured along the back tangent is 250+00, and compute the stations of the TS, SC, CS, and ST.
17. A roadway goes from tangent alignment to a 275 m circular curve by means of a 100 m long spiral transition curve. The deflection angle between the tangents is 60°. Use formulas to compute Xs, Ys, p, and k. Assume that the station of the P.I., measured along the back tangent is 200+00, and compute the stations of the TS, SC, CS, and ST.
18. A circular curve with a radius of 350 m is connected by 60 m spiral transition curves to tangents with a deflection angle of 0.349 rad. If the station of the TS is 105+40, determine the station of the ST.
19. A horizontal curve is connected by two spiral transition curves to tangents with a deflection angle of 0.26 rad. Stations of critical points are as follows: TS, 105+00; SC, 105+80; CS, 107+50; ST, 108+30. The roadway is a two-lane road with one 3.6 m lane in each direction. If the difference in grade between the centerline and edge of traveled way in the superelevation transition is exactly 1/200, at what speed can the curve be taken with no side friction?
20. A horizontal curve is connected by two spiral transition curves to tangents with a deflection angle of 0.30 rad. Stations of critical points are as follows: TS, 308+00; SC, 308+40; CS, 310+40; ST, 310+80. The roadway is a two-lane highway with one 3.6 m lane in each direction. If the difference in grade between the centerline and edge of traveled way in the superelevation transition is exactly 1/200, what is the maximum speed that can be maintained on the curve if the side friction is limited to 0.10?
21. The allowable side friction factor for horizontal curves with a design speed of 100 km/h is 0.12. (a) What superelevation rate would you use for a curve with a design speed of 100 km/h and a radius of 420 m? Round to the nearest whole percent. (b) A spiral transition curve is used to go from a normal crown slope with 2% cross-slopes to full superelevation for the curve described above. If the maximum difference in grade between the centerline and the edge is 1/200 and the roadway consists of two 3.6 m lanes, what is the minimum length of spiral? Round up to the next integral multiple of 20 m.