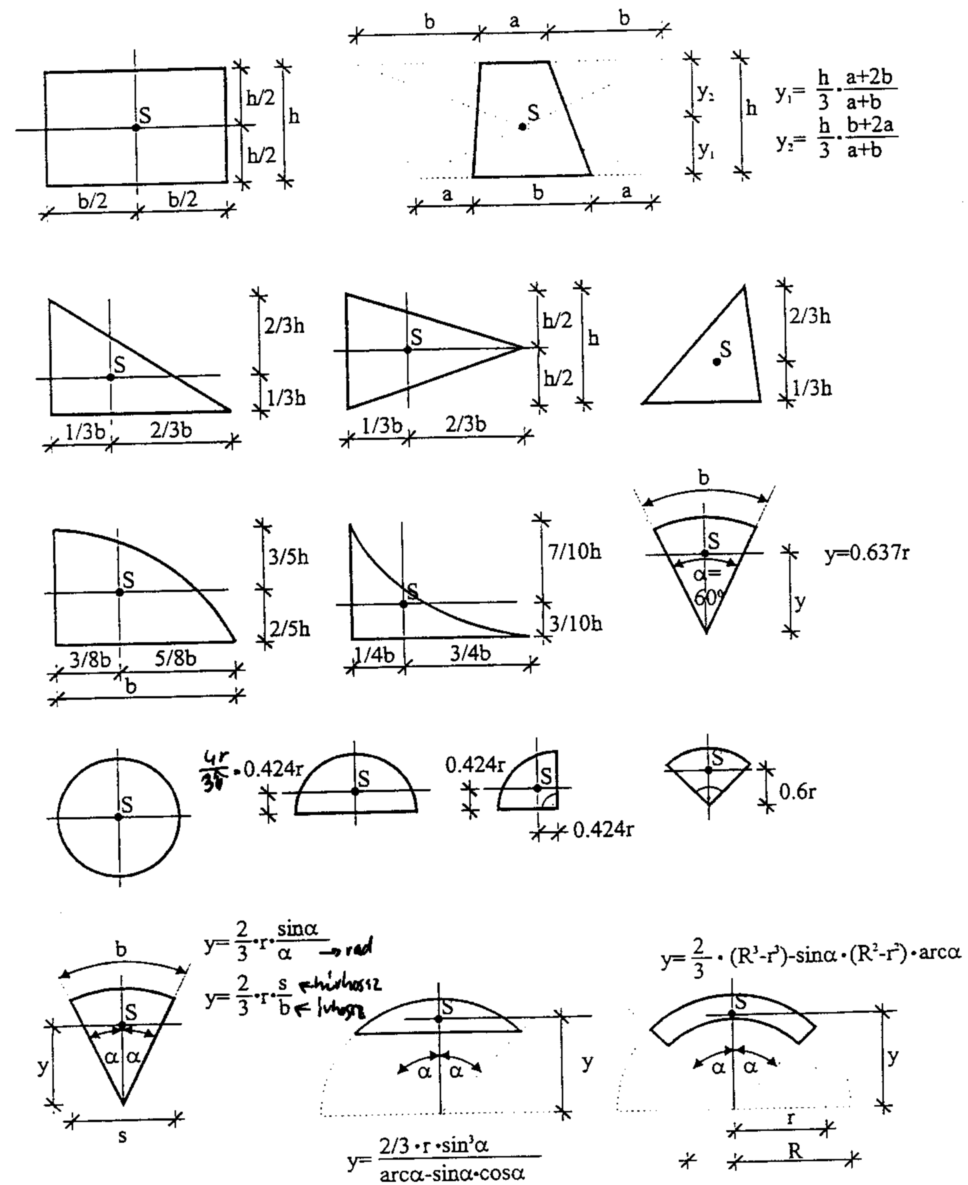
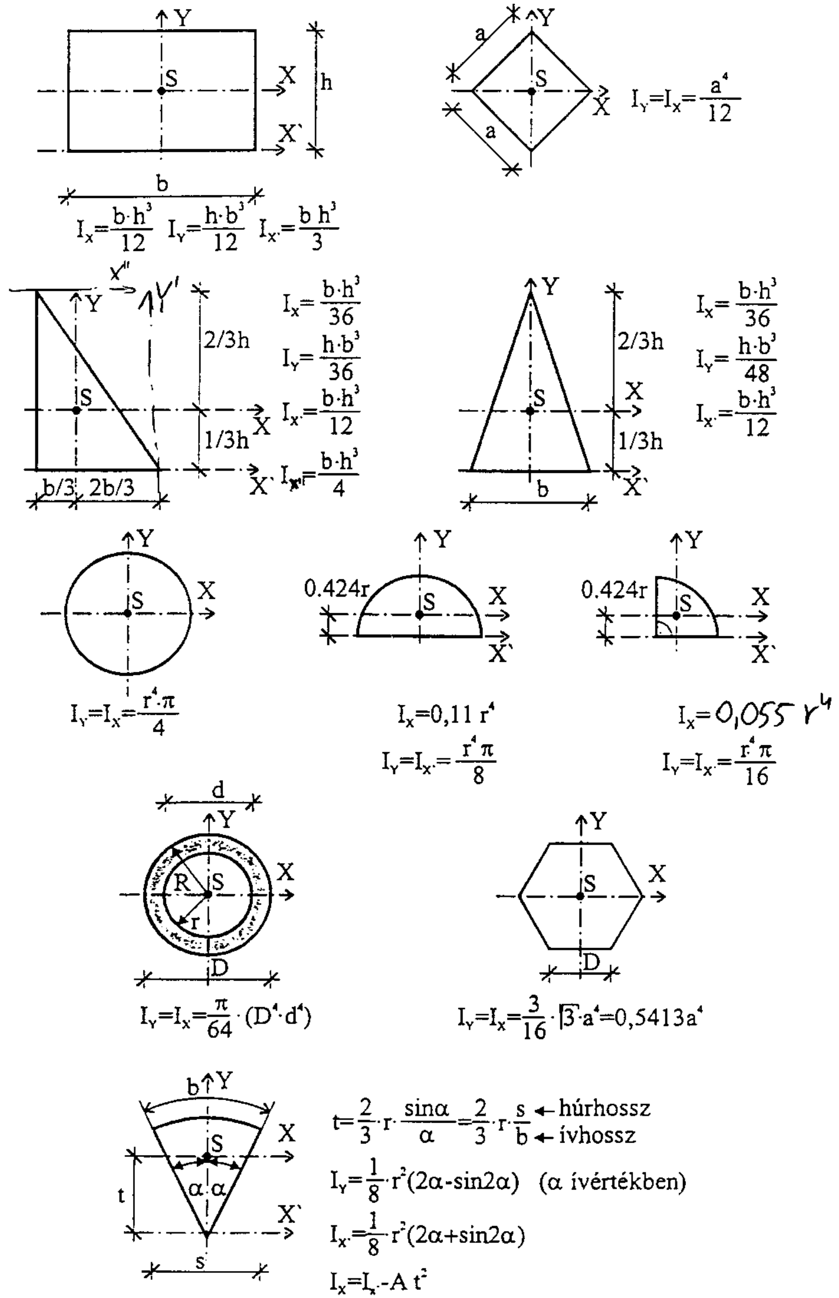
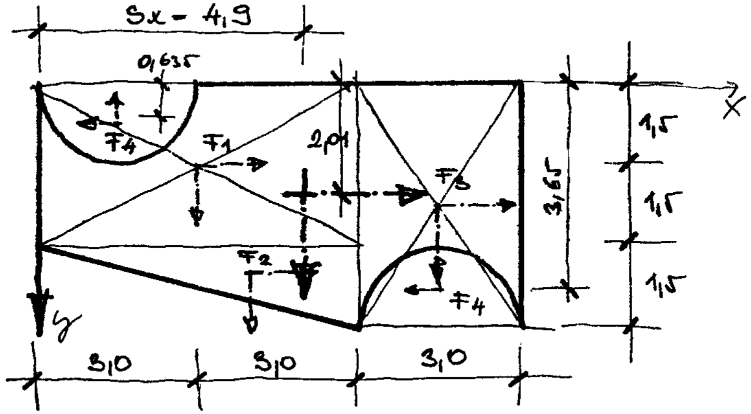
Alapvető síkidomok súlypontjának helye



Egyszerű síkidomok inercianyomatékai



SÚLYPONT SZÁMITÁS
(TÁBLAI GYAKORLAT)



$$F_1 = 6 \times 3 = 18 \text{ cm}^2$$

$$F_2 = \frac{6 \cdot 1{,}5}{2} = 4{,}5 \text{ cm}^2$$

$$F_3 = 3 \cdot 4{,}5 = 13{,}5 \text{ cm}^2$$

$$-F_4 = \frac{1{,}5^2 \pi}{2} = 3{,}54 \text{ cm}^2$$

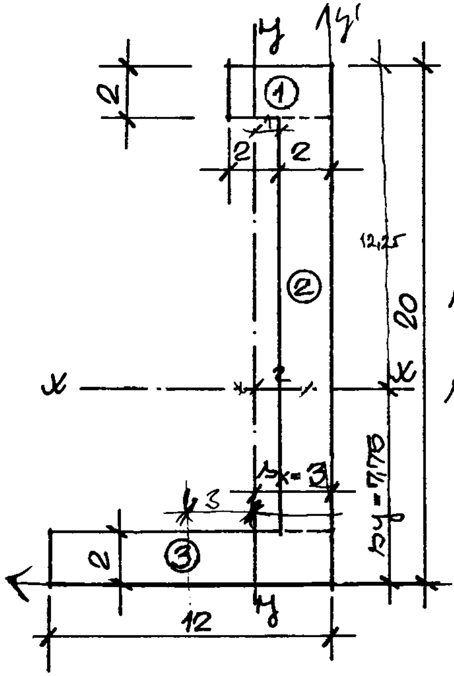
$$X_S = \frac{3 \cdot 18 - 1{,}5 \cdot 3{,}54 + 4 \cdot 4{,}5 + 7{,}5 \cdot 13{,}5 - 7{,}5 \cdot 3{,}54}{28{,}92}$$

$$X_S = \frac{54 - 5{,}3 + 18 + 10{,}12 - 26{,}5}{28{,}92} = \frac{141{,}4}{28{,}92} = 4{,}9 \text{ cm}$$

$$y_S = \frac{1{,}5 \cdot 18 - 0{,}635 \cdot 3{,}54 + 3{,}5 \cdot 4{,}50 + 2{,}25 \cdot 13{,}5 -}{28{,}92} \; \frac{- 3{,}65 \cdot 3{,}54}{28{,}92}$$

$$y_S = \frac{27 - 2{,}25 + 15{,}75 + 30{,}4 - 12{,}90}{28{,}92}$$

$$y_S = \frac{58{,}00}{28{,}92} = 2{,}01 \text{ cm}$$



	A cm²	y' cm	S_y cm³	x' cm	S_x cm³
①	8	19	152	2	16
②	32	10	320	1	32
③	24	1	24	6	144
Σ	64	—	496	—	192

$$s_y = \frac{S_y}{\Sigma F} = \frac{496}{64} = 7{,}75 \text{ cm}$$

$$s_x = \frac{S_x}{\Sigma F} = \frac{192}{64} = 3{,}00 \text{ cm}$$

$$I_x = \left(\frac{4 \cdot 2^3}{12} + 8 \cdot 11{,}25^2\right) + \left(\frac{2 \cdot 16^3}{12} + 32 \cdot 2{,}25^2\right) + \left(\frac{12 \cdot 2^3}{12} + 24 \cdot 6{,}75^2\right)$$

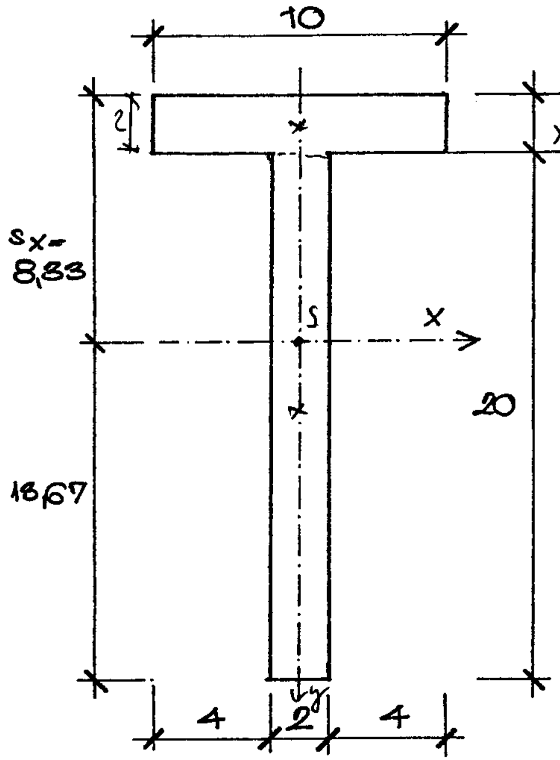
$$I_x = \left(\frac{32}{12} + 8 \cdot 126{,}66\right) + \left(\frac{2 \cdot 4096}{12} + 32 \cdot 5{,}06\right) + \left(\frac{12 \cdot 8}{12} + 24 \cdot 45{,}5\right)$$

$$I_x = (2{,}66 + 1012{,}5) + (682{,}66 + 162) + (8 + 1093{,}5)$$

$$I_x = 1015{,}46 + 843{,}92 + 1100$$

$$I_x = 2961{,}33 \text{ cm}^4$$

$$I_y = \frac{2 \cdot 4^3}{12} + 8 \cdot 1^2 + \frac{16 \cdot 2^3}{12} + 32 \cdot 2^2 + \frac{2 \cdot 12^3}{12} + 24 \cdot 5^2 = 669{,}3 \text{ cm}^4$$



SÍKIDOMOK INERCIANYOMATÉKA

$J_x = ?$

$J_y = ?$

$J_x' = ?$

$$A = 10\cdot 2 + 20\cdot 2 = 60\ cm^2$$

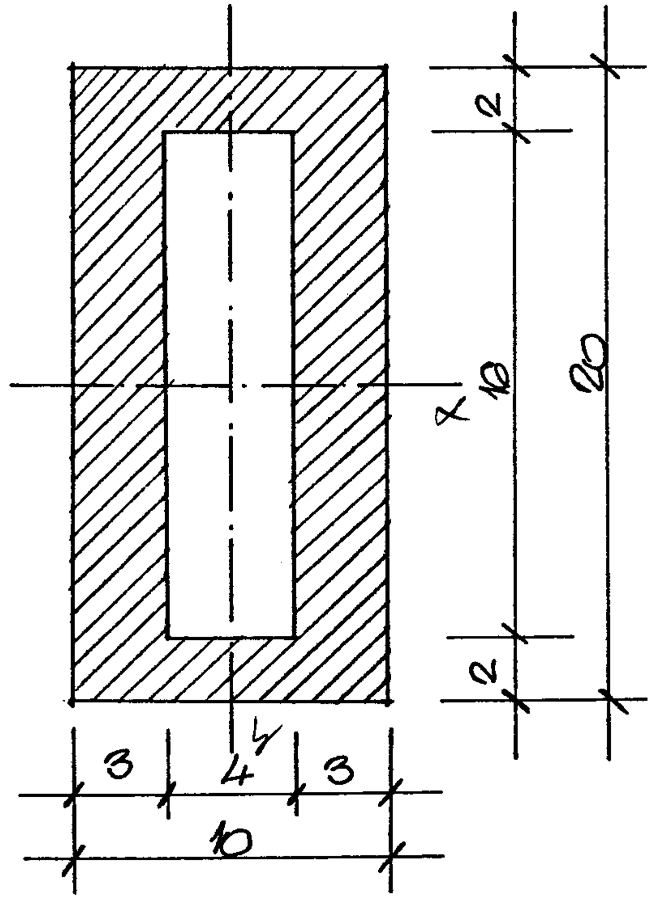
$$S_x = \frac{10\cdot 2\cdot 1 + 2\cdot 20\cdot 12}{60} = 8{,}83\ cm$$

$$J_x' = \frac{10\cdot 2^3}{3} + \frac{2\cdot 20^3}{3} = 5359{,}99\ cm^4$$

$$J_x = 5359{,}99 - 60\cdot 6{,}33^2 = 2955{,}86\ cm^4$$

$$J_y = \frac{2\cdot 10^3}{12} + \frac{20\cdot 2^3}{12} = 180\ cm^4$$

$$J_x = \frac{10\cdot 2^3}{12} + 20\cdot 7{,}33^2 + \frac{2\cdot 20^3}{12} + 40\cdot 3{,}67^2 = \underline{\underline{2953{,}33\ cm^4}}$$

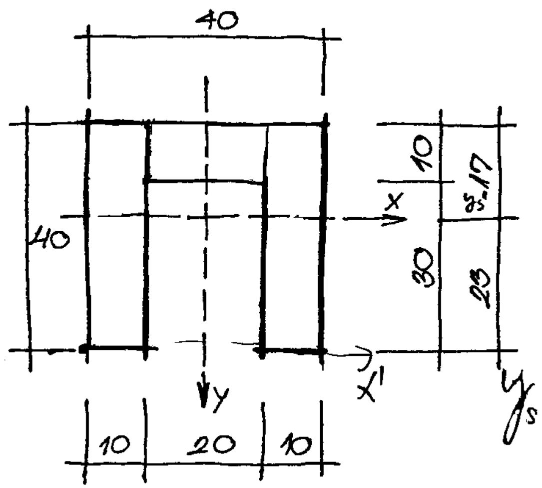


$$J_x = \frac{10\cdot 20^3}{12} - \frac{4\cdot 16^3}{12} = \frac{80000}{12} - \frac{16384}{12}$$

$$\underline{J_x = 5301{,}33\ cm^4}$$

$$J_y = \frac{20\cdot 10^3}{12} - \frac{16\cdot 4^3}{12} = \frac{20000}{12} - \frac{1024}{12}$$

$$\underline{J_y = 1581{,}33\ cm^4}$$



$J_x = ?$

$J_{x'} = ?$

$J_y = ?$

$F = 40\cdot 40 - 20\cdot 30 = 1000\ cm^2$

$$y_s = \frac{20\cdot 10\cdot 5 + (40\cdot 10\cdot 20)2}{1000} = 17\ cm$$

$$J_x = \left[\frac{40\cdot 40^3}{12} + 1600\cdot 3^2\right] - \left[\frac{20\cdot 30^3}{12} + 600\cdot 8^2\right] = 144\,333{,}33\ cm^4$$

vagy

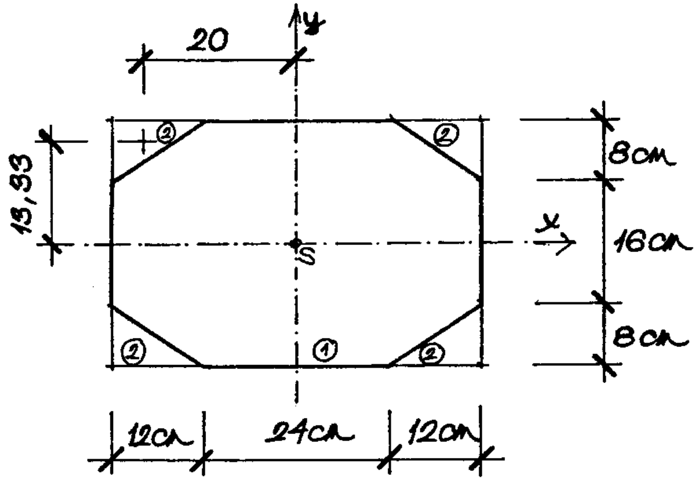
$$J_x = \left[\frac{40\cdot 10^3}{3} + 2\left(\frac{10\cdot 30^3}{3}\right)\right] - 1000\cdot 7^2 = 144\,333{,}33\ cm^4$$

$$J_y = \frac{40\cdot 40^3}{12} - \frac{30\cdot 20^3}{12} = 193\,333{,}33\ cm^4$$

$$J_{x'} = 144\,333{,}33 + 1000\cdot 23^2 = 673\,333{,}33\ cm^4$$

vagy

$$J_{x'} = 2\left(\frac{10\cdot 40^3}{3}\right) + \frac{20\cdot 10^3}{12} + 200\cdot 35^2 = 673\,333{,}32\ cm^4$$



$$J_x = \frac{48\cdot 32^3}{12} - 4\left(\frac{12\cdot 8^3}{36} + \frac{12\cdot 8}{2}\cdot 13{,}33^2\right) =$$

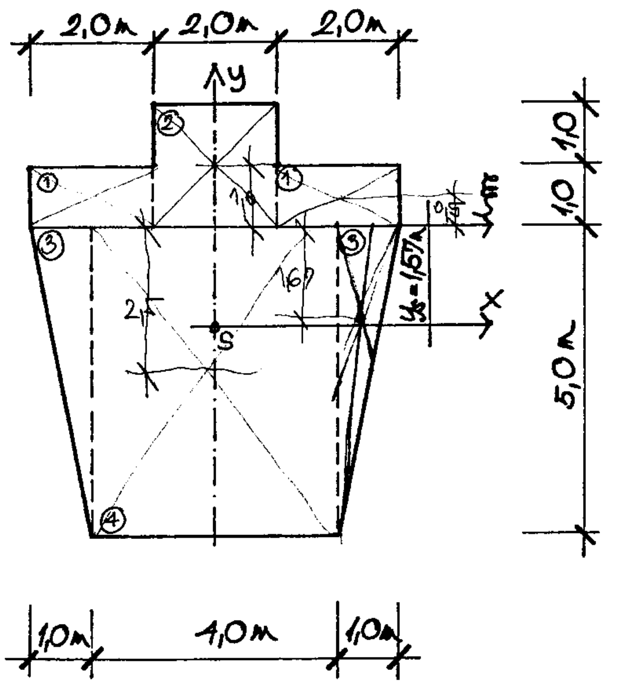
$$= 131\,200 - 4(170{,}67 + 8540) = 131\,200 - 34\,842{,}67 =$$

$$\underline{J_x = 96\,357{,}33\ cm^4}$$

$$J_y = \frac{32\cdot 48^3}{12} - 4\left(\frac{8\cdot 12^3}{36} + \frac{12\cdot 8}{2}\cdot 20^2\right) =$$

$$= 295\,000 - 4(384 + 19\,200) = 295\,000 - 78\,336 =$$

$$\underline{J_y = 216\,664{,}00\ cm^4}$$



$F_1 = 2\ cm^2$
$F_2 = 4\ cm^2$
$F_3 = 2,5\ cm^2$
$F_4 = 20\ cm^2$

$$y_S = \frac{S_F}{F} = \frac{4\cdot 0,5 + 4\cdot 1 - 5\cdot 1,67 - 20\cdot 2,5}{4,0+4,0+5,0+20,0} =$$

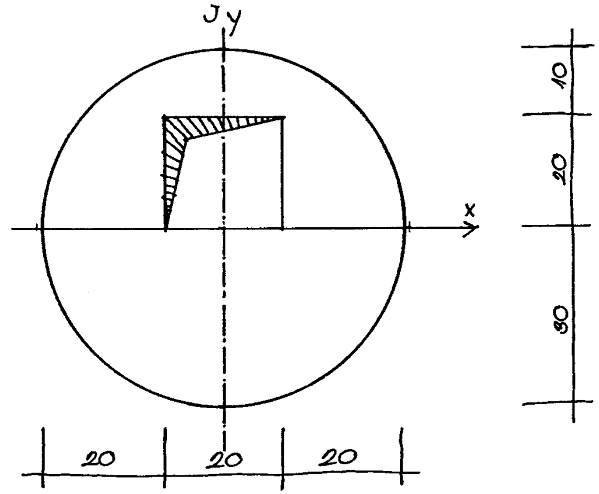
$$= \frac{2+4-8,33-50,0}{33,0} = \underline{-1,57\ m}$$

$$J_x = J_F - F y_S^2 = \frac{4\cdot 1^3}{3} + \frac{2\cdot 2^3}{3} + \frac{2\cdot 5^3}{12} + \frac{4\cdot 5^3}{3} - 33,0\cdot 1,57^2 =$$

$$= 1,33 + 5,33 + 20,83 + 166,67 - 81,33 = \underline{112,83\ m^4}$$

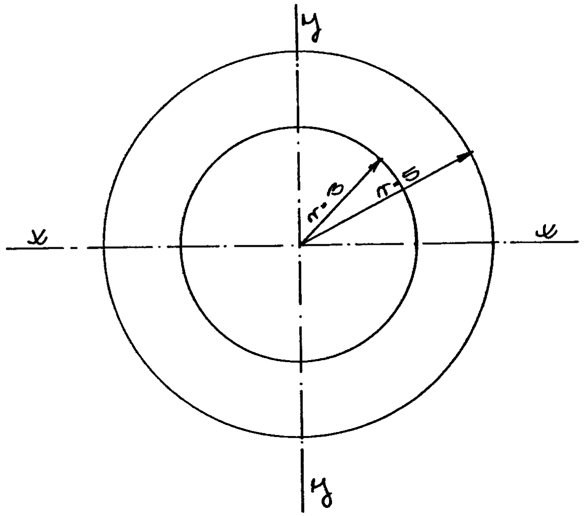
$$J_y = \frac{1\cdot 2^3}{12} + \frac{1\cdot 6^3}{12} + 2\left(\frac{5\cdot 1^3}{36} + \frac{5}{2}\cdot 2,33^2\right) + \frac{5\cdot 4^3}{12} =$$

$$= 0,67 + 18,0 + 27,48 + 26,67 = \underline{72,82\ m^4}$$



$$J_x = \frac{30^4\cdot\pi}{4} - \frac{20^4}{3} = 582\,839,18\ cm^4$$

$$J_y = \frac{30^4\cdot\pi}{4} - \frac{20^4}{12} = 622\,839,18\ cm^4$$

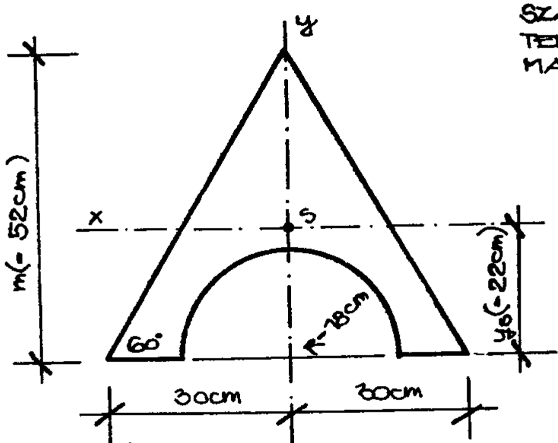


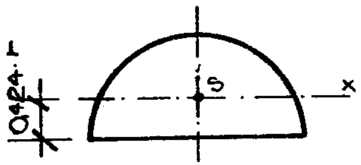
$$J_x = J_y = \frac{\pi\cdot 10^4}{64} - \frac{\pi\cdot 6^4}{64} = \frac{\pi(10^4-6^4)}{64} =$$

$$= 0,0491(10\,000 - 1296) = 0,0491\cdot 8704 =$$

$$= \underline{427,37\ cm^4}$$

SZÁMÍTSUK KI AZ „x' SÚLYPONTI TENGELYRE VETT INERCIANYOMATÉKOT!





$J_x = 0{,}1098\ r^4$

$m = \sqrt{3} \cdot 30 = 52\ cm$

$$y_S = \frac{\frac{60 \cdot 52}{2} \cdot \frac{52}{3} - \frac{18^2 \cdot 3{,}14}{2} \cdot 0{,}424 \cdot 18}{\frac{60 \cdot 52}{2} - \frac{18^2 \cdot 3{,}14}{2}} = \frac{27\,000 - 3885}{1560 - 509} =$$

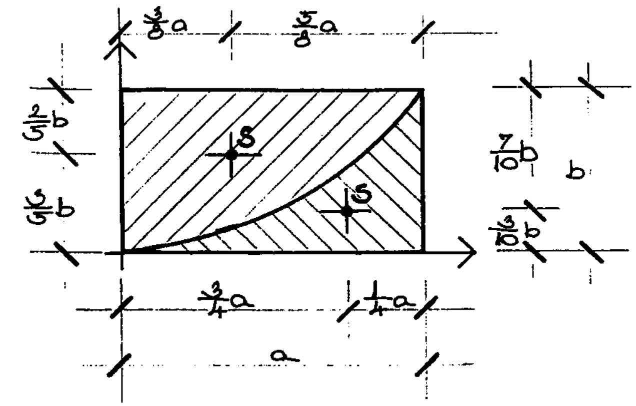
$$= \frac{23\,115\ cm^3}{1\,051\ cm^2} = \underline{\underline{22\ cm}}$$

$0{,}424 \cdot 18 = 7{,}64\ cm$
$22 - 17{,}3 = 4{,}7\ cm$
$22 - 7{,}64 = 14{,}36\ cm$

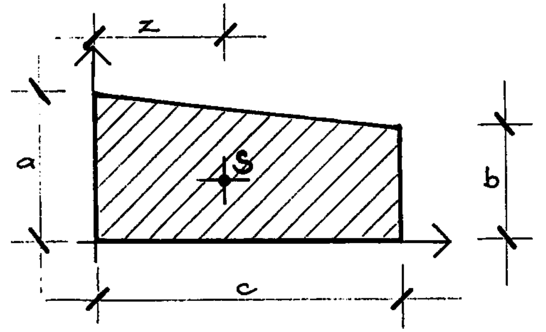
$$J_x = \underbrace{\frac{60 \cdot 52^3}{36}}_{234\,346} + \underbrace{4{,}7^2 \cdot 1560}_{34\,460} - (\underbrace{0{,}1098 \cdot 18^4}_{11\,520} + \underbrace{14{,}36^2 \cdot 509}_{104\,960}) =$$

$$\underline{\underline{J_x = 151\,759\ cm^4}}$$

másodrendű parabola súlypontjának helye x,y koordináta rendszerben

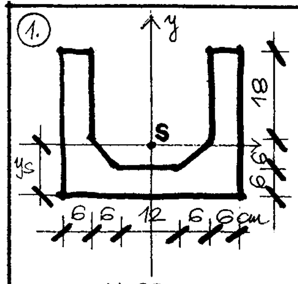


trapéz súlypontjának helye



$$z = \frac{c}{3} \cdot \frac{a + 2b}{a + b}$$

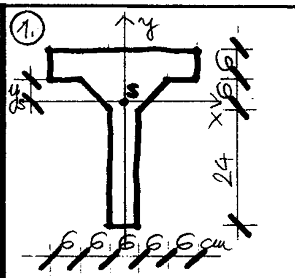
1.



$y_s = 11{,}33$ cm

$I_x = ?$ $I_y = ?$ $I_p = ?$

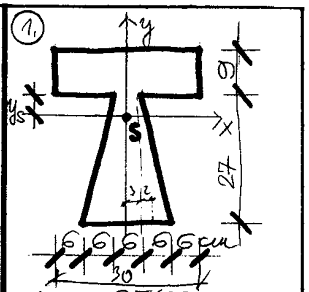
1.



$y_s = 5{,}64$ cm

$I_x = ?$ $I_y = ?$ $I_p = ?$

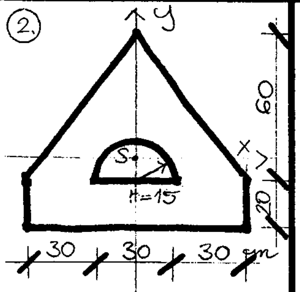
1.



$y_s = 6{,}545$ cm

$I_x = ?$ $I_y = ?$ $I_p = ?$

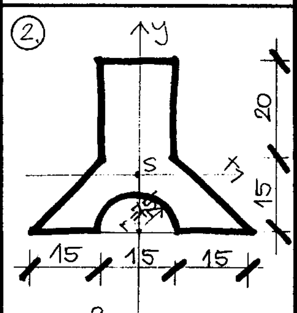
2.



$y_s = ?$

$I_x = ?$

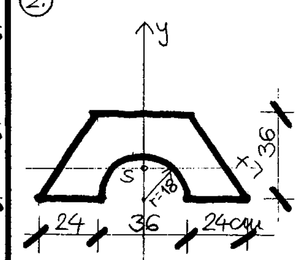
2.



$y_s = ?$

$I_x = ?$

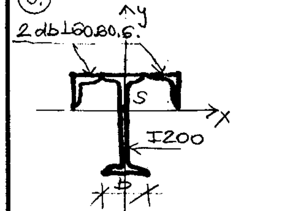
2.



$y_s = ?$

$I_x = ?$

3.

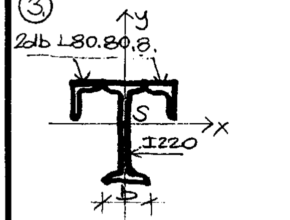


$y_s = ?$

$I_x = ?$

$I_y = ?$

3.

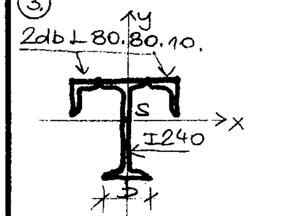


$y_s = ?$

$I_x = ?$

$I_y = ?$

3.

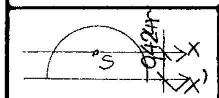


$y_s = ?$

$I_x = ?$

$I_y = ?$

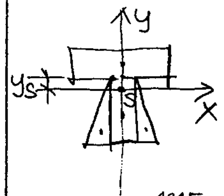
MEGADANDÓK:



$I_x = 0{,}11 r^4$

$I_{x'} = \frac{r^4\pi}{8}$

I	A cm^2	I_x cm^4	I_y cm^4	b cm	e	A cm^2	$I_x = I_y$ cm^4	$e_x = e_y$ cm
200	33,5	2140	117	9	L60×60×6	6,91	22,8	1,69
220	39,6	3060	162	9,8	L80×80×8	12,3	72,3	2,26
240	46,1	4250	221	10,6	L80×80×10	15,1	87,5	2,34



$A_{\square} = 30 \cdot 9 = 270$

$A_{\Pi} = 6 \cdot 27 = 162$

$2A_{\Delta} = 6 \cdot 27 = 162$

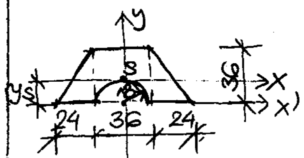
$A = 594\ cm^2$

$$y_s = \frac{-\underset{1215}{270 \cdot 4{,}5} + \underset{2187}{162 \cdot 13{,}5} + \underset{2916}{162 \cdot 18}}{594} = \frac{3888}{594} = \boxed{6{,}545\ cm} \text{ (adott)}$$

$$\boxed{I_x} = \underset{7290}{\frac{30 \cdot 9^3}{3}} + \underset{39366}{\frac{6 \cdot 27^3}{3}} + 2 \cdot \underset{59049}{\frac{6 \cdot 27^3}{4}} - \underset{25445{,}19}{594 \cdot 6{,}545^2} = \boxed{80249{,}81\ cm^4}$$

$$\boxed{I_y} = \underset{20250}{\frac{9 \cdot 30^3}{12}} + \underset{486}{\frac{27 \cdot 6^3}{12}} + 2\left(\underset{162}{\frac{27 \cdot 6^3}{36}} + \underset{2025}{81 \cdot 5^2}\right) = \boxed{25110\ cm^4}$$

$$\boxed{I_p} = I_x + I_y = \boxed{105359{,}81\ cm^4}$$



$A_{\square} = 36 \cdot 36 = 1296\ cm^2$

$2A_{\Delta} = 24 \cdot 36 = 864$

$-\ A_{\frown} = \frac{1}{2} \cdot 18^2\pi = 508{,}94$

$A = 1651{,}06\ cm^2$

$0{,}424 \cdot 18 = 7{,}632\ cm$

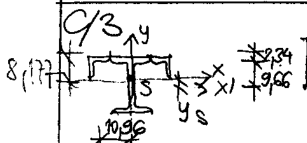
$$S_{x'} = \underset{23328}{1296 \cdot 18} + \underset{10368}{864 \cdot 12} - \underset{3884{,}23}{508{,}94 \cdot 7{,}632} = 29811{,}77\ cm^3$$

$$\boxed{y_s} = \frac{S_{x'}}{A} = \boxed{18{,}056\ cm}$$

$$I_x = \underbrace{\underset{559872}{\frac{36^4}{3}} + 2 \cdot \underset{186624}{\frac{24 \cdot 36^3}{12}} - \underset{41223{,}98}{\frac{18^4\pi}{8}}}_{I_{x'}} - \underbrace{\underset{538277{,}15}{1651{,}06 \cdot 18{,}056^2}}_{A \cdot y_s^2}$$

$$\boxed{I_x = 166994{,}87\ cm^4}$$

C/3



$$\boxed{y_s} = \frac{2 \cdot 15{,}1 \cdot 9{,}66}{46{,}1 + 2 \cdot 15{,}1} = \frac{291{,}732}{76{,}3} = \boxed{3{,}82\ cm}$$

$$\boxed{I_x} = 4250 + 46{,}1 \cdot 3{,}82^2 + 2\left(87{,}5 + \overset{1204{,}99}{15{,}1 \cdot 5{,}84^2}\right) = \boxed{6127{,}70\ cm^4}$$

$$\boxed{I_y} = 221 + 2\left(87{,}5 + 15{,}1 \cdot 10{,}96^2\right) = \boxed{4023{,}67\ cm^4}$$