Highway and Railway Design 1 **EXERCISES 3.** (for Lecture 5. of **Dr. A. Timár)**

A. Traffic flow

1. Example

Given five observed speed of vehicles (60 km/h, 35 km/h, 45 km/h, 20 km/h, and 50 km/h) on a given urban road section, what is the average speed ($\underline{v}$) of the flow?

Solution

$\underline{v}$ = (1/5)*(60+35+45+20+50) = **42** km/h

2. Example

Given that 40 vehicles pass a given point on a road in 1 minute and traverse a length of l=1 kilometer, what is the flow (q), density (k), average speed ($\underline{v}$) and average headway ($\underline{h}$)?

Solution

Compute the values of traffic flow *(q)* and traffic density *(k)* from the equation: $q = k*\underline{v}$

Traffic flow or volume: q = 3600*40 / 60 s = **2400** veh/h
Traffic density: k = 40/1 = **40** veh/km

Find *average speed* of the traffic flow:

$q = k*\underline{v} = 2400 = 40*\underline{v}$
$\underline{v}$ = **60** km/h

Compute *average headway*:

$k = 1/\underline{h} = 40$
$\underline{h}$ = 0.025 km = **25** m

3. Example

The spot speeds (expressed in km/h) observed at a road section are 66, 62, 45, 79, 32, 51, 56, 60, 53 and 49 respectively. What is the mean/average speed and the median speed (expressed in km/h) of the traffic flow? Which one is greater than the other?

Solution

Median speed is the speed at the middle value in series of spot speeds that are arranged in ascending order. 50% of speed values will be *greater* than the median 50% will be *less* than the median. Ascending order of spot speed studies are: 32,39,45,51,53,56,60,62,66,79

Mean speed: $\underline{v}_{mean}$ = 1/10*(32+39+45+51+53+56+60+62+66+79+) = 543/10 = **54.3** km/h
Median speed: $\underline{v}_{median}$ = (53 + 56) / 2 = **54.5** km/h

The value of median speed is greater in this case, than that of the mean speed.

4. Example

Four vehicles are travelling at constant speeds between sections ***X*** and ***Y*** (280 meters apart) with their positions and speeds observed at an instant in time. An observer at point ***X*** observes the four vehicles passing point ***X*** during a period of 15 seconds. The speeds of the vehicles are measured as

88, 80, 90, and 72 km/h respectively. Calculate the flow, density, average speed, and average headway of the traffic flow.

Solution

Compute *flow*: $q = N\,(3600/t_{measured}) = 4\,(3600/15) = \underline{\mathbf{960}}$ veh/h

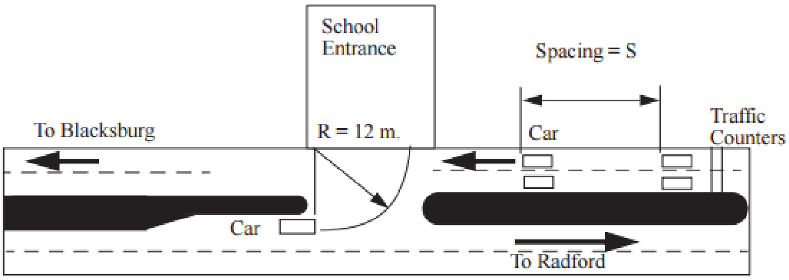
Compute *density*: $k = N/L = 4*1000/280 = \underline{\mathbf{14}}$ veh/km

Compute *average speed*: $\underline{v} = 1/N \sum_{n=1}^{N} * v = ¼\,(72+90+80+88) = \underline{\mathbf{82.5}}$ km/h

Compute *average headway*: $\underline{h} = 1/k = 1/14 = 0.07$ km $= \underline{\mathbf{70}}$ m

5. Example

The Blacksburg Middle school board hires you as a transportation engineer to ease complaints from parents driving vehicles and making a left turn to the school entrance during the peak hour in the morning (see Figure below). The road is divided and has a left turn queuing island allowing cars to stop before making the turn. Measurements at the road by the town engineer indicate that traffic flow in this section has a maximum/jam density (k_{jam}) of 70 vehicle/lane-km and the free flow speed of v_f = 50 km/h (restricted by the speed limit).



The typical acceleration model for a car is known to be:

$$a = 4.0 - 0.1*v$$

where

a is the acceleration of the car (in m/s^2) and v is the vehicle speed in m/s. During the morning peak period, traffic counters at the site measure an average of 20 vehicles per kilometer per lane traveling from Radford to Blacksburg (see Figure).

(a) Find the typical spacing (S) and the average headway ($h; m$) between vehicles traveling from Radford to Blacksburg during the morning peak period.
(b) Find if the average headway (h) allows a typical driver to make a left turn if the driver has a perception/reaction time of 0.5 seconds. The radius of the curve to make a left turn is 12 meters. According to the relevant standards, the critical vehicle length (L) is 5.8 meters.

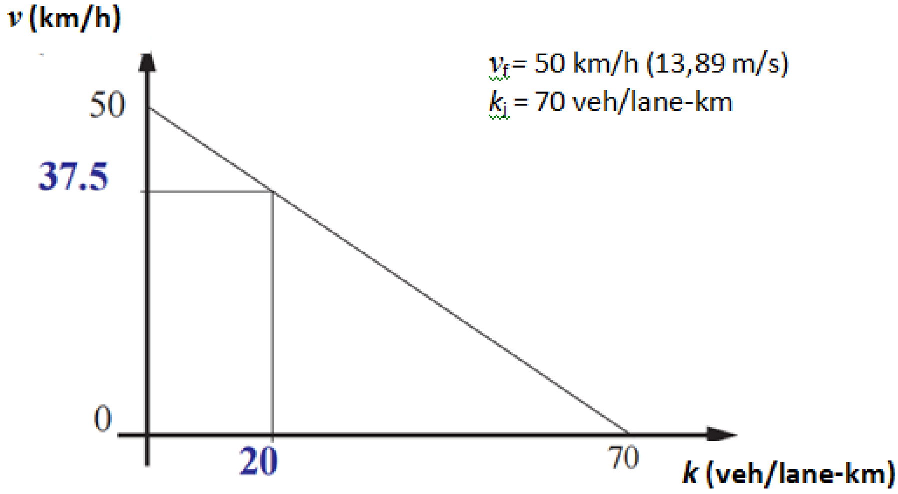
Solution to part **(a)**

Find the spacing (S) between vehicles. Since the density of the traffic flow is known to be 20 veh/lane-km, the headway is computed as the reciprocal of the density:

$$S = 1/k = 1/20 = 0.05 \text{ km}$$

$$S = \underline{\mathbf{50}} \text{ m}$$

To find the headway we need to figure out how fast the cars are traveling on the road. The Speed-Density *Fundamental Diagram* to be used to estimate the speed when ***k*** = 20 vehicle/lane-km, is shown below:



$v = v_f - (v_f/k_j)*k = 50 - (50/70)*20 =$ **<u>35.71</u>** k/h

Travelling at 35.71 km/h (9.92 m/s) the *headway* (*h; expressed as time*) between successive cars is

h = 50/9.92 = **<u>5.04</u>** seconds

Solution to part **(b)**

To check if the turning vehicle can make a safe maneuver, check the time to turn against the headway (*h*) calculated in part (**a**). Account for the reaction time of the turning vehicle.

The time available to execute a safe turn is (*h*) - 0.5 seconds to account for reaction time:

$t_{available}$ = 5.04 – 0.5 = **<u>4.54</u>** seconds

Technically we should use the gap between two successive vehicles to estimate the time to turn left. In this case we have to subtract the time traveled by the oncoming vehicle to cover its car length at 9.92 m/s

t_{gap} = 5.04 – 0.5 – 5.8/9.92 = **<u>3.96</u>** seconds

The distance traveled by a vehicle with a linearly-varying acceleration model is

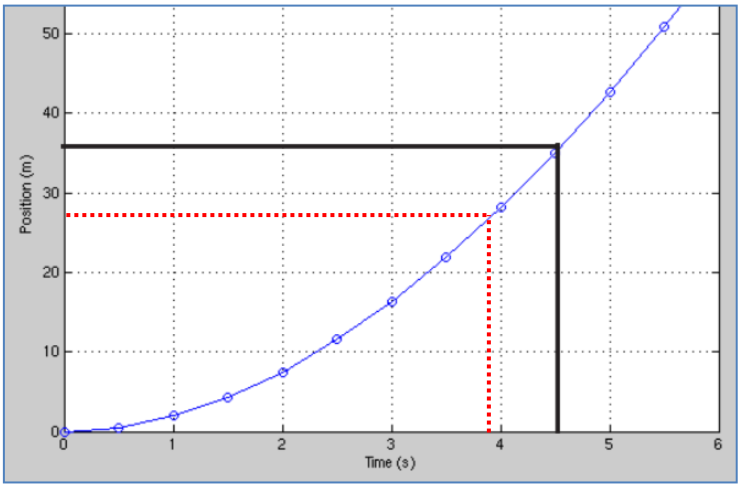
$$S = \frac{k_1 t}{k_2} - \frac{k_1}{k_2^2}(1 - e^{-k_2 t}) + \frac{v_0}{k_2}(1 - e^{-k_2 t})$$

Note that ***t*** is either 3.96 or 4.54 seconds (depending on your assumption on when the stopped vehicle starts the left turn). Using values of k_1, k_2 as 4.0 and 0.1, respectively, the left turning vehicle travels 35.6 meters in 4.54 seconds and 28 meters in 3.96 seconds.

A plot of distance (*d*) traveled versus time is shown in the following diagram on next page. The total distance to be traveled in the left turn maneuver to reach a safe point is:

$$d = \frac{2\pi R}{4} + L = \frac{2\pi(12)}{4} + 5.8 = 24.65 \text{ meters}$$

So, the vehicle *can* execute the turn *safely*.



6. Example

Two types of speed are used in traffic analyses: (*i*) time-mean speed, and (*ii*) space-mean speed. The time-mean speed ($\underline{v}_t$) is defined in the following way:

$$\underline{v}_t = 1/N * \sum_{i=1}^{N} v_i$$

where

v_i represents recorded speed of the *i*-th vehicle. We see that the time-mean speed can be calculated by calculating the *arithmetic* time mean speed.

The space-mean speed is the average speed that has been used in the majority of traffic models. Let us note a section of the highway whose length equals *D*. We denote by t_i the time needed by the *i*-th vehicle to travel along this highway section. The v_s space-mean speed is defined in the following way:

$$\underline{v}_s = \frac{D}{\frac{1}{N} \cdot \sum_{i=1}^{N} t_i} = \frac{D}{\underline{t}}$$

where the expression

$$\frac{1}{N} \cdot \sum_{i=1}^{N} t_i$$

represents the average travel time $\underline{t}$ of the vehicles travelling along the observed highway section.

Measurement points are located at the beginning and at the end of the road section whose length equals 1 km (see Figure below). The recorded speeds and travel times are shown in the following Table.

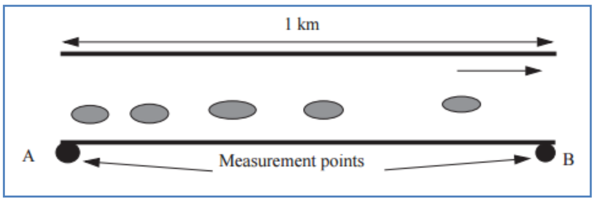


Figure (Example 6.) *Road section under scrutiny*

Vehicle number	Speed at point A [km/h]	Travel time between point A and point B [sec]
1	80	45
2	75	50
3	62	56
4	90	39
5	70	53

Table (Example 6): *Recorded speeds and travel times*

Speeds of the five vehicles are recorded at the beginning of the section (point A). The vehicle appearance at point A and point B were also recorded.

Calculate the time-mean speed and the space-mean speed. Which one is greater?

Solution

The time-mean speed $\underline{v}_t$ at point ***A*** is:

$$\underline{v}_t = 1/N * \sum_{i=1}^{N} v_i$$

$$\underline{v}_t = 1/5*(80+75+62+90+70) = 1/5*377 = \underline{\mathbf{75.4}} \text{ km/h}$$

The space-mean speed represents measure of the average traffic speed along the observed highway section. The space-mean speed is:

$$\underline{v}_s = \frac{D}{\frac{1}{N} \cdot \sum_{i=1}^{N} t_i}$$

The total travel time for all five vehicles is:

45 + 50 + 56 + 39 + 53 = 243 [sec] = 243 / 3600 [h] = **0.0675** h

The space-mean speed is calculated as:

$$\underline{v}_s = 1 / (1/5)*0.0675 = \underline{\mathbf{74.07}} \text{ km/h}$$

The *time-mean speed is greater* than the space-mean speed *in this case*.

7. Example

The results of a speed study are given in the form of a frequency distribution table below. Find the values of time mean speed and space mean speed.

speed range	frequency
2-5	1
6-9	4
10-13	0
14-17	7

Solution

The time mean speed and space mean speed can be found out from a frequency table to be constructed (see next page). First, the average speed is computed, which is the mean of the speed range. For example, for the first speed range, average speed, v_i = (2+5)/2 = 3.5 seconds. The volume of flow q_i for that speed range is same as the *frequency*. The terms $v_i * q_i$ and q_i / v_i could be also tabulated, and their summations in the last row.

Sl. No.	speed range	average speed (v_i)	flow (q_i)	$q_i\,v_i$	$\frac{q_i}{v_i}$
1	2-5	3.5	1	3.5	2.29
2	6-9	7.5	4	30.0	0.54
3	10-13	11.5	0	0	0
4	14-17	15.5	7	108.5	0.45
	total		12	142	3.28

Time mean speed can be computed as,

$$v_t = \Sigma v_i * q_i / \Sigma q_i = 142/12 = \underline{\mathbf{11.83}} \text{ m/s.}$$

Similarly, space mean speed can be computed as, $v_s = \Sigma q_i / (\Sigma q_i/v_i) = 12/3.28 = \underline{\mathbf{3.65}}$ m/s

8. Example

According to the speed-flow *fundamental diagram* of a given type of road, the average speed of the traffic flow ($\underline{v}$; km/h) is expressed in function of the traffic volume (q; PCU/h) as

$$\underline{v} = v_{free} - 0.00002q^2$$

vhere v_{free} = 95 km/h and the *capacity* of the road is about 2000 PCU/h (Passenger Car Unit/hour)

(a) Compute the average speed of the traffic flow when the flow/capacity ratio reaches 85%
(b) Compute the percentage of this average speed compared to the free flow speed ($\underline{v}/v_{free}$)
(c) What is the average speed when capacity is reached?

Solution

(a) The average speed is $\underline{v}$ = 95 – 0,00002 . 1700.1700 = 95- 57,8 = **<u>37,2</u>** km/h
(b) The percentage is 37,2 / 95 = **<u>39,1</u>**%
(c) The average speed at capacity (q_{max}) is $\underline{v}$ = 95 – 0.00002*2000*2000 = 95-80 = **<u>15</u>** km/h

9. Example

Inspection of a motorway data set reveals a *free flow speed* of v_{free}=95 km/h, a *jam density* of 80 vehicles per km per lane and an observed *maximum flow* of 2000 vehicles per hour. Determine the linear equation for speed under these conditions, and determine the speed and density at maximum flow conditions. How do the theoretical and observed conditions compare?

Solution

$v_s = v_{free} - (v_{free}/k_j)*k$ (km/h)
v_s = 95 – (95/80) k = 95 – 1.188*k
q = v_s*k
q = 95*k-1.188*k^2
dq/dk = 95 –2(1.188)k
95 = 2(1.188)*k = 2.376*k
k= **<u>40</u>** (veh/lane-km)

At half of jam density k = 40 = k_j/2

v_m= 95 - 95/80*40 = 47.5 km/h = v_f/2 = half of free flow speed
q= v_s *k
q = 47.5*(40) = **1900** veh/h > 2000 veh/h

The value of maximum traffic volume observed under real field conditions remains *below* the theoretical capacity value.